TEXT-BOOKS OF SCIENCE

ADAPTED FOR THE USE OF

TISANS AND STUDENTS IN PUBLIC AND SCIENCE SCHOOLS

MACHINE DESIGN

THE ELEMENTS

OF

MACHINE DESIGN

BY

W. CAWTHORNE UNWIN

B.Sc., Mem. Inst. Civil Engineers

PROFESSOR OF HYDRAULIC AND MECHANICAL ENGINEERING AT THE
ROYAL INDIAN CIVIL ENGINEERING COLLEGE

REVISED AND ENLARGED

TENTH EDITION

LONDON

LONGMANS, GREEN, AND CO.

AND NEW YORK : 15 EAST 16th STREET

1888

THE THIRD SCIENCE is that of MACHINE DESIGN. This has been removed by Redtenbacher from its incorrect position as a part of Applied Mechanics, and established on a footing of its own. Its province is to show how the parts of the machine are to be proportioned so as to resist deformation. In order to accomplish this fully, they must be considered both with reference to the external forces acting on the machine, and the corresponding molecular forces within its substance.

The former are assumed as determined by theoretical mechanics (for example, the steam pressure upon a piston, or the water pressure on the vanes of a turbine); these define the requirements of the parts as to strength. The latter, the molecular forces, transmit the force action from part to part (for example, from the piston rod to the connecting rod, or from toothed wheel to toothed wheel), and cause also friction and wear. The science of Machine Design applies the results of research in these two directions to the special problems with which it deals. When it solves these problems in accordance with technological requirements, it forms a really technical science.

REULEAUX, *Theoretische Kinematik.*

PREFACE

TO

THE FOURTH EDITION.

IT MAY perhaps be inferred from the need of a Fourth Edition of this Treatise within less than five years from its first publication, that it has proved useful to those for whom it was intended. That an English Text-book of Machine Design was likely to be serviceable was very apparent to the author, but to write such a Text-book satisfactorily involves not only much practical knowledge, but considerable labour in comparing the dimensions which workshop experience has shown to be necessary, with those deduced on mechanical principles from the forces acting in the machine, so far as they are known. Probably no one is more aware than the author how far this Treatise falls short of what such a Treatise might be. It is pleasant, therefore, to have reason to think that, in spite of defects and omissions, it has not altogether failed in its purpose.

The author was, indeed, warned, in a not unkindly review, that from the redundancy of symbolical expressions his work would be regarded as a mere algebraical puzzle. But it appears that elementary knowledge of applied mechanics is not quite at so low a level amongst practical engineers as the reviewer supposed. When the reviewer

adds that the work, though 'unique of its kind,' 'will remain a closed book to very many practical men of little erudition but of inquiring minds, to whom it would otherwise have been of the greatest value and as good a companion as their Molesworth or other formulæ books,' it can only be replied that there may be practical engineers to whom the mathematical parts of the work will be difficult, but that is not a sufficient reason for omitting them, and that it would have been much easier to write a 'formula book,' but that it was precisely to avoid doing this that a large part of this work was written.

Long experience has enabled engineers to proportion the special machines which are daily manufactured in a very perfect way, and no great improvement is to be expected from a theoretical study of their proportions. The empirical rules, current in the workshop and drawing office, are sufficient for their design. Although some rules of this kind are given in the present Treatise, it is not merely or mainly a collection of such rules. All intelligent mechanical design must be based on a knowledge of scientific principles. If an engineer often rightly guides himself by induction from instance to instance, it is none the less desirable that a more general knowledge should control his inferences. The author's primary object was not to furnish ready-made rules of thumb, useful as these often are, but to explain the principles available as guides in machine design, by as many illustrative examples of their application as possible. The first question should always be, what are the considerations to be attended to in different cases. It is of quite secondary importance to evolve formulas which save the designer the trouble of thinking. But then the explanation of principles

does involve much that may appear obscure or difficult, to those who care only about the result and nothing about the reasoning which leads to it.

That this Treatise does contain a good many mathematical expressions is due, in part, to the fact that much has been condensed into a small space, and that the symbolical expression of the reasoning is the simplest and briefest. But the mathematics employed are, with few exceptions, of a very simple kind, and ought to present no difficulty to any one fairly acquainted with ordinary algebra and trigonometry. The use of the rules given has been made easy by copious tables.

To some it may appear that the book is too theoretical. But every engineer acts upon a theory of some kind, in proportioning machines. If a piston rod for a 48-inch cylinder is made of twice the diameter of that for a 24-inch cylinder, a theory of the piston rod's resistance is acted upon, and in fact a theory which is only approximate and safe within certain limits, for the resistance of the rod is not independent of its length, and the forces due to the inertia of the piston which act on the rod are not proportional to the piston's diameter. Hence, theory is essential to any systematic treatment of the subject, and all that ought to be required is that the theory should be accurate and free from useless refinements. It is not legitimate, or safe, to render the solutions of practical problems easier, by ignoring some essential conditions. Even when all the elements of the problem cannot in practice be taken into the reckoning, it is still important that the designer should bear them in mind.

In order to avoid constant repetition, a uniform plan is adopted, as to the units employed, which is only departed

from in a few cases for special reasons. Wherever there is no express statement to the contrary, the units adopted are as follows :—

Dimensions are in inches.

Loads or forces are in lbs.

Stresses are in lbs. per sq. in.

Fluid pressure is in lbs. per sq. in.

Velocities and accelerations are in feet per second.

Work is in foot lbs.

Speeds of rotation are in revolutions per minute, or in angular velocity per second.

Statical moments (as bending and twisting moments) are in inch lbs.

A more consistent and scientific system of units could easily be adopted, but it would involve a departure from the modes of reckoning current in the workshop.

In the second edition a somewhat stricter use of scientific terms was attempted, in Chapters II. and III. For the suggestion of this and of several other corrections then made, the author is indebted to Mr. A. G. Greenhill, M.A.

The fault of many of the terms commonly used in the workshop and in books dealing with the subject of the strength of materials is, that they are applied to express both the forces acting on a structure and the deformations which are produced. Thus, compression means in ordinary usage either the stress acting on a bar or the strain due to its action. There is a further ambiguity arising from the use of the same words for a quantity and an intensity. Thus elongation and compression are used either for the whole deformation or for the deformation per unit of length.

An attempt has been made to avoid some of the ambi-

guities arising from this double use of the same terms. The following short scheme may be useful for reference:

$$\frac{\text{Stress}}{\text{Strain}} =$$

$$\frac{\text{Tension}}{\text{Extension}} \text{ or } \frac{\text{Pressure}}{\text{Compression}} \text{ or } \frac{\text{Shearing stress}}{\text{Shearing strain}} =$$

Corresponding Elasticity.

$$\text{Also Extension} = \frac{\text{Elongation}}{\text{original length}}$$

$$\text{and Compression} = \frac{\text{Contraction}}{\text{original length.}}$$

Thanks are due to Mr. Heys, of Manchester, who read the chapter on toothed gearing, and made some suggestions noted in the text; to Messrs. Pearce, of Dundee, who supplied data of their rope gearing; to Messrs. Tullis, of Glasgow, who afforded information about leather belting; and to Messrs. Jackson, of Manchester, who gave information about some special forms of toothed gear.

In the present edition the chapter on riveting has been almost entirely rewritten in accordance with the results of an examination of experiments on riveted joints in the author's Report to the Institution of Mechanical Engineers. Besides additions scattered through the book, part of the theory of the strength of journals and shafts; the method of drawing worm and wormwheel teeth; the theory of piston rings; the account of Zeuner's valve diagram; and the chapters on chains, gearing chains, and lubricators, are new.

CONTENTS.

PAGE

CHAPTER XV.

CHAPTER XVI.

CHAPTER XVII.

APPENDIX.

ELEMENTS OF MACHINE DESIGN.

CHAPTER I.

MATERIALS USED IN MACHINE CONSTRUCTION.

CAST IRON.

1. UNDER the term cast iron may be included every kind of iron produced direct from the ore, which will not temper or weld. The materials thus comprehended in one class are very various in quality, appearance and strength, and in the purposes to which they are applicable. Cast iron is in general fusible at a lower temperature, contains more carbon, and is less pure than wrought iron or steel. The cast irons of commerce are generally divided into six classes. At one end of the series is placed the whitest cast iron, a silvery and excessively hard material; at the other the greyest cast iron, which is much softer, and owes its dark appearance to particles of graphite. The whiter cast irons are used only for conversion into wrought iron. The greyer irons, classed as Nos. 1, 2 and 3, are used also for foundry purposes. The greyer iron is less fusible than the whiter varieties, but it is more fluid when molten, and it has the property of expanding a little at the moment of solidification, so as to form accurate and smooth castings. The greyest iron is deficient in strength. Hence most castings are composed of mixtures of Nos. 1, 2, and 3 iron, in varying proportions. The larger

the casting and the stronger it requires to be, the less is the proportion of No. 1 which is used.

Cast iron usually contains 3 to 5 per cent. of carbon. In white iron this is entirely combined with the iron. In grey iron from 0·6 to 1·5 per cent. is combined, and the remainder, 2·9 to 3·7 per cent. crystallises separately as graphite.

Cast-iron machine parts are formed by pouring the melted cast iron into moulds. A *pattern* is first made of the exact shape of the casting required. A *mould* is then formed from this in foundry sand or loam. Then the molten iron is poured into the mould. After solidification the sand is cleared away.

The patterns are commonly made of yellow pine, or, when small, of mahogany. Metal patterns are used when a great number of similar castings are required. As the cast iron contracts about $\frac{1}{8}$th of an inch per foot in each direction, the pattern is made larger than the required casting in that proportion. The amount of contraction varies with the quality of the iron and the size of the casting, and this sometimes gives rise to much difficulty and trouble. Passages and apertures in castings which are so small that the sand would not resist the scouring action of the flowing metal, are formed of loam, in wooden moulds termed core-boxes, and are baked before being used. Simple cylindrical parts can be moulded in loam, without the use of a core-box. Thus, the core of a pipe is formed of loam, plastered on to a hollow metal core-bar. By rotating the core-bar and strickling off the superfluous loam with a sharp-edged board, the exact cylindrical form is obtained. The moulds for large cylinders are formed of loam, plastered over roughly built brick cylinders, and strickled to the required form.

Although simple forms are more easily moulded and cast than more complex forms, the skill of the moulder enables him, when necessary, to mould castings of very complicated and difficult shapes. Hence, the cast parts of machines may be more complicated in form than those which are forged.

Castings, however, do not retain an altogether sharp and accurate impression of the mould. The corners of castings are usually somewhat blunt and ragged, deep hollows partially filled up, and straight lines slightly twisted. Hence, for appearance sake, castings should have broad and rounded surfaces with well-rounded edges and filleted hollows. Architectural mouldings are not suitable for castings.

2. Cast iron is stronger than wrought iron under pressure, and much weaker under tension. Hence, it is more suitable for compressed than for stretched machine parts. Within a limited range of stress, it is tougher than wrought iron, or undergoes a greater deformation. But its range of deformation is not great. Hence, it is not so safe as wrought iron when subjected to suddenly applied impulses.

When castings are contracted for, it is usual to stipulate that test bars shall be cast at the same time and of the same metal as the castings. These test bars are very commonly $3\frac{1}{2}$ ft. $\times$ 2 ins. $\times$ 1 inch. They are laid on supports 3 ft. apart, with the deeper side vertical, and loaded at the centre till they break. Such bars should carry from $1\frac{1}{4}$ to $1\frac{7}{8}$ tons before breaking, and will deflect before fracture from 0·2 to 0·5 inch. Generally it is desirable that the iron should be ductile, and the deflection should not be less than 0·3 inch. The tensile strength of cast iron varies from 7 to $11\frac{1}{2}$ tons per square inch.

3. The special difficulty and danger in the use of cast iron is its liability to be put into a state of internal stress, in consequence of its contraction when cooling. That contraction varies with the size and thickness of the casting, and with the quality of the iron. Thus it has been found that thin locomotive cylinders contract only $\frac{1}{16}$th of an inch per foot. Heavy pipe castings and girders contract $\frac{1}{8}$th inch in 12 inches, or $\frac{1}{8}$th inch in 15 inches. Small narrow wheels contract as little as $\frac{1}{32}$th inch per foot, while large and heavy wheels contract $\frac{1}{10}$ inch per foot or more. If some parts of a casting contract more than others, the thick parts, for

instance, more than the thin parts, the casting is twisted and strained. If some parts of a casting solidify while others are still fluid, the former attain nearly their final dimensions, while the contraction of the latter has still to be effected. That contraction therefore strains the parts already set, and their resistance to deformation gives rise to stresses in the parts which are contracting. Thus a condition of initial stress is induced, sometimes great enough to fracture the casting without the application of any external cause, and in all cases reducing the effective strength of the casting. The danger of initial stress is less when the form of the casting is simple and the thickness uniform and not excessive. It appears that the initial stress is to some extent gradually removed by molecular yielding, the alteration going on for months after the casting is made.

Suppose a casting of the form shown at *A*, Fig. 1. The

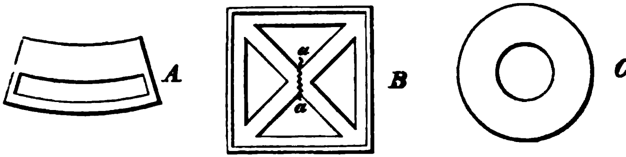


Fig. 1.

thin side would solidify, while the greater body of heat in the thick part still retained it in a fluid condition. When the thick part contracted, it would necessarily curve the bar and induce compression in the thin part, and a corresponding extension in the thick part. In a panel of the form shown at *B*, with a thin but rigid flange, the contraction of the diagonals takes place more slowly than that of the rim surrounding them, and is very liable to cause fracture at *a a*. In a thick cylinder, such as a press cylinder, Fig. 1, *C*, the outer layers solidify and begin contracting first. The contraction of the inner layers, after that of the outer layers is completed, induces pressure in the outer layers; and the rigidity of the outer layers, causing a resistance to

the contraction of the inner layers, puts them into tension. Such a cylinder will not bear so great a bursting pressure as if there were no initial strain. In fact, to obtain the greatest resistance to an internal bursting pressure, the reverse distribution of initial stress is necessary. This has sometimes been obtained by a water core, or hollow core having a water circulation through it. The interior is then cooled most rapidly. Compression of the inner layers and extension of the outer layers is the result of this mode of cooling. Castings in the form of wheels and pulleys often give much trouble. In pulleys which have a thin but rigid rim, the rim contracts first, and the subsequent contraction of the arm breaks it by tension along the line *a a a*, Fig. 2. In some cases, however, the rim breaks across near the arm, at *a b*. This appears to be due to the arms setting first. They then form a rigid abutment resisting the contraction of the rim,

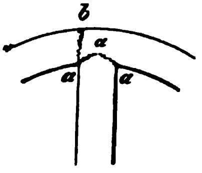


Fig. 2.

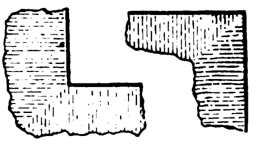


Fig. 3.

and bending stress is produced in the rim, causing fracture to begin outside and extend inwards.

It is because of these incalculable initial strains, that cast iron is an unreliable material, where great strength is required, in structures of irregular form. The danger may be partially removed by the skill of the founder, who, by various devices, ensures as far as he can an uniform rate of cooling. But generally cast-iron structures must have excessive dimensions in order to ensure safety.

At sharp corners, a plane of weakness is formed, in consequence of the way in which the crystals arrange themselves, normally to the surfaces through which heat is trans-

mitted. This is one reason why all corners should be well rounded. Fig. 3 shows roughly the crystalline structure.

4. *Chilling.*—When castings are rapidly cooled during solidification, the separation of the graphite from the iron is prevented. The casting has then a silvery fracture and is extremely hard. To effect this chilling, as it is termed, the mould is made of a thick block of cast iron, the surface in contact with the molten iron being protected by a wash of loam. The iron mould abstracts the heat much more rapidly than a sand mould.

5. *Malleable Cast Iron.*—This is made by surrounding a casting with oxide of iron or powdered red hæmatite, and keeping it at a high temperature, for a time varying with the size of the casting, from two or three, to thirty or forty hours, or even longer. Part of the carbon is eliminated, and the casting is converted, partially or wholly, into a tough material resembling wrought iron. Malleable castings stand blows much better than ordinary castings, but they should only be hammered when cold. The decorative parts of iron-work and pinions of wheels are often thus treated.

Wrought Iron.

6. Wrought iron is a silvery metal, fusing with difficulty, moderately hard, very strong and very tough. It is obtained from cast iron by eliminating the greater part of the carbon, and during the process passes into a pasty condition, so that it cannot be cast into an ingot. At a temperature of 1,500° or 1,600° Fahr. wrought iron softens, and can then be welded, a property of great importance and value. Wrought iron is used for parts of machines requiring strength and toughness, and such parts should generally have as simple a form as possible. Wrought-iron parts are first shaped by hammering or rolling at welding heat, and are then reduced to the exact form required by cutting tools. In some cases dies or swages are used to facilitate the forging of difficult

forms. Large wrought-iron structures are built up of bars or plates riveted together. Wrought iron easily oxidises, and must be polished bright and oiled, or painted.

The different qualities of wrought iron are commercially distinguished as merchant bar, best iron, double best, and treble best. These terms refer to the amount of working the iron has received in manufacture, and are only rough indications of quality. To ensure a given quality the iron used should be tested. Its strength is usually determined by subjecting it to tensile stress. Its ductility and toughness may be deduced from its elongation and contraction of area before rupture. Workmen test its toughness by bending it over a sharp corner with the hammer.

7. The forms in which wrought iron is most easily procured are the following:—

Bar iron. Round bars $\frac{1}{8}$ in. to 7 ins. diameter. Square iron up to 5 ins. or 6 ins. each side. Flat iron from $\frac{1}{4}$ in. thick and $\frac{1}{2}$ in. to 6 ins. wide, to $1\frac{1}{2}$ in. thick and 3 to 10 ins. wide. Lengths usually 20 to 30 ft.

Plates $\frac{3}{16}$ to 1″ thick, and usually not exceeding 24 sq. ft. area. Angle iron, Tee iron, and double Tee iron, in bars, usually not exceeding 8 ins. in the sum of the widths.

Various other forms are made, as half-round iron, channel iron, grate bar iron.

The quality of wrought iron varies greatly, and for some purposes strength is most important, while for others capability of being worked under the hammer without cracking or losing strength is more important. The following is a rough classification of the qualities usually met with:—

(*a*) Iron easily worked hot, and hard and strong when cold; used for rails.

(*b*) Common iron, used for ships, bridges, and sometimes for shafting.

(*c*) Single, double, and treble best iron, from Staffordshire and other parts, where similar qualities are made.

The single or double best is used for boilers. Double and treble best are used for forging.

(*d*) Yorkshire iron, from Lowmoor, Bowling, or other forges where only fine qualities are made. The best Yorkshire iron is very reliable, and uniform in quality. It is used for tyres, for difficult forgings, for furnace plates exposed to great heat, for boiler plates which require flanging, &c.

(*e*) Charcoal iron. Very ductile, and of the best quality.

The ultimate tensile strength of wrought iron ranges from 18 to 28 tons per sq. in. Generally bar iron is stronger than plate iron, and bars of simple round or square section are stronger than bars of L, T, I, or other more complex sections. For ordinary round bar iron the tensile strength is about 25 tons per sq. in.; for angle iron about 20 to 24 tons; iron wire increases in strength as the diameter is smaller, the tenacity ranging from 31½ to 50 tons per sq. in. Plate iron has the peculiarity that the strength is much greater when the fracture is perpendicular to the direction of rolling than when it is parallel to it, and this is true both of tensile and shearing resistance. The elongation is also greater in the former case. The mean tenacity when the fracture is transverse to the direction of rolling is about 22·3 tons per sq. in., and when the fracture is parallel to the direction of rolling 20·1 tons. Occasionally the strength in the latter case has been observed to fall as low as 12 or 15 tons.

Wrought iron elongates about $\frac{1}{10.000}$ of its length for each ton per sq. in. of tension up to the limit of elasticity. Beyond that limit it elongates much more rapidly, the chief part of the increase being due to permanent set. When a bar is broken by tension, the transverse area of the bar is reduced in the neighbourhood of the fracture, and, according to Mr. Kirkaldy, this contraction of area is the most reliable test of the toughness of the material. The contraction is about 10 per cent. in plates, 15 per cent. in T or L iron, and 20 per cent. in round or square bars.

The following table, prepared by Mr. Kirkaldy for the Government of India, indicates very clearly the range of quality which may be stipulated for in contracting for wrought iron :—

Kind of Material.	Class C.		Class D.		Class E.		Class F.		Class G.	
	Tenacity.	Contraction.	Tenacity.	Contraction.	Tenacity.	Contraction.	Tenacity.	Contraction.	Tenacity.	Contraction.
Round & square bars	26·4	45	25·2	35	24·6	30	23·6	25	22·8	20
Flat iron . .	25·2	40	24·6	30	23·6	25	22·8	20	21·8	16
L and T iron . .	24·6	30	23·6	22	22·8	18	21·8	15	20·8	12
Plate (across) . .	21·8	12	19·8	9	19.0	7	17·8	5	16·8	3
Plate (mean) . .	22·8	15	21·2	12	20·4	9½	19·3	7½	18·4	5½

8. *Case Hardening*.—The surface of wrought iron may be hardened by partially converting it into steel. This can be effected to a slight extent by making the surface bright, heating it to a red heat, then rubbing it with prussiate of potash, and quenching in water. It is far more completely effected by heating the iron in a close box, filled with bone dust and cuttings of horn and leather.

9. *Cold Rolled Iron.*—Wrought iron, rolled cold under great pressure, gets a smooth polished surface, and is found to have a greatly increased tenacity. Its ductility and toughness are, however, much diminished. Hammering iron when cold produces a similar effect. Annealing, or heating the iron to red heat and allowing it to cool slowly, restores it to its original condition.

All mechanical compression of iron when cold appears to increase its strength at the expense of its toughness and ductility. On the other hand, annealing reduces the strength, but increases the ductility and toughness. In rolling or hammering when hot, mechanical compression and annealing are going on simultaneously.

Steel.

10. Steel is distinguished from wrought and cast iron by having been cast into a malleable ingot during the process of manufacture. Under the term steel are included materials differing greatly in quality. The softer kinds, which contain least carbon, approach wrought iron in character, having equal or greater toughness, greater strength, and the same capacity of welding. The harder qualities have less toughness, but much greater strength, and are less easily welded. All materials to which the term steel is strictly applicable may be defined as capable of being cast into a malleable ingot, and they are rendered harder and less tough, and have a higher limit of elasticity, when heated to red heat and suddenly cooled. When hardened, they can again be softened by heating to a temperature determined by the colour of the oxide which forms on the surface, the process being termed tempering or 'letting down.' Steel of soft quality is used for the same purposes as wrought iron; but it is more liable to injury than the latter, in the various processes of forging, punching, &c., to which it is subjected in the workshop. It is also more variable in quality, and in large masses is liable to concealed defects. In small masses, steel is generally preferable to wrought iron, and the difficulties attending its use will probably be overcome.

Steel is more fusible than wrought iron, and can be cast roughly to the form required, but not so easily and accurately as cast iron. When cast it is porous, and contains cavities, and it is only after being forged that its soundness can be relied on. Sir J. Whitworth has introduced a method of casting under pressure, which may probably remove the difficulties of producing sound steel castings. In welding steel, it is important that the two pieces to be united should contain the same amount of carbon. If they do not, their welding temperatures are different. The smithing of steel

is more difficult than that of wrought iron, and it is more liable to injury from over-heating, and for this reason iron rivets are preferable to steel rivets. If pure iron were combined with carbon, the physical properties of the steel produced would depend on the proportion of carbon combined with the iron, the mildest steels containing about 0·15 to 0·4 per cent. of carbon, and the hardest 1·4 to 1·6. Actual steel, however, contains other ingredients, such as silicon and phosphorus, which influence its physical properties, having a similar effect, in hardening the steel, to a certain amount of carbon.

All straining of the material by hammering, punching, &c., has a more injurious action on steel than on wrought iron, but the injury may be removed by subsequent annealing. When the pieces are not of great thickness they may be simply heated to a cherry red heat and allowed to cool in the air on a homogeneous surface of uniform conducting power, and the annealing will be found to be practically perfect.

Bauschinger has found the strength of steel to increase with the percentage of carbon nearly in the following ratio :—Breaking strength $= 39\ (1 + C^2)$ tons per sq. in., where C is the percentage of carbon. In steel plates the strength in the direction of rolling and transverse to it is nearly the same. The steel plates used in construction have a tenacity of about 30 to 35 tons per sq. in. Good hard cast steel has a tenacity of 50 tons per sq. in.

COPPER.

11. Copper is a reddish metal, of inferior tenacity to wrought iron, but of greater ductility. It can be cast, but is seldom used in that way, as it fills the mould badly, and the castings are porous and unsound. It is usually rolled into plates and hammered to shape. It can be welded if proper precautions are taken, or joints in this material can be united by brazing. Its tenacity is rather variable, and is

about 15 tons after rolling. It is an expensive material and is chiefly used for pipes which require to be bent cold, for bolts in positions where corrosion must be prevented, and for firebox plates where its ductility and power of resisting great heat are of value. After hammering cold it loses ductility and requires to be annealed. Its tenacity diminishes with increase of temperature to a greater extent than is the case with wrought iron.

Bronze or Gun-metal.

12. Bronze or gun-metal is harder and less malleable than copper. It is fusible, and makes excellent castings. It varies in quality according to the proportion of tin. Thus :—

Soft gun-metal contains .	8 tin to	92 copper.
Hard gun-metal . . .	18 „	82 „
Bell metal, from . . .	23½ „	76½ „
to . . .	23 „	77 „

Some zinc is often added to facilitate casting. Ordinary bronze is not uniform in texture. Whitish spots of alloy, rich in tin, are distributed through the mass. It has been found that when it is rapidly cooled after casting, the composition is more uniform, the density greater, and the strength and toughness are increased. This rapid cooling is best effected by using thick cast-iron moulds or chills, the process being analogous to the chilling of cast iron. The best alloy for guns contains 8 to 10 parts of tin and 100 of copper. Such an alloy, when cast in sand moulds, breaks with about 11 tons per sq. in. of tension, and its limit of elasticity is reached at 5·6 tons. Cast in chills, its tenacity is 17·6 tons, and its limit of elasticity is raised to 6·7 tons per sq. in.

The friction between bronze and wrought iron is moderate and regular, and the bronze being softer wears most rapidly. Hence it is very suitable for the steps upon

which rotating pieces are supported. Gun-metal for bearings often contains 82 per cent. of copper and 18 per cent. of tin. The softest bronze is used for cocks and small fittings. Bronze is tougher than cast iron, and is sometimes used for gearing subjected to shocks.

BRASS.

13. Brass contains from 66 per cent. copper and 34 per cent. zinc to 70 per cent. copper and 30 per cent. zinc. A little lead is often added. Common brass for cheap brass-work contains a larger proportion of zinc. Muntz metal, which can be rolled hot, contains 60 per cent. of copper and 40 per cent. of zinc, or sometimes 66 per cent. of copper, 33 of zinc, and one of lead. It is used for sheathing-plates for ships and for the tubes of locomotives. Brass is extensively used on account of its easy working and good colour. It is cheaper but less strong and tough than gun-metal. The tenacity of brass is from 8 to 13 tons per sq. in. and that of Muntz metal somewhat greater.

WHITE BRASS.

14. Various alloys have been used for bearings containing large proportions of tin or lead. The alloys in which tin is the chief ingredient contain 40 to 90 parts of tin, 5 to 17 parts of antimony, and 1·5 to 17 parts of copper. Alloys in which lead is the chief ingredient contain 66 to 88 parts of lead with 4 to 20 parts of antimony and 12 to 20 parts of tin. The object of trying these very various alloys is to obtain a metal for bearings which is cheaper and softer than ordinary gun-metal, and which works with less friction. The friction depends on the way in which the step wears. If it is soft and of uniform texture, and wears with a smooth and polished surface, the friction may be expected to be small.

The following table gives the composition of some alloys which have been used for railway and other bearings.

PARTS BY WEIGHT.

Lead	70	—	42·5	37·5	—	—	—	84
Zinc	—	82	42·5	—	—	—	—	—
Tin	—	—	—	37·5	66·7	90	85	—
Antimony	20	11	15	25·0	11·1	7	10	16
Copper	10	7	—	—	22·2	3	5	—

Some of these alloys are fusible at a low temperature, and are cast in position round a smooth mandril. Then they do not require turning. Most of them are too soft to be used for large bearings; in such cases, a thin sheet of the alloy is cast in recesses in an ordinary gun-metal step. One objection to very soft alloys is that they crush, and clog the oil channels.

PHOSPHOR BRONZE.

Under this name an alloy has been introduced composed of copper and tin with a small proportion of phosphorus, which is likely to be of great service in machine construction. It appears to owe its properties, in part, to the great care exercised in its manufacture, and the accurate proportions of the constituents. Its qualities can be varied at will, so that it may be either very strong and hard, or, with less strength, ductile and very tough. Unlike ordinary bronze, it can be re-melted without deterioration of quality.

As to its strength and ductility, various tests show a tenacity of from 22 tons per square inch in the softer qualities, to 33 tons in the hardest. The elastic limit of the former is about 5 tons, and the latter 25. The former elongate 30 per cent. or more before fracture, and the latter 3 to 4 per cent. The contraction of area at fracture ranges from 4 to 30 per cent. Unannealed wire (16 B. W. G.)

broke with from 102 tons per square inch to 151 tons per square inch, and the same wire after annealing carried from 48 to 74 tons per square inch.

In some Belgian experiments, railway axle bearings of phosphor bronze were found to wear much longer than gun-metal bearings, and this bronze has also been used for the large crank bearings of marine engines, where ordinary gun-metal has failed. Its great strength and toughness render it especially suitable for gearing subjected to shocks. It has been used in place of steel for tools, in gunpowder factories, and it can be drawn into wire and used for rigging and perhaps for wire rope belting.

15. *Protection of Iron from Corrosion.*—One of the difficulties in the use of iron or steel is the corrosion to which these materials are liable. In many cases, as, for instance, in steam boilers, the corrosion, if allowed to proceed, may greatly weaken and endanger the structure. The corrosion is most rapid on surfaces which are alternately wet and dry, and less rapid on surfaces entirely covered by water. Cast iron obtains in the sand mould a covering of silicates which, if unbroken, is less liable to corrosion than clean surfaces of the metal. Cast iron and steel are more rapidly attacked in sea-water than wrought iron. The acids present in some woods (as, for instance, oak) cause rapid corrosion of iron in contact with them. Hence in oak copper bolts are generally used. The modes of protecting iron from corrosion are as follows:—(1) Heating the iron to 310° F. and immersing it in a bath of pitch maintained at a temperature of at least 210°. A little oil is generally added to the pitch. This process, known as Doctor Angus Smith's, is commonly employed for protecting water pipes. The pitch used is coal tar, from which the naphtha has been removed by distillation. (2) A tar varnish for application to surfaces which cannot be heated, consists of tar with a little tallow and resin. (3) Painting with oil paint, especially with paints which have oxide of iron as a basis. (4) Certain transparent varnishes

are manufactured which protect clean iron surfaces without altering their appearance. (5) Mr. Barff protects iron by forming on its surface a coating of magnetic or black oxide of iron. This is effected by subjecting the iron for some time to the action of superheated steam at a high temperature. (6) Temporary protection is obtained by a coating of tallow. (7) The most complete protection is obtained by immersing the iron in a bath of melted zinc, a process which is termed 'galvanising.'

Where iron is in contact with a metal electro-negative to it, and both are immersed in water, there is a voltaic action which causes rapid corrosion. If the water contains acids, as is the case sometimes with the feed water of boilers, the action is still more rapid. The irregular corrosion known as pitting and furrowing, is probably due to portions of the surface exposed being electro-negative to others, either from want of homogeneity in the material, or from other causes. On the other hand, if a metal electro-positive to iron is placed in contact with it, the iron is protected from corrosion. Thus boilers are now sometimes protected by suspending inside them blocks of zinc. The zinc gradually disappears, but the iron is protected.

SECTIONAL SHADING.

Fig. 4 shows the sectional shading adopted in this Treatise to indicate the materials most commonly used.

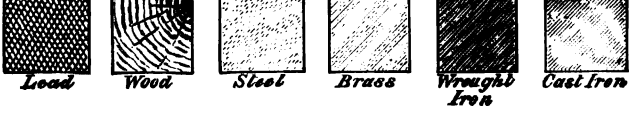


Fig. 4.

CHAPTER II.

ON THE STRAINING ACTIONS TO WHICH MACHINES ARE SUBJECTED.

16. *Load.*—By the load on any member of a machine is meant the aggregate of all the external forces acting on it; including the weight of the member itself or of other parts supported by it, the reactions of friction or inertia called into play by its motion, and the pressures transmitted through it.

We may distinguish (1) the *useful load*, or the actions arising out of the useful work transmitted; (2) the *prejudicial resistances* due to friction of parts of the machine, or to work expended otherwise than at the working point. (3) Reactions due to the weight of parts of the machine. (4) Reactions of inertia due to changes in velocity of motion. (5) Centrifugal forces due to changes of direction of motion. (6) Force due to alterations of temperature.

In each part of the machine the straining action varies with the fluctuations of the useful load, and with the variations of velocity and position of the different parts of the machine. Each member of the machine must be capable of sustaining the maximum straining action for that part of the machine. If, in consequence of variation of position or velocity, the kind of straining action is different at different times, it must be capable of sustaining the maximum straining action of each kind. Lastly, as will be more fully explained presently, the amount of variation of the straining action affects the endurance of the material, and therefore requires also to be considered.

17. *Factor of Safety.*—In most cases, some of the kinds of straining action above enumerated produce a comparatively small effect, and are for simplicity omitted, in estimating the stresses to which a part of a machine is subjected. In many cases, some of the straining actions cannot be determined. The aggregate amount of straining action taken into consideration is therefore multiplied by a *factor of safety*, before estimating the strength of the piece; and in this way a rough allowance is made for the straining action neglected, and a margin of resistance is provided for contingencies, not foreseen in designing the structure. The amount of that factor of safety is fixed by practical experience in similar cases.

The following table gives factors of safety which have been adopted in certain cases, for different materials. They are the ratios of the calculated working stress to the statical breaking stress, and include an allowance for ordinary contingencies:—

Material.	Factors of safety for			
	Dead Load.	Live Load.		
		In temporary structures.	In permanent structures.	In structures subjected to shocks.
Wrought iron and steel	3	4	4 to 5	10
Cast iron	3	4	5	10
Timber	—	4	10	—
Brickwork	—	—	6	—
Masonry	20	—	20 to 30	—

These numbers fairly represent practice based on experience in many actual cases, but they are not very trustworthy. It will be shown presently that the ratio of the safe working stress to the breaking stress depends on the range of variation of stress.

18. *Steady or Dead Load, and Variable or Live Load.*—A steady load is one which is invariable during the life of the structure, and which produces a permanent and un-

varying amount of straining action. The weight of a fixed part of a machine is such a dead load. A variable or live load is a load which is alternately imposed and removed, and which produces a constantly varying amount of straining action. A steady load can generally be very exactly estimated, and when the load is entirely of this kind, a comparatively low factor of safety affords a sufficient guarantee of security. A live load is often less easily estimated, and a load of this kind produces much more injurious effects on a structure than a dead load of the same amount. Hence, for a double reason, a higher factor of safety must be used for a live than for a dead load.

A suddenly applied load is a load imposed on an unstrained structure, without velocity, but at one instant. Practical cases only approximate to these conditions. Such a load accumulates, in deflecting or elongating the structure, a certain amount of energy of motion, which is ultimately expended in increasing the deformation beyond the amount due to a steady load. If the stress does not exceed the limit of elasticity of the material, a suddenly applied load produces twice the stress which would be produced by the same load gradually applied or resting on the structure.

If the load impinges on the structure with an amount of energy of motion previously accumulated, the stress produced will exceed that due to the same load applied steadily, to an extent which depends on the original energy of motion. Such a load may be termed an impulsive load.

19. *Strain.*—Every load which acts on a structure produces a change of form which is termed the strain due to the load. The strain may be either a vanishing or elastic deformation, that is, one which disappears when the load is removed; or a permanent deformation or set, which remains after the load is removed. In general, machine parts must be so designed that, under the maximum straining action, there is no sensible permanent deformation.

Stress and Strength.—The molecular actions within the

material of a structure, which are called into play by external forces, and which resist its deformation, are termed stresses. In all materials the stresses are sensibly proportional to the strains, so long as no considerable permanent set is produced. If, however, the straining action is so great as to produce a permanent change of form, then the strains increase more rapidly than the stresses. For any given material subjected to gradually increasing straining action there is found to be a certain limit, more or less clearly marked, within which the stresses are sensibly proportional to the strains, and beyond which that proportionality sensibly ceases.[1] The intensity of stress corresponding to that limit, or stress per unit of area when that limit is reached, is called the elastic strength of the material. A bar subjected to straining action producing that intensity of stress, is said to be strained to the elastic limit.

Thus, for iron and steel it is found that the ratio of the stresses and strains is approximately constant, and there is no considerable permanent set, so long as the intensity of stress does not exceed from $\frac{1}{2}$ to $\frac{4}{7}$ of the breaking stress. We may place the elastic strength for new bars between these limits. The condition that a machine or structure should suffer no permanent deformation enables us to fix on the intensity of stress corresponding to the elastic limit, as the maximum stress which should in any case be permitted.

[1] Prof. Kennedy has shown that in a bar of mild steel subjected to tension for the first time after manufacture there are three marked points at which its behaviour changes. A small permanent set is observable with apparatus sufficiently delicate with a load of about $\frac{1}{3}$ or $\frac{2}{5}$ of the ultimate breaking load. Nevertheless, up to a load of about $\frac{1}{2}$ the ultimate breaking load the extensions are sensibly proportionate to the tensions. The accumulated permanent set up to that point is about $\frac{1}{25}$ of the total extension. With greater loads the elastic extension and the set increase more rapidly until a load equal to nearly $\frac{7}{10}$ of the breaking load is reached. At that point there is an abrupt permanent extension or breaking down of the material.

It has been inferred that if a machine part is subjected to a steady load, whose amount can be exactly determined, it will be sufficiently strong, if the intensity of stress due to that load is less than the elastic strength of the material. But that if the straining action can be only partially determined, or if the straining action is variable, then the stress corresponding to that straining action must be less than the elastic strength of the material.

Although this rule appears at first sight tolerably definite, it is more vague than is desirable. Admitting that for any given bar there is a tolerably well marked limit of stress, below which the ratio of the stresses and strains is very nearly constant, and beyond which that constancy markedly ceases, yet that limit is not a fixed one, but appears to change with every incident in the life of the bar. Thus by straining a bar beyond the elastic limit, that limit is sensibly raised, and the more so, judging from such experiments as have been made, the more often and the longer it has been overstrained. But it is not clear that the safe working resistance is thus increased. So again it is probable that straining a bar in tension alters its elastic limit for compression. The amount of variation of the elastic limit which thus occurs and the limit of that variation are unknown. The most that can be concluded at present is that for a perfectly steady load the stresses corresponding to the elastic limit, as ordinarily understood and determined, are proved by experience to be safe, and that in other cases the stress must be less, in a proportion more or less clearly indicated by the factors of safety which experience has led constructors to adopt.

Safe Working Intensity of Stress.—According to the views hitherto generally held, the stress corresponding to the elastic limit divided by a factor of safety gives the permissible working intensity of stress, due to those straining actions which are taken into account in estimating the strength of the structure. Although this is generally termed

the greatest safe stress, it is in most cases less than the actual maximum stress. The resistance corresponding to the greatest safe stress may be termed the working strength of the piece. It will be seen presently that the more modern view of the strength of materials makes the permissible stress depend on the ultimate and not on the elastic stress of the material, although the stress due to the working load is still always less than the elastic stress.

20. *Ultimate Strength.*—If the straining action on a bar is gradually increased till the bar breaks, the load which produces fracture is called the ultimate or breaking strength of the bar. Ultimate strength is for different materials more or less roughly proportional to the elastic strength. Now experiments on ultimate strength are much more easily made than experiments on the elastic strength, and the results are more definite. For most materials there are more numerous experiments on ultimate than on elastic strength ; and for certain forms of machine parts only experiments on ultimate strength are available. Thus we know nothing, either theoretically or experimentally, of the elastic strength of cylindrical boiler flues, but only that at a certain limit of straining action they collapse altogether. In such a case, we may ensure the safety of a structure by taking care to multiply the actual straining action by a factor sufficiently large to allow, not only for unforeseen contingencies and the neglected causes of straining action, but also for the difference between the elastic and ultimate strength. The actual straining action multiplied by this factor, which is still termed a factor of safety, is then equated to the ultimate strength of the structure. The value of the factor of safety must, as in other cases, be determined by practical experience. There is as yet no theory of the resistance of materials in circumstances in which the stresses are not proportional to the strains; but in all cases of ultimate strength the limit has been passed within which that proportionality continues. Hence, in determining the ultimate strength of structures, we are de-

pendent on empirical formulæ, derived from experiments necessarily extending over a limited range of cases. In applying such formulæ to cases beyond the range of the experiments there is always a doubt whether or not they are strictly true. For this reason it is less satisfactory to determine the strength of a structure in this way, than to fix a limit to the working stress.

21. *On the Peculiar Action of Live Loads.*—The researches of Wöhler, since repeated by Spangenberg, show that the safety of a structure, subjected to a varying amount of straining action, depends on the *range of variation* of stress to which the structure is subjected, and on the number of repetitions of the change of load. It has been, hitherto, assumed that it depends only on the maximum intensity of the stress; but this must now be considered to be erroneous. Every machine, subjected to a constant variation of load, must be designed to resist a practically infinite number of changes of load. In order that it may do so, the greatest intensity of stress must be less than for a steady load, and less in some ratio which depends on the amount of variation the stress undergoes in its successive changes.

A steady load has already been defined as one which remains invariable during the life of the structure. Let the intensity of stress required to fracture a given material, under a steady load, be denoted by K, so that K is what is commonly termed the breaking strength of the material. In designing a machine part to sustain a steady load, the greatest safe stress is generally taken at about $\frac{1}{2}$ to $\frac{1}{3}$ K. With a live or variable load, it has been usual to take a higher factor of safety, and to restrict the working stress to $\frac{1}{4}$ or $\frac{1}{6}$ K, or to some other limit, ascertained by practical experience in special cases. Wöhler's researches show that this is not a scientific way of dealing with the question. Suppose that under the action of the live load the stress varies from $\sigma_{\text{max.}}$ to $\sigma_{\text{min.}}$, and that the range of variation $= \Delta = \sigma_{\text{max.}} - \sigma_{\text{min.}}$ In using this expression, if tensions

are reckoned positive, pressures must be reckoned negative, so that if the two stresses are of different sign, the range of stress is equal to their sum $[\sigma_{\text{max.}} - (-\sigma_{\text{min.}}) = \sigma_{\text{max.}} + \sigma_{\text{min.}}]$. Let the number of changes of load be indefinitely great. Then Wöhler's researches show that fracture will occur, for some value of $\sigma_{\text{max.}}$ less than κ, and so much smaller, the greater the range of stress Δ. Hence, in designing a structure for such a varying load, the ultimate strength is to be taken at some value k less than κ, which is determined with reference to Δ.

For example, Wöhler found that a bar was equally safe to resist varying bending, and tensile straining actions, repeated for an indefinite time, when the maximum and minimum stresses had the following values :—

For Wrought Iron.

	Pounds per sq. in.		
	σ max.	σ min.	Δ
In tension only	18713 to	31	18682
In tension and compression alternately	8317 to	− 8317	16634

For Cast Steel.

In tension only	34307 to	11436	22871
In tension and compression alternately	12475 to	− 12475	24950

These experiments are sufficient to show that the greatest safe stress depends very much on the range of variation of stress, being much greater when a bar is subjected to stress of one kind than when subjected alternately to stresses of opposite kinds, and in both cases being less than for a steady load with no variation.

Unfortunately, Wöhler's experiments, although extensive, do not furnish decisive rules for practical guidance. They afford an explanation of the apparently high factors of safety, which, in certain cases, experience has shown to be necessary, but they are not complete enough to indicate precisely the factor of safety to be chosen in different cases. Nor indeed could rules be obtained, without the most careful comparison of the results of researches, of the kind begun

by Wöhler, with the actual stresses found to be safe in practice, in a great variety of cases.[1]

Let, as before, K be the breaking strength per unit of section, for the given material, and for a load once gradually applied. Let $k_{\text{max.}}$ be the breaking strength for the same material subjected to a variable load ranging between the limits $k_{\text{max.}}$ and $\pm k_{\text{min.}}$, and repeated an indefinitely great number of times. $k_{\text{min.}}$ is $+$ if the stress is of the same kind as $k_{\text{max.}}$ and K, and $-$ if the stress is of the opposite kind, and it is supposed that $k_{\text{min.}}$ is not greater than $k_{\text{max.}}$ Then the range of stress is $\Delta = k_{\text{max.}} \mp k_{\text{min.}}$, the upper sign being taken if the stresses are of the same kind, and the lower if they are different. Hence Δ is always positive.

Then Wöhler's experiments appear to suggest a rule of the following kind, for the relation between K and $k_{\text{max.}}$

$$k_{\text{max.}} = \frac{\Delta}{2} + \sqrt{K^2 - n \Delta K}.$$

If $\Delta = 0$, we get $k_{\text{max.}} = K$, the load being then a steady one. Further, by choosing a suitable value for n, we can make the decrease of $k_{\text{max.}}$ for increasing values of Δ correspond with the observed values in Wöhler's experiments.

[1] Wöhler's experiments agree with and confirm the earlier experiment of Sir W. Fairbairn, communicated to the Royal Society, on the effect of continuous changes of load on a riveted girder. In Germany the breaking strength receives different names according to the conditions in which the piece is placed. The stress at which fracture occurs by a single application of a steady or very gradually applied load is called the *statical breaking strength* (Tragfestigkeit). If after each application of a load, the bar reverts to its original condition, and if all stresses are in the same sense (that is all tensions, compressions, or shears in one direction) then the greatest stress which can be sustained for a specified number of repetitions is termed the *primitive strength* (Ursprungfestigkeit). If the stresses are alternately of opposite senses, that is alternately tensions and compressions, or shears in opposite directions, the stress which can be sustained is termed the *vibration strength* (Schwingungsfestigkeit). See Weyrauch, Proc. Inst. Civil Eng. vol. lxiii.

For iron and steel the average value of n appears to be about 1·5.

The special cases most useful to consider are the following :—

(1.) The load is invariable ; then $\Delta = 0$.

(2.) The load is entirely removed and replaced. Then $k_{\text{min.}} = 0$ and $\Delta = k_{\text{max.}}$

(3.) The stress is alternately a compressive and tensile stress of the same magnitude. Then $k_{\text{max.}}$ and $-k_{\text{min.}}$ are equal in magnitude, and $\Delta = 2\,k_{\text{max.}}$

For these cases the formula gives the following values :—

	Greatest Stress.	Least Stress.	Δ	
(1.)	$k_{\text{max.}}$	$k_{\text{max.}}$	0.	Then $k_{\text{max.}} = K$.
(2.)	$k_{\text{max.}}$	0	$k_{\text{max.}}$	Then $k_{\text{max.}} = 2(\sqrt{n^2+1}-n)K$.
(3.)	$k_{\text{max.}}$	$-k_{\text{max.}}$	$2k_{\text{max.}}$	Then $k_{\text{max.}} = \frac{1}{2n}K$.

Putting $n = 1·5$, we get, in Case 1, $k_{\text{max.}} = K$; in Case 2, $k_{\text{max.}} = 0·6054\,K$; in Case 3, $k_{\text{max.}} = \frac{1}{3}K$.

Launhardt has proposed a formula of a somewhat different form, which is discussed in Weyrauch's 'Dimensionberechnung.' In determining the values of the constants for Launhardt's formula, Weyrauch obtains from Wöhler's experiments the following values for the breaking stresses with indefinitely repeated loads in the cases given above :—

	Steady load. Case 1.	Stress of one kind. Case 2.	Stress of opposite kinds Case 3.
Wrought iron	45,000	30,000	15,000
Krupp's axle steel . . .	84,600	46,500	25,000

We shall obtain almost exactly the same values by taking, in the formula above, $n = 1·42$ for wrought iron and $n = 1·66$ for steel,—values which do not differ greatly from the mean value assumed above.

The stresses thus obtained, being breaking stresses,

must be divided by the factor of safety to find the safe working stresses. For such structures as bridges, Weyrauch selects 3 as the proper factor of safety, and the same factor would appear to be sufficient for machinery in cases where the actual straining action is known. Where contingencies have to be allowed for, a larger factor must be taken. Thus wrought iron under a steady load may be loaded up to 15,000 lbs. per sq. in. With a load alternately replaced and removed, the stress should not exceed 10,000 lbs., and with a stress constantly reversed, as in the case of axles, the stress should not exceed 5,000 lbs.

21*a*. *Fatigue of Materials.*—In many cases materials are subjected to impulsive loads, and a gradual deterioration of strength is observed. Thus a crane chain sometimes breaks while carrying a load which it has often before carried safely. In part, this deterioration of strength may be due to the ordinary action of a live or repeated load; but it appears to the author to be more often due directly to the gradual loss of the power of elongation, in consequence of the slow accumulation of the permanent set. Suppose a crane chain carrying a load, W, surges, so that the load falls a distance h, and let the elongation of the chain under the action of this impulsive load be l. Then the work done by the load in falling is $W(h+l)$. The work absorbed in elongating the chain is $k\,p\,l\,A$, where A is the area of the section of the chain, p is the maximum intensity of the stress induced, and k is a constant, which would be $\frac{1}{2}$, if the stress p were within the elastic limit, but which lies between $\frac{1}{2}$ and 1, if p exceeds the elastic resistance. Equating these, we get—

$$W(h+l) = k\,p\,l\,A$$

$$p = \frac{W(h+l)}{k\,l\,A}.$$

Hence, if h is not small compared with l, the stress p will be

greater, the smaller the elongation of the chain l. In a new chain, l will include the permanent set as well as the elastic elongation. In an old chain, which has already elongated permanently, and thus become less tough, the power of elongation before fracture is diminished. Hence the stress p induced by a load W (capable of producing stresses somewhat beyond the elastic limit) increases for any given chain as the chain gets older, and may ultimately reach the breaking stress. It is believed that annealing a crane chain restores its power of elongation and its original power of resisting impulsive loads. The term *fatigue*, which has been somewhat loosely applied to various kinds of deterioration of resistance, may be conveniently restricted to this, which is a removeable deterioration.

22. *Straining Action due to Power transmitted.*—When HP horses' power are transmitted through a link or connecting rod moving with velocity v, in ft. per second, the straining force, parallel to the axis of the rod, due to the work transmitted, is

$$P = \frac{550 \text{ HP}}{v} \text{ lbs.}$$

There will be in this case other straining actions, due to the reactions of the supporters of the link, if the link is not moving parallel to its axis.

When HP horses' power are transmitted through a rotating piece, making n revolutions per second, the twisting moment, about the axis of the piece, is given by the equation

$$M = \frac{550 \text{ HP}}{2 \pi n} = 1050{\cdot}4 \frac{\text{HP}}{n} \text{ inch lbs.}$$

Or if N = revolutions per minute,

$$M = 63024 \frac{\text{HP}}{\text{N}} \text{ inch lbs.}$$

Straining Actions due to Variations of Velocity.—When a heavy body is accelerated or retarded, straining actions are

produced, due to its inertia. If w lbs. acquire an increase of velocity $d\ v$, in the time $d\ t$, the stress due to acceleration estimated in lbs. weight is

$$-\frac{w}{g} \cdot \frac{d v}{d t}.$$

Graphic Method of determining the Stress due to Inertia.— In most cases the algebraic expression of the velocity of machine parts is rather complicated, but it is easy to draw a curve representing very approximately the velocity at any instant. Suppose such a velocity curve, having for abscissa the distance moved, and for ordinate the velocity, to have been drawn. Then the acceleration per unit of mass at any point of the path is the subnormal of the velocity curve at that point.

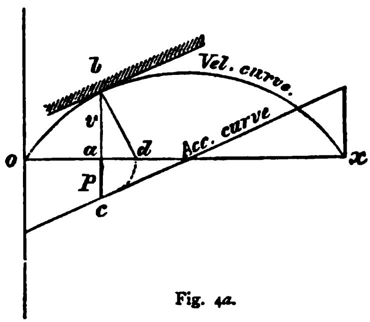


Fig. 4*a*.

Thus, let Ox be the path of a heavy piston, and let Obx be the velocity curve ; that is, when the piston has moved through a distance Oa, let ab be the velocity. If now the normal bd is drawn to the curve, ad represents the acceleration per unit of mass, at the moment the piston is at a, and if ad is twisted to the position ac, c is a point in a curve representing the accelerations. The scale of the acceleration curve is the same as the scale of the velocity curve. The normal bd is easily drawn with sufficient accuracy by using a straight edge and set square. If W is the weight of the piston, and p is the ordinate ac, then $\frac{W}{g} p$ is the total acceleration. This is the straining action due to inertia mentioned in the last paragraph, and is + or − according

as the motion is accelerated or retarded. This construction is due to Dr. Proll.[1]

Straining Effects due to Change of Direction of Motion.—When a mass is forced to move in a curved path, it exerts, in consequence of its inertia, a force equal and opposite to the constraint which deflects it. If a mass of weight w moves in a circular path of radius r, with the angular velocity ω, its centrifugal force, which is equal and opposite to the force deviating it, is

$$\frac{w}{g}\omega^2 r.$$

23. *Resilience. Resistance to Impulsive Loads.*—The quantity of work expended in deforming a bar (provided the stress does not exceed the elastic limit) is equal to the product of the deformation, and the mean load producing it. Thus, if a bar is elongated or deflected a feet, by a force gradually increased from nothing to P, the work done in deformation is $\frac{1}{2}Pa$ ft. lbs. W pounds moving with velocity v have $\frac{W}{g}\cdot\frac{v^2}{2}$ ft. lbs. of work stored in them. Hence the relation between the impulsive load and the resistance of the bar, when the direction of the impulse coincides with the direction of the deformation, is

$$\frac{W}{2g}v^2 = \tfrac{1}{2}Pa.$$

If a bar is twisted, the work done is equal to half the twisting moment, multiplied by the angle of torsion.

The work done in deforming a bar up to the elastic limit is termed the resilience of the bar.

[1] It is easy to see that $\frac{dv}{dt} = \frac{dv}{ds}\cdot\frac{ds}{dt} = v\frac{dv}{ds}$. But $\frac{dv}{ds}$ is the trigonometrical tangent of the angle between the tangent to the curve and ox, or, what is the same thing, the angle abd. Putting $v = ab$, $\frac{dv}{ds} = \frac{ad}{ab}$. Consequently $\frac{dv}{dt} = v\frac{dv}{ds} = ad = p$.

CHAPTER III.

RESISTANCE OF STRUCTURES TO DIFFERENT KINDS OF STRAINING ACTION.

PHYSICAL CONSTANTS FOR ORDINARY MATERIALS.

24. THE Table given on pp. 32, 33 shows the elastic and ultimate strength of different materials, when the stress is simple tension, pressure, or shearing stress. The values given are either average, or maximum, mean, and minimum values, selected from the most trustworthy experiments.[1] In different specimens of the same material, and even in different pieces of the same bar or plate, there are often considerable differences of elasticity and strength, and the judgment of the engineer must be relied on, in deciding how far average values of this kind are applicable in any given case. The first two columns of the Table relate to direct stresses produced by straining actions normal to the sections considered. The third relates to tangential stress produced by straining action parallel to the section. The elastic strength is that stress per unit of area at which the strains cease to be sensibly proportional to the stresses, and cannot in practice be determined with great exactness. The ultimate strength is the intensity of stress at the moment preceding rupture, and this also depends in some degree on the manner of carrying out the experiment. The more rapid the loading of the bar, and the less vibration induced, the greater is the

[1] The Tables of Grashof, Rankine, Weisbach, Kirkaldy, and Reuleaux have been consulted, in selecting the values in this Table.

TABLE I.—*Ultimate and Elastic Strength of different*

Material.	Breaking Strength. Tension.	Compression.	Shearing.
Cast iron	30,500	130,000	...
	17,500	95,000	28,500
	10,800	50,000	...
Wrought iron bars	67,000	...	...
	57,600	50,000	50,000
	33,500	...	...
Wrought iron plates, with fibre	50,700	...	...
,, ,, across fibre	46,100	...	...
,, ,, mean	48,400	...	...
Steel boiler plates	66,000	...	...
Rivet steel	65,000	...	55,600
Soft steel, unhardened	100,000	...	...
	80,000	...	...
	60,000	...	...
Soft steel, hardened	120,000	...	...
Cast steel, untempered	150,000	...	...
	120,000	...	...
	84,000	...	...
Cast steel, tempered	...	...	...
Copper	33,000	58,000	...
Brass, yellow	17,500	10,500	...
Gun-metal	52,000	...	...
	36,000	...	...
	23,000	...	...
Muntz metal	49,000	...	...
Phosphor bronze	58,000	...	...
Cast zinc	7,500	...	...
Lead	1,900	7,300	...
Tin	4,700	...	...
Wood, pine	12,000	6,000	650
,, oak	15,000	10,000	2,300
Leather	4,200	...	...

Materials, and Moduli of Elasticity, in lbs. per sq. in.

Elastic Strength.			Direct. E.	Transverse. G.
Tension.	Compression.	Shearing.		
...	...	...	23,000,000	...
10,500	21,000	7,900	17,000,000	6,300,000
...	...	...	14,000,000	...
...	...	...	...	...
24,000	24,000	20,000	29,000,000	10,500,000
...	...	...	...	...
...	...	...	25,000,000	...
...	...	...	27,000,000	...
20,000	20,000	15,000	26,000,000	9,500,000
31,000	...	...	29,500,000	...
...	...	...	30,670,000	...
...	...	...	...	...
35,000	...	26,500	30,000,000	11,000,000
...	...	...	...	...
70,500	...	53,000	30,000,000	11,000,000
...	...	...	...	...
80,000	...	64,000	30,000,000	11,000,000
...	...	...	...	...
190,000	...	145,000	36,000,000	13,000,000
4,300	3,900	2,900	15,000,000	5,600,000
6,950	...	5,200	9,170,000	3,440,000
...	...	...	...	..
6,200	...	4,150	9,900,000	3,700,000
...	...	...	...	...
...	...	...	...	...
19,700	...	14,500	14,000,000	5,250,000
3,200	...	...	...	...
1,500	...	...	720,000	270,000
...	...	...	...	...
...	...	...	1,400,000	90,000
...	...	...	1,500,000	82,000
...	...	...	25,000	...

load carried before rupture ensues. Nevertheless, if the experiment is carried out with proper care, the breaking strength is a definite measure of the properties of the material. The elastic and breaking strength are expressed in lbs. per sq. in.

A modulus of elasticity is the ratio of the intensities of stress and strain of some given kind, when the elastic limit is not passed. Thus the modulus of direct elasticity of a material is the ratio of the stress p, per unit of section of a bar, to the extension or compression, l, per unit of length, produced by the stress, when the bar is not subjected to lateral constraint. That is, the modulus of direct elasticity $= \text{E} = \frac{p}{l}$, where p is expressed in lbs. per sq. in., and l in inches per inch of length. The bar is supposed to be free laterally. The modulus of transverse elasticity is the ratio of the shearing stress q per unit of area to the distortion n; the distortion being measured by the tangent of the difference of the angles of an originally square particle before and after the stress is applied. Hence the modulus of transverse elasticity $= \text{G} = \frac{q}{n}$. The ratio $\frac{\text{G}}{\text{E}}$ for ordinary materials of construction is about $\frac{3}{8}$ to $\frac{2}{5}$.

25. *Working Stress.*—It has been pointed out that the working stress of a material must be less than the elastic strength, to allow for straining actions which cannot be taken into account, for imperfections of workmanship and for other sources of danger. The Table on p. 35 gives values of the ordinary working stress allowed in designing machinery, in which the load is of the nature of a live or varying load. Parallel with these have been placed theoretical values of the working stress in the following cases:—(1) Structures subjected to tension alone; (2) Structures subjected to compression alone; (3) Structures subjected to both tension and compression of equal intensity. (See Art. 21.)

TABLE II.—*Ordinary Working Stress.*

Material.	Safe limit of stress in lbs. per sq. in.			Theoretical limit of stress.			Weight of a cub. ft. in lbs.
	Tension f_t	Compression f_c	Shearing f_s	Tension.	Compression.	Tension and Compression.	
Cast iron	3,600	10,400	2,700	5,250	28,500	3,000 ?	450
Wrought-iron bars	10,400	10,400	7,800	17,280	15,000	9,000	480
,, ,, plates	10,000	10,000	7,800	14,520	...	8,000	487
Soft steel, untempered	17,700	17,700	13,000	24,000	...	13,000	480
Cast steel, untempered	52,000	52,000	38,500	36,000	...	20,000	496
Copper	3,600	3,120	2,300	9,900	17,400	5,500 ?	550
Brass	3,600	...	2,700	5,250	3,150	3,000	518
Gun-metal	3,120	...	2,400	10,800	...	6,000	546
Phosphor bronze	9,870	...	7,380	17,400	...	9,700	...

RESISTANCE TO SIMPLE TENSION AND COMPRESSION.

26. A bar is in tension or compression when the load acts parallel to its axis, and the stress on any section of the bar is uniformly distributed or not, according as the line of action of the load does or does not pass through the centre of figure of that section. Cases in which the stress is a varying stress will be treated as cases of compound stress. At present only cases of uniformly distributed stress are considered.

Let A B, fig. 5, be a section, of area a (in sq. ins.), on which a load P (in lbs.) acts, normally to the section. Then

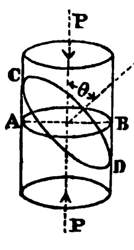


Fig. 5.

the intensity of normal or direct stress is $f=\frac{P}{a}$ (in lbs. per sq. in.). If the section C D is not perpendicular to the direction of the load, let θ be the angle between the normal to the section and the direction of P. Then the stress on C D consists of a normal or direct stress $f_n=f\cos^2\theta$, and a tangential or shearing stress $f_t=f\sin\theta\cos\theta$.

To determine the section of a bar for a given load P, a value must be selected from the preceding Tables for the working stress f, suitable for the material and the kind of loading to which the bar is subjected. Then the section of the bar normal to the direction of P is

$$a=\frac{P}{f} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Very long bars bend under the action of a longitudinal compressive force, and must be treated by special rules (§ 37).

From the definition of the modulus of elasticity already given, it is obvious that the extension or compression l, of a uniform bar of length L in inches, is given by the equation

$$l=\frac{f}{E}L \quad . \quad . \quad . \quad . \quad . \quad (1a)$$

where f is the intensity of stress on sections perpendicular

to the axis of the bar, and E is the modulus of direct elasticity for the material of the bar. This equation ceases to be accurate if f exceeds the limit of elasticity.

Work done in Extending or Compressing a Bar.—During the extension of the bar from L to $L+l$ the stress increases from zero to f proportionately to the elongation. Hence the mean stress during the operation is $\frac{1}{2}f$, and consequently the work done in extending the bar is

$$W=\tfrac{1}{2}f\,a\,l=\tfrac{1}{2}f^2\frac{aL}{E}\text{ inch lbs.} \qquad . \qquad . \qquad (1\,b)$$

That is, for a given intensity of stress the work varies as the volume of the bar. The same formulæ are applicable for compression. The bar is assumed to be uniform in section, and not strained beyond the elastic limit.

Resistance of thin Cylinders to an internal bursting Pressure.—Consider a thin cylindrical shell of diameter d, length l, and thickness t, in inches, subjected to a uniform internal pressure of p lbs. per sq. in. Let the cylinder be cut by a diametral plane $a\,b\,c\,d$, fig. 6. The resultant force P acting on either side of that plane $=p\times$area $a\,b\,c\,d$. Hence, $P=p\,d\,l$. The molecular tensions which resist the bursting force act at $a\,b$ and $c\,d$, and are equal to the intensity of stress induced × area of $a\,b$ and $c\,d$. Putting f for the intensity of tensile stress, the total force resisting the bursting pressure is $2\,f\,t\,l$. Equating the load and resistance

$$2\,f\,t\,l=p\,d\,l$$

$$f=\frac{p\,d}{2\,t} \qquad . \qquad . \qquad . \qquad (2)$$

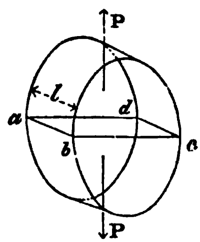


Fig. 6.

If the cylinder consists of riveted plates, the section $a\,b\,c\,d$ should be taken, so as to pass through the rivet holes. Then the area of the rivet holes must be deducted from

$2\,t\,l$, before equating the internal and external forces. If the cylinder is thick relatively to its diameter, the mean stress is unaltered, but the inner layers are more severely strained than the outer layers. In that case the thickness necessary to resist a bursting pressure p, with a maximum intensity of stress f, is found by Grashof to be

$$t=\frac{d}{2}\left\{-1+\sqrt{\frac{3f+2p}{3f-4p}}\right\} \quad . \quad . \quad . \quad . \quad (3)$$

it being assumed that p is less than $\frac{3}{4}f$. If $\frac{p}{f}$ is a small ratio—

$$t=\frac{d\,p}{2\,f}\left(1+\frac{5}{8}\frac{p}{f}\right) \text{ very nearly} \quad . \quad . \quad . \quad . \quad (3a)$$

In a thin spherical shell, the tension is half as great as in a thin cylindrical shell of the same diameter and thickness, exposed to the same pressure.

In a cylindrical shell the intensity of longitudinal stress is only half as great as the intensity of circumferential stress.

Resistance to Bending.

27. A bar is subjected to simple bending when the following conditions are fulfilled :—(1) The axis of the bar is straight; the axis of the bar being a line connecting the centres of figure of parallel transverse sections ; (2) The bar is symmetrical about a plane passing through the axis ; (3) All the external forces act in such a plane of symmetry normally to the axis. If these conditions are not fulfilled, the action of the straining forces is more complex, and some cases in which this happens will be considered under the head of Compound Stress.

Consider the case represented in fig. 7, where, in the lower figure, the flexure is exaggerated for the sake of clearness. In this case, a bar originally straight, and having transverse sections symmetrical about the plane of the paper, in which the bending forces act, is subjected to flexure, under the action of two equal couples of forces applied to

its ends. Then the curvature from c to d is circular, and the effect of the bending is to lengthen the upper parts of the bar, and to shorten the lower parts. If the flexure is very small, so that the straining forces are sensibly parallel, a plane normal to the paper, through the axis of the bar, will divide the parts in tension from those in compression. The length of the bar measured along that surface will be unaltered by the flexure, and hence it is termed the neutral surface.

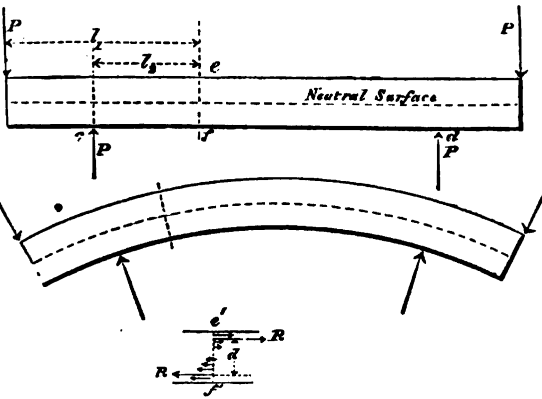


Fig. 7.

The amount of the bending action, at any section ef of the bar, is measured by the resultant moment of the straining forces on either side of that section, which is termed the bending moment. Taking the forces to the left of ef, the bending moment is $\text{P}\,l_1 - \text{P}\,l_2$. The molecular stresses in the bar, developed by the external actions, form at any section a couple, whose moment is equal and opposite to the bending moment, and which is termed the moment of resistance of the section. The action of the molecular stresses is represented at $e'f'$. The tensions above and the compressions below the axis have resultants R, R, whose moment is $\text{R}\,d$. Equating this to the bending moment

$$\text{P}\,(l_1 - l_2) = \text{R}\,d \quad . \quad . \quad . \quad . \quad (4)$$

In other cases the action is a little more complex. Suppose the force P acts at the end of a bar (fig. 8) solidly fixed at the other end, and let it be required to find the straining action at *ef*. Equilibrium is not disturbed, if we introduce two equal and opposite forces P' P'', in the direction *e f*. Then the action of P on the section *e f* is equivalent to that of a couple, P, P'', and an unbalanced force P'. The couple has a moment $P\,l$, which produces simple bending, and is in equilibrium with a couple formed by molecular stresses at *e f*, parallel to the axis of the bar, precisely similar to those described in the previous case. The remaining force P' produces a shearing stress on the section *ef*. The two actions are independent, and the bar must be strong enough to resist both the bending moment and the shearing action. In a large number of cases the amount of material necessary to resist the bending moment is much more than sufficient to resist the shearing action, so that the latter may be left out of consideration.

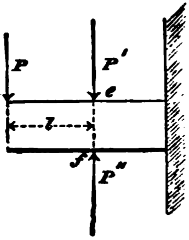


Fig. 8.

If several forces act to the left of *e f*, we may take their resultant, and then proceed as if only a single force required to be dealt with.

It will alter nothing in the conditions of the stresses of the bar in fig. 7, if we suppose it to form part of a longer bar bent to a complete circle of the same curvature, by the action of the external forces. It can then be seen that fibres originally straight in the unstrained bar become coaxial circles in parallel planes in the strained bar, and plane transverse sections become plane radial sections, across which there is no shearing stress, but a bending moment only, the resultant of the tensions and pressures of the fibres.

Let ρ be the radius of the layer of fibres which are

neither extended nor compressed by the bending of the bar. Then the length of all the fibres before bending was $2\pi\rho$. After bending, a fibre at radius $\rho+y$ has the length $2\pi(\rho+y)$, and its extension (or compression if y is negative) is $2\pi y$. Assuming the formulæ in § 26, the stress f due to this extension is given by the equation

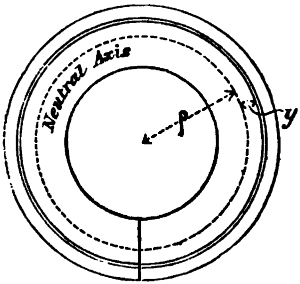


Fig. 8a.

$$2\pi y = \frac{f}{E} 2\pi\rho.$$

Hence
$$f = \frac{Ey}{\rho}.$$

The total stress on an element of area a, at radius $\rho+\gamma$, is therefore $fa = \frac{Eya}{\rho}$, and the total stress on the whole section is $\Sigma\left(\frac{Eya}{\rho}\right)$. But since the pressures and tensions across the section form a couple,

$$\Sigma\left(\frac{Eya}{\rho}\right) = 0;$$

or since $\frac{E}{\rho}$ is a constant,

$$\Sigma ya = 0.$$

This equation is only true if the distances y are measured from a line passing through the centre of figure of the section. Hence the neutral axis of the bar passes through the centres of figure of the cross sections.

The moment of the stress fa about the neutral axis of the section is $fay = \frac{Ey^2a}{\rho}$. The total moment of the couple formed by the tensions and pressures at the section is $\Sigma\left(\frac{Ey^2a}{\rho}\right)$ or $\frac{E}{\rho}\Sigma ay^2$. Now the quantity Σay^2 is known

as the moment of inertia of the section, and is usually denoted by the symbol I. Hence, putting M for the moment of the actions producing bending on one side of the section, which are in equilibrium with the stresses at the section,

$$M = \frac{E\,I}{\rho},$$

which expresses the relation between the bending moment and the curvature of the bar. Let $f_t\, f_c$ be the tension and pressure at points distant y_t and y_c from the neutral axis. Then

$$f_t = \frac{E}{\rho} y_t \text{ and } f_c = \frac{E}{\rho} y_c.$$

Therefore

$$M = f_t \frac{I}{y_t} = f_c \frac{I}{y_c}.$$

$$f_t = M \frac{y_t}{I} \text{ and } f_c = M \frac{y_c}{I}.$$

It is generally necessary to find the greatest tension and pressure in the bar, and we must then take for y_t and y_c the distances of the parts of the bar farthest from the neutral axis on the extended and compressed sides. Then the quantities $\frac{I}{y_t}$ and $\frac{I}{y_c}$ may be termed the moduli of the section with respect to tension and pressure, and putting z_t and z_c for these moduli, the equations may be more simply written

$$M = f_t z_t = f_c z_c.$$

28. Let M be the moment of the external forces on one side of any transverse section, or bending moment, estimated relatively to the section.

Z=the modulus of the section, that is a function of the dimensions of the section, which is proportional to the moment of resistance of the section. The value of z for various sections is given in the following tables.

f_t and f_c=the greatest safe working stress in tension and compression for the material of the bar. Then the bar will be safe, if

$$\left.\begin{array}{r}\text{M is not greater than } f_t z \\ \text{and also is not greater than } f_c z\end{array}\right\}$$

If we put fz for the lesser of the two values of the moment of resistance, the bar will be of adequate strength when

$$M = fz \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and the greatest stress due to bending is

$$f = \frac{M}{z} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

29. When the section is not symmetrical about the neutral surface, z has two values, z' corresponding to the part above, and z'' to the part below the neutral surface. Then two cases arise :—

(1) The part of the bar above the neutral axis in tension, the part below in compression,

$$\text{Moment of resistance} = f_t z' \text{ or } f_c z''.$$

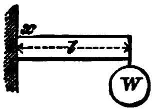
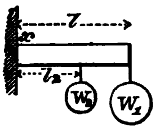
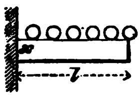
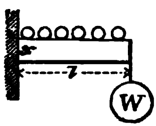
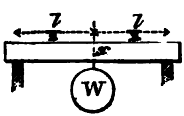
(2) The part of the bar above the neutral axis in compression, the part below in tension,

$$\text{Moment of resistance} = f_c z' \text{ or } f_t z''.$$

The smaller of the two values is to be taken in either case. If the straining forces act successively in opposite directions, the least of the four values is the effective moment of resistance.

In many cases, bars subjected to bending are necessarily uniform in section. Then it is only necessary to consider the greatest bending moment, and to design the section of the bar for that moment. In other cases, the bar varies in section, and the moment of resistance at each section must be, at least, equal to the bending moment at that section. The section at which the bending moment is greatest is sometimes termed the dangerous section. The Table on p. 44 gives, for various loads and modes of support, the greatest bending moment; the position of the dangerous section; the greatest shearing force; and the working load corresponding to a given moment of resistance fz, at the dangerous section.

TABLE III.—*Bending Moment and Shearing Force corresponding to different Loads and for different modes of Support.*

		Greatest bending moment. (at x)	Working load for given moment of resistance	Greatest shearing force	Remarks
		BEAMS FIXED AT ONE END.			
I.		$W l$	$W=\frac{fz}{l}$	W	Loaded at free end.
II.		$W_1 l_1 + W_2 l_2$		$W_1 + W_2$	More than one load.
III.		$w\frac{l^2}{2}$	$w=\frac{2fz}{l^2}$	$w l$	Uniform load, w lbs. per in. run.
IV.		$\frac{w l^2}{2} + W l$		$l + W$	Load partly uniform, partly concentrated.
		BEAMS SUPPORTED AT BOTH ENDS.			
V.		$W\frac{l}{4}$	$W=4\frac{fz}{l}$	$\frac{W}{2}$	Loaded at centre.

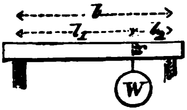
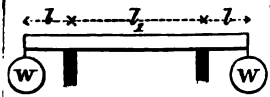
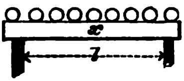
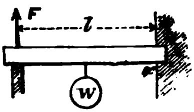
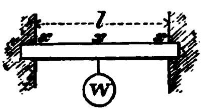
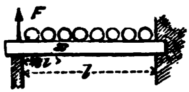
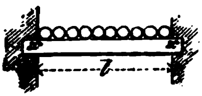
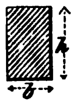
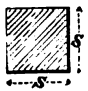
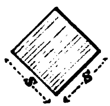
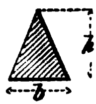
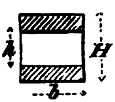
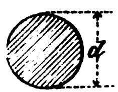
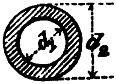
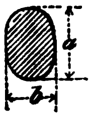
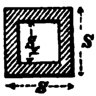
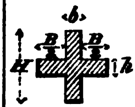
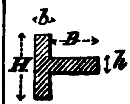
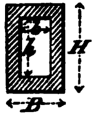
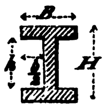
		Greatest bending moment (at x)	Working load for given moment of resistance	Greatest shearing force	Remarks
VI.		$W\frac{l_1 l_2}{l}$	$W=\frac{fzl}{l_1 l_2}$	$\frac{Wl_1}{l}$ & $\frac{Wl_2}{l}$	Load not at centre.
VII.		Wl	$W=\frac{fz}{l}$	W	Two equal couples. Uniform bending moment, and no shearing force between the supports.
VIII.		$\frac{wl^2}{8}$	$w=\frac{8fz}{l^2}$	$\frac{wl}{2}$	Uniformly distributed load. w lbs. per in. run.
		BEAMS FIXED AT ONE END AND SUPPORTED OR FIXED AT THE OTHER.			
IX.		$\frac{3}{16}Wl$	$W=\frac{16fz}{3\,l}$		Load at centre. Pressure on support = $F=\frac{5}{16}W$.
X.		$\frac{Wl}{8}$	$W=\frac{8fz}{l}$		Load at centre. Equal bending moments at ends and centre.
XI.		$\frac{wl^2}{8}$	$w=\frac{8fz}{l^2}$		Greatest bending moment at fixed end.
XII.		$\frac{wl^2}{12}$	$w=\frac{12fz}{l^2}$		Greatest bending moment at ends.

TABLE IV.—*Area and Modulus of different forms of Section.*

The plane of bending is supposed parallel to the side of the page. Where two values are given for the modulus, z′ is applicable to the upper part, z″ to the lower part, of the section.

	Form of section		Area of section A	Modulus of section Z $\left(\frac{I}{y}\right)$
I.	Rectangle		$b\,h$	$\frac{1}{6} b\,h^2$
II.	Square		s^2	$\frac{1}{6} s^3$
III.	Square		s^2	$0{\cdot}118\,s^3$
IV.	Triangle		$\frac{1}{2} b\,h$	$z' = \frac{1}{24} b\,h^2$ $z'' = \frac{1}{12} b\,h^2$
V.	Pierced rectangle		$b\,(\mathrm{H} - h)$	$\frac{1}{6}\frac{b}{\mathrm{H}}\,(\mathrm{H}^3 - h^3)$
VI.	Circle		$\frac{\pi}{4} d^2 = {\cdot}785\,d^2$	$\frac{\pi}{32} d^3 = {\cdot}0982\,d^3$

	Form of section		Area of section A	Modulus of section Z
VII.	Hollow circle		$\frac{\pi}{4}(d_2^2 - d_1^2)$	$\frac{\pi}{32}\frac{d_2^4 - d_1^4}{d_2}$
VIII.	Ellipse		$\frac{\pi}{4} b a$	$\frac{\pi}{32} ba^2 = \cdot 0982\, ba^2$
IX.	Hollow ellipse		$\frac{\pi}{4}(b a - b_1 a_1)$	$\frac{\pi}{32}\frac{b a^3 - b_1 a_1^3}{a}$
X.	Hollow square		$s^2 - s_1^2$	$\frac{1}{6}\frac{s^4 - s_1^4}{s}$
XI.	Cross or Tee	 	$b\text{H} + \text{B}h$	$\frac{b\text{H}^3 + \text{B}h^3}{6\text{H}}$
XII.	Hollow rectangle or I with equal flanges	 	$\text{BH} - bh$	$\frac{\text{BH}^3 - bh^3}{6\text{H}}$

30. *Continuous Beams.*—When a beam rests on more than two supports, the ordinary statical conditions of equilibrium do not suffice to determine the reactions of the supports. Recourse may then be had to a relation due to Clapeyron, which is termed the Theorem of three moments. Let fig. 9 represent two consecutive spans of a beam resting

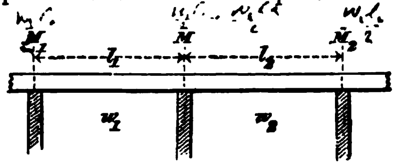


Fig. 9.

on several supports, at the same level; let l_1, l_2, be the lengths of the spans; w_1, w_2, the loads per unit of span; M_1, M, M_2, the bending moments over the supports. Then

$$8\,(l_1+l_2)\,M+4\,l_1\,M_1+4\,l_2\,M_2=w_1\,l_1^3+w_2\,l_2^3 \quad . \quad . \quad (7)$$

This theorem furnishes, for a beam of n spans, $n-1$ equations. In addition to these, the condition that a beam simply supported at the ends, has no bending moment at the ends, furnishes two additional equations, $M_0=0$, $M_n=0$. There are then $n+1$ equations, to determine the $n+1$ bending moments at the points of support. By then reversing the ordinary process, the reactions can be found, from the bending moments and loads. The following are some of the simplest results of applying this theorem to beams uniformly loaded with w lbs. per inch run.

	Reactions at supports
Beam of 2 equal spans	$\frac{3}{8}\,w\,l$; $\frac{5}{4}\,w\,l$; $\frac{3}{8}\,w\,l$.
„ 3 „ . . .	$\frac{4}{10}\,w\,l$; $\frac{11}{10}\,w\,l$; $\frac{11}{10}\,w\,l$; $\frac{4}{10}\,w\,l$.

31. *Relative Economy of different forms of Section.*—The weight of a bar is proportional to its sectional area, its resistance to bending to its section modulus. Of two bars of different forms, subjected to the same loading, that will be

the more economical of material which, with a given value of the modulus of resistance z, has the lesser sectional area A. Hence, the more economical the form of the bar, the greater will be the ratio $\frac{z}{A}$.

In a prismatic bar, of circular or rectangular section, only the material at the extreme top and bottom of the section is fully strained. Nearer the neutral surface the material is less strained, and at the neutral surface it is not strained at all by the direct stresses due to bending. Such a bar would be made stronger, by removing some of the material from the neighbourhood of the neutral surface towards the top and bottom of the section. We thus arrive at the excellent form of section known as the I or double T section. The material is chiefly collected in the top and bottom flanges, which bear nearly the whole of the direct stresses due to bending; the remainder forms a vertical web, whose chief function is to resist the shearing action.

32. *Flanged Sections when both Flanges are strained to the Working Limit.*—In order that the stress at the stretched edge of the bar may be at the working limit of tension, and the stress at the compressed edge may be at the working limit of pressure, we must have

$$f_t z' = f_c z'' \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where z' is the modulus, corresponding to the part in tension, and z'', that corresponding to the part in compression, and f_t, f_c, are the working intensities of tension and pressure resistance. If $f_t = f_c$, then $z' = z''$, or the modulus must be the same, both for the upper and lower parts of the section, and this will be the case when the section is symmetrical about the neutral surface.

If f_t and f_c are not equal, both edges cannot be fully strained when $z' = z''$, and the material is not used in the most economical way. In that case, it is better to adopt a section unsymmetrical with respect to the neutral surface.

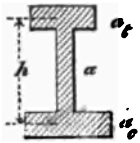


Fig. 10.

Let a_t be the area of the tension flange, a_c the area of the compression flange, and a the area of the web of a beam of I-shaped section. Let h be the depth, measured from centre to centre of the flanges. Then the required condition is nearly fulfilled, when

$$f_t\, a_t = f_c\, a_c$$

$$a_t = \frac{M}{f_t\, h} \text{ and } a_c = \frac{M}{f_c\, h} \quad . \quad . \quad . \quad . \quad (9)$$

Further, if F is the total shearing action, and f_s the safe shearing stress, the strength of the web is sufficient when,

$$a = \frac{F}{f_s} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

For practical reasons, especially in cast beams, a has often to be made of larger area than is given by this equation.

In the foregoing equations the resistance of the web to bending is neglected. If this is taken into account we get the following equations, which are taken from Rankine :—

CASE I.—f_c greater than f_t, as in cast iron.

$$a_t = \frac{f_c}{f_t}\, a_c + \frac{f_c - f_t}{2 f_t}\, a \quad . \quad . \quad . \quad . \quad (11)$$

CASE II.—f_t greater than f_c, as in wrought iron and steel.

$$a_c = \frac{f_t}{f_c}\, a_t + \frac{f_t - f_c}{2 f_c}\, a \quad . \quad . \quad . \quad . \quad (12)$$

and the moduli of the section are in either case

$$z_c = h \left\{ a_c + \left(2 - \frac{f_t}{f_c} \right) \frac{a}{6} \right\} \text{ for compression ;}$$

$$z_t = h \left\{ a_t + \left(2 - \frac{f_c}{f_t} \right) \frac{a}{6} \right\} \text{ for tension.}$$

Lastly, the moment of resistance which is to be equated to the bending moment is

$$M = f_c z_c = f_t z_t.$$

In consequence of the tendency in wrought-iron beams to a vertical or lateral buckling of the compressed flange, the working stress, in compression, is sometimes taken $\frac{2}{8}$ths less than the working stress in tension. In cast iron, the working resistance, in compression, may be taken three or four times as great as the resistance in tension.

33. *Distribution of Bending Moment and Shearing Action.* —When the magnitude and position of the greatest bending moment and shearing action are known, the bending moment and shearing action, at any other point, can often be found very easily, by a simple graphic construction. Let O A be equal, on any scale, to the span of a beam. If, at any point, *a*, a perpendicular is erected, and *a b* is made equal, on any scale, to the bending moment at the section of the beam which corresponds to *a*, *b* is a point in a curve, termed the curve of bending moments.

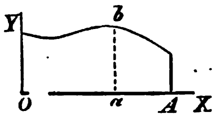
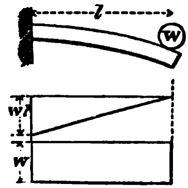
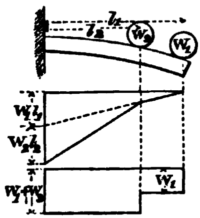
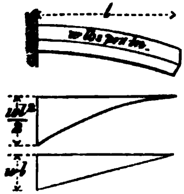
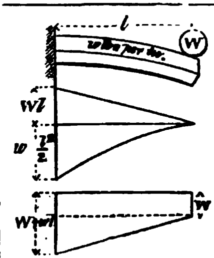
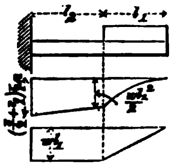
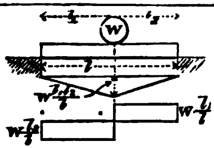
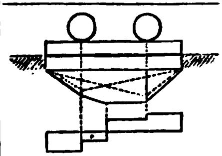
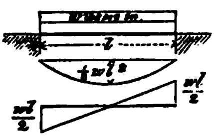


Fig. 11.

Similarly, if *a b* were made equal to the shearing action at *a*, then *b* would be a point in a curve of shearing action. The curves might be constructed, by finding the moments and the shearing actions at a sufficient number of points, setting them off on a diagram in the way just described, and then connecting the points of the curve, so found, by a line. In the simpler cases of loading, however, these curves can be more simply constructed. When the curve (which in some cases becomes a straight line) is drawn, the moment, or shearing action, at any point is obtained by scaling off the ordinate corresponding to that point. The following Table (p. 52) gives the form of the curves of bending moment and shearing action, in the simpler cases.

TABLE V.—*Distribution of Bending Moment and Shearing Force.*

	Loading	Diagram of load, and of bending moment and shearing force curves	Bending moment curve	Shearing force curve
BEAMS ENCASTRE AT ONE END.				
I.	Load at free end		Straight line	Straight line
II.	Two loads		Broken line	Broken line
III.	Uniform load		Parabola, vertex at free end, axis vertical	Straight line
IV.	Uniform load and load at free end		Obtained by combining the curves in I. and III.	

	Loading	Diagram of load, and of bending moment and shearing force curves	Bending moment curve	Shearing force curve
V.	Partial uniform load		Parabola, with vertex at free end and straight line	Broken line
		BEAMS SUPPORTED AT BOTH ENDS.		
VI.	Single load		Broken line	Broken line
VII.	Two loads		Obtained by adding ordinates due to each separate load	
VIII.	Uniform load		Parabola, vertex at centre, axis vertical	Straight line

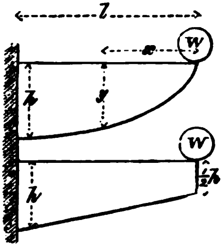
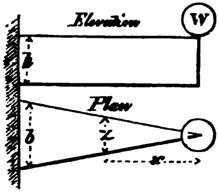
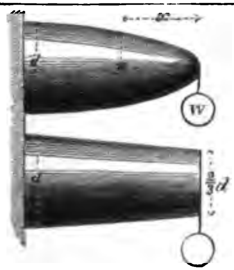
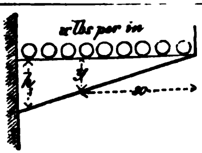
The cases given in this Table should be compared with the corresponding cases in Table III.

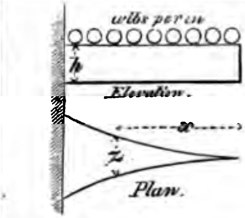
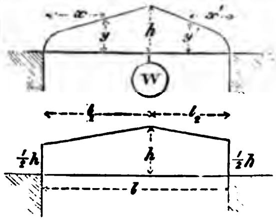
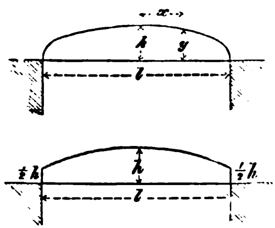
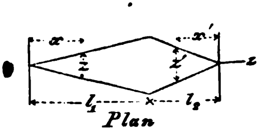
34. *Beams of Uniform Resistance to the direct Stresses due to Bending.*—Except in one special case, the bending moment varies at different points in the length of the beam. At the point where the bending moment is greatest, the section must be designed for that maximum moment. For practical reasons, it is frequently necessary to make the beam, or bar, uniform, and then the section where the bending moment is greatest determines the section of the rest of the bar. In other cases, the section of the bar may be diminished in parts where the bending moment is less, and material is then economised. The best distribution of material, so far as the direct stresses are concerned, is that which fulfils the condition

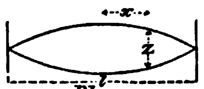
$$\mathrm{M}=f\mathrm{Z} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

for every transverse section, M being the bending moment at any section, and Z the modulus of that section. Beams so designed are often termed beams of uniform strength. The theoretical form thus obtained requires, in some cases, to be modified for practical reasons. Approximate forms fulfilling the necessary conditions are given with the theoretical forms in the following Table. Table VI. gives some examples, partly selected from Reuleaux.

TABLE VI.—*Forms of Beams of Uniform Strength.*

		Longitudinal elevation of beam	Form of transverse section	Bounding lines of elevation or plan	Equation for determining the dimensions
BEAMS FIXED AT ONE END.					
I.	Load at free end		Rectangle of uniform breadth, b, and variable depth, y	Straight line and parabola. Approximate form, a truncated pyramid	$y^2 = \frac{6\,W}{bf}x$
II.			Rectangle of uniform depth, h, and variable breadth, z	Straight lines forming a wedge	$z = \frac{6\,W}{h^2 f}x$
III.			Circle of variable diameter, y	Cubic parabola. Approximate form, a truncated cone	$y^3 = \frac{32\,W}{\pi f}x$
IV.	Uniform load		Rectangle of uniform breadth, b, and variable depth, y	Straight lines forming a wedge	$y^2 = \frac{3\,w}{fb}x^2$

		Longitudinal elevation of beam	Form of transverse section	Bounding lines of elevation or plan	Equation for determining the dimensions
V.	Uniform load		Rectangle of uniform depth and variable breadth, z	Parabolas with vertex at free end	$z = \frac{3\,w}{f\,h^2}x^2$
Beams supported at each end.					
VI.	Single load		Rectangle of uniform breadth, b, and variable depth, y	Two parabolas and straight line. Approximate form, truncated pyramids	$y^2 = \frac{6\,W\,l_2}{b\,f\,l}x$ $y'^2 = \frac{6\,W\,l_1}{b\,f\,l}x'$
VII.	Uniform load		Rectangle as above (w lbs. per inch uniformly distributed)	Ellipse and straight line. Approximate form, circular arc and straight line	$y^2 = \frac{3\,w}{4\,f\,b}(l^2 - 4\,x^2)$
VIII.	Single load		Rectangle of uniform depth, h, and variable breadth, z	Straight lines	$z = \frac{6\,W\,l_2}{h^2 f\,l}x$ $z' = \frac{6\,W\,l_1}{h^2 f\,l}x'$

		Longitudinal elevation of beam	Form of transverse section	Bounding lines of elevation or plan	Equation for determining the dimensions
IX.	Uniform load	 *Plan*	Rectangle as above. (Load, w lbs. per inch, uniformly distributed)	Two parabolas	$z=\frac{3(l^2-4x^2)w}{4fh^2}$

A beam, supported at each end, is equivalent to two beams encastre at the point where the bending moment is greatest. The forms given for beams encastre at one end, may be used for each segment of a beam supported at both ends.

Resistance and Deflection of Springs.

34*a*. *Straight Springs.*—The best form for a straight spring, or that which gives the greatest deflection for a given strength, is the form shown in Case VI. Table VI., the greatest depth, h, being at the centre, where the load is applied. The breadth is uniform. Let l be the length of the spring, h its depth, and b its breadth, in inches. Let P be the force acting on the spring, and δ the deflection due to P

$$\delta=\frac{\text{P}l^3}{\text{E}bh^3}$$

where E is the modulus of elasticity of the material. The deflection should not exceed $\frac{1}{10}$th of the length.

Spiral Springs.—Let r be the radius of the cylindrical surface passing through the centre of the coils of the spring; n the number of coils; d the diameter of the wire of which the spring is made; G the coefficient of transverse elasticity of the material; f the greatest safe shearing stress; W the load acting axially, and not exceeding the greatest safe load; δ the extension or compression due to W; W_1 the

greatest safe steady load, and δ_1 the corresponding extension or compression. Then, according to Rankine :—

$$\delta = \mathrm{W}\frac{64\,n\,r^3}{\mathrm{G}\,d^4}$$

$$\mathrm{W}_1 = \frac{0{\cdot}196\,f\,d^3}{r}$$

$$\delta_1 = \frac{12{\cdot}566\,n\,f\,r^2}{\mathrm{G}\,d}$$

Values of G are given in Table I.

Resistance to Shearing.

35. An action which causes sliding parallel to the section considered is termed a shearing action. Thus, the pressure of the cutting edges of an ordinary shearing machine, fig. 12, induces a shearing stress in the plane *a b*. The mean intensity of the shearing stress is the shearing force P, divided by the area *a* of the section *a b* of the bar. In the case shown, the forces P P act exactly in the plane of the section, and the shearing stress is uniformly distributed, and at all parts of the section$=\mathrm{P}\div a$. But if the forces P P do not act exactly in the plane of the section, the bar tends to bend as well as to shear (fig. 13). The effect of this is to alter the distribution of the shearing action at all sections between *a* and *b*. Near the middle of the section the shearing stress is greater than the mean shearing stress, and at the upper and lower boundary of the section it becomes zero. A rivet connecting two plates (fig. 14) is almost always in shear, and a bolt is very often so. A cotter or key is similarly intended to resist shear. In these cases, the shearing forces do not act in the plane of the section *b c*, but along the centres of

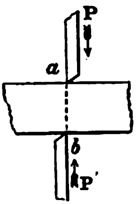


Fig. 12.

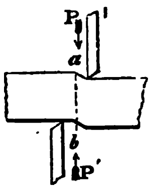


Fig. 13.

the plates connected. In consequence, however, of the rigidity and friction of the edges $a\,b$ and $c\,d$ of the plates, the points of application of the forces $P\,P'$ on the surface of the rivet may very nearly approach the plane $b\,c$, and then the shearing stress is uniformly distributed on $b\,c$. If, however, the rivet fits very loosely in the rivet holes, fig. 15, the rivet bends and the distribution of stress becomes more or less unequal. For rivets, it is usual to assume that they fit their holes tightly, and that the shearing stress is simply $f_s = P \div a$. But for bolts and cotters, it is safer to assume that the stress is unequally distributed, and the maximum stress may then reach the values

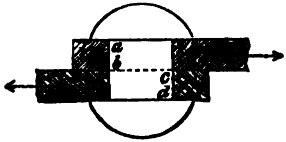


Fig. 14.

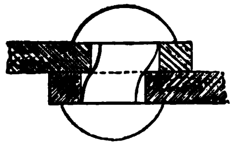


Fig. 15.

$f_s = \frac{3}{2}\,\frac{P}{a}$ if the section is rectangular and P perpendicular to one side.

$= \frac{4}{3}\,\frac{P}{a}$ if the section is circular or elliptical.

$= \frac{9}{8}\,\frac{P}{a}$ if the section is square and P acts parallel to a diagonal.

Resistance to Torsion.

36. A bar is subjected to simple torsion when two equal and opposite couples act upon it in two planes perpendicular to its axis, instead of being, as in the case of bending, in the plane of the axis. When the bar is subjected to straining action of this kind, any two transverse sections rotate slightly relatively to each other, and on any one transverse section the stress is a simple tangential or shearing stress,

varying in intensity as the distance from the centre of the bar, where it is zero, to the circumference, where it is greatest. Of the two couples one, P P′, is usually due to motive forces applied to the bar. The other, P″ P‴, is due to the passive reaction of the parts to which the bar is attached, or to the resistances which are being overcome. Further, of the two forces P, P′, constituting the former

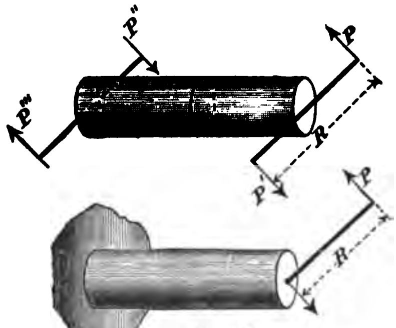


Fig. 16.

couple, one of the two forces, for instance, P′, may be due to the reaction of a support or bearing of the shaft, and it then acts at the centre of the shaft, as shown in the lower figure.

The amount of straining action at any section *a b*, is measured by the moment of the couple on either side of the section. In this case that moment, termed the twisting moment, is $T = P R$. If several couples act on one side of the section, the algebraic sum of the moments of all those couples is to be taken, right-handed couples being considered positive, and left-handed couples negative.

When the bar is kept in rotation overcoming a resistance, and the amount of work transmitted is known, the twisting moment is easily found. Let H P be the number of horses' power transmitted, N the number of revolutions of the bar

per minute. Then the work expended in inch lbs. per minute is $12 \times 33000 \times \text{H P}$, and this is equal to the twisting moment T in statical inch lbs., multiplied by the angular motion $2\pi\text{N}$ of the bar in the same time. Hence

$$\text{T} = \frac{12 \times 33000 \times \text{H P}}{2\pi\text{N}} = 63024 \frac{\text{H P}}{\text{N}} \text{ inch lbs.} \quad . \quad (14)$$

The moment of resistance of any section to twisting is proportional to the greatest stress at any part of the section, and to a function of the dimensions, which is termed the modulus of the section with respect to torsion. Let f be the greatest shearing stress, and z_t the modulus :—

$$\text{T} = f z_t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

For cylindrical bars of diameter d,

$$z_t = \frac{\pi}{16} d^3 = \frac{d^3}{5 \cdot 1} = 0 \cdot 196\, d^3.$$

For hollow cylindrical bars having d_1, d_2, for outside and inside diameters,

$$z_t = \frac{\pi}{16} \cdot \frac{d_1^4 - d_2^4}{d_1} = 0 \cdot 196 \frac{d_1^4 - d_2^4}{d_1}.$$

For bars of square section, s being the side of the square,

$$z_t = 0 \cdot 208\, s^3.$$

It is sometimes necessary to know the angle through which one end of a bar rotates, relatively to the other end, that is the rate of twist when subjected to torsion. For a cylindrical bar, let d be the diameter, l the length and G the modulus of transverse elasticity; then the angle of torsion is, in circular measure,

$$\theta = \frac{2\text{T}l}{\text{G}z_t d} = \frac{2fl}{\text{G}d} \quad . \quad . \quad . \quad . \quad (16)$$

where f is the greatest actual stress due to the twisting moment. Values of G are given in Table I.

RESISTANCE TO COMBINED COMPRESSION AND BENDING.

37. When a bar, of moderate length, is subjected to a thrust, acting in the direction of its axis, the stress on each cross section is a simple pressure. The working strength of the bar is given by the equation

$$P_1 = fA \quad . \quad . \quad . \quad . \quad . \quad (17)$$

where P_1 is the greatest safe load, A the sectional area of the bar, and f the safe working compressive resistance of the material.

38. If, however, the bar is of great length, it gives way, ultimately, under the action of a thrust, by lateral bending; the stress, at the section where fracture occurs, being a compound stress, due both to the longitudinal pressure, and the curvature of the bar. Rules for the ultimate resistance of long bars to compression were first obtained theoretically by Euler and experimentally by Professor Hodgkinson. Hodgkinson's formulæ have been very generally used in this country, in designing compression bars. These rules are inconvenient in form, and they can only be extended to many cases of common occurrence, by theoretical assumptions, which are probably only approximately true. An expression more convenient in form than Hodgkinson's was proposed by Gordon, and was afterwards modified by Rankine, so as to be applicable to bars of all forms of section. The reasoning on which this rule is based is, however, not satisfactory. All these rules are intended to give the ultimate strength of the bars, and in applying them it is necessary to divide the resistance thus calculated by an arbitrary factor of safety. Actual compression bars are not intended to be loaded beyond their elastic limit; and hence it may be urged, with some reason, that the theoretical rules of Euler, which Hodgkinson discarded, as not agreeing with his experiments on ultimate strength, are more strictly applicable to

the circumstances in which compression bars are used, than Hodgkinson's rules. They are also simpler, and include all cases. Euler's rules assume the elasticity of the bar to be unimpaired. In that case, no increase of the load would *directly* cause bending, but a point is reached at which the equilibrium of the bar becomes unstable. With less loads, the bar, if bent, will restore itself to straightness by its elastic resistance to bending; with greater loads, it is unable to do so, and if any flexure is produced, however slight, that flexure will be increased by the action of the load, until the bar breaks. The greatest load for which the bar remains stable, is the measure of the strength of the bar. The working load may be $\frac{1}{n}$th of the load thus calculated, where n is a factor of safety.

Let E be the modulus of direct elasticity of the material of the bar; I, the moment of inertia of the section of the bar, estimated with respect to an axis passing through the centre of gravity of the section, and at right angles to the plane in which the bar most easily bends; λ, the length of an arc of the curved bar, measured between two points of contrary flexure. Then

$$P = \pi^2 \frac{IE}{\lambda^2} \quad . \quad . \quad . \quad . \quad . \qquad (18)$$

where P is the greatest load consistent with stability or fracturing load of the long bar. If l is the whole length of the bar, then $\lambda = m\,l$, where m is a constant depending on the mode in which the ends of the bar are fixed. The values of m are given in the following table.

The greatest working load should not exceed nP, where n is about $\frac{1}{4}$ for wrought iron or steel, $\frac{1}{6}$ for cast iron, and $\frac{1}{10}$ for wood. Hence the working load is

$$P_2 = \frac{\pi^2}{m^2} \cdot \frac{n\,EI}{l^2} \quad . \quad . \quad . \quad . \quad . \qquad (19)$$

The formulæ in Table VII. give the fracturing load of

the long bar or column, supported in different ways, when E is the modulus of elasticity. But if n E is substituted for E, we get the safe working load.

	E=	n E=
Wrought iron	29,000,000	5,800,000
Steel ...	30,000,000	6,000,000
Cast iron ...	17,000,000	2,800,000
Wood (hard)	1,500,000	150,000
„ (soft)	1,400,000	140,000

Values of the modulus of elasticity E for some other materials are given in Table I., p. 32.

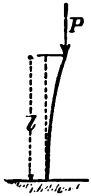
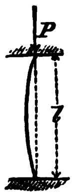
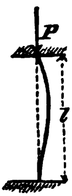
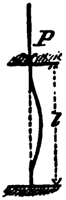
Let Z be the smallest modulus of the section of the bar, values of which are given in Table IV. p. 47; r the distance from the centre of figure of the section to the edge of the bar, measured parallel to the plane of bending.[1]

$$I = zr \quad . \quad . \quad . \quad . \quad . \quad (20)$$

	Moment of Inertia = I =
Circular section (diam. $=d$)	$0{\cdot}0491\, d^4$
Annular section (diams. $=d_1, d_2$)	$0{\cdot}0491\,(d_1^4 - d_2^4)$
Square section (length of side $=s$)	$\frac{1}{12} s^4$
Rectangular section (longer side b, shorter h) . .	$\frac{1}{12}\, b\, h^3$
Cross-shaped section (Fig. XI. Table IV.) if bending is parallel to H	$\frac{1}{12}(b\mathrm{H}^3 - \mathrm{B}h^3)$

[1] The relation of I and Z is explained in § 27.

TABLE VII.—*Strength of Long Compression Bars. Greatest Load consistent with Stability.*

		Mode of fixing.	Length of arc λ in terms of total length	Greatest load P consistent with stability
I.		One end free, the other fixed.	$\lambda = 2l$	$\frac{\pi^2}{4} \frac{EI}{l^2}$
II.		Both ends free, but guided in the direction of the load.	$\lambda = l$	$\pi^2 \frac{EI}{l^2}$
III.		One end fixed, the other free, and guided in direction of the load.	$\lambda = \frac{l}{\sqrt{2}}$	$2\pi^2 \frac{EI}{l^2}$
IV.		Both ends fixed in direction.	$\lambda = \frac{l}{2}$	$4\pi^2 \frac{EI}{l^2}$

39. In applying the above rules, it is assumed that the length is so great that $P_2 < P_1$. For bars of moderate length, when—

λ is less than 24 d or 28 h for wrought iron;
" " 10 d or $11\frac{1}{2}$ h for cast iron;
" " $11\frac{1}{3}$ d or 13 h for wood;

the rules in Table VII. are not applicable. The following empirical rules, suggested by Grashof, may then be used:

For a bar free at both ends as in Case II.

$$P_3 = \frac{k_1 A I}{C A l^2 - I} \text{ or } = \frac{k_2 A I}{C A l^2 - I} \quad . \quad . \quad . \quad (21)$$

the lesser value given by these equations is to be taken. For a bar fixed at both ends as in Case IV.

$$P_3 = \frac{4 k_1 A I}{C A l^2 - 4 I} \text{ or } = \frac{4 k_2 A I}{C A l^2 - 4 I} \quad . \quad (21a)$$

In these equations A is the sectional area of the bar perpendicular to the axis; I is the moment of inertia of the section about an axis through the centre of gravity, and perpendicular to the plane in which bending is most likely to occur; l is the length, and the other quantities are constants, the values of which are given in the following table. P_3 is the greatest safe load, the lesser of the two values being always taken.

	$C=$	$k_1=$	$k_2=$
Steel . . .	·00009	14220	14220
Wrought iron .	·00009	8500	8500
Cast iron . .	·00027	2840	11360
Wood . . .	·00022	850	700

40. *Resistance of thin cylinders to an external collapsing pressure.*—When a thin cylinder, rigidly supported at the ends, is subjected to a uniform external pressure, it gives way by buckling, or collapse. There is, at present, no theory of this mode of yielding, but experiments were made by Sir W. Fairbairn, from which the following rules were deduced.

Let t be the thickness, d the diameter, and l the length between the rigidly-supported ends of a cylinder, subjected to a uniform external pressure of p lbs. per sq. in. Then collapse takes place when

$$p=9{,}672{,}000\frac{t^{2\cdot19}}{l\,d} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

This can only be solved by logarithms; and in a logarithmic form the equation becomes

$$\log.\ p=6\cdot9855+2\cdot19\ \log.\ t-\log.\ (l\,d) . \quad . \quad . \quad (23)$$

All the dimensions are in inches. A formula, which is probably still more reliable, was deduced from the same experiments by M. Love. His formula is

$$p=5{,}358{,}150\frac{t^2}{l\,d}+41906\frac{t^2}{d}-1323\frac{t}{d}. \quad . \quad . \quad . \quad (24)$$

The limits of length for which these formulæ are applicable are not known. The factor of safety may be 6 or 8.

40*a*. The author has recently re-examined[1] Sir W. Fairbairn's experiments, and has found that the length of the tube influences the number of segments into which the circumference of the tube divides in collapsing. When this is taken into account the laws of collapse are found to be related to those of the resistance of long columns. The following are the formulæ arrived at.

For tubes with a longitudinal lap joint—

$$p=7{,}363{,}000\frac{t^{2\cdot1}}{l^{0\cdot9}\,d^{1\cdot16}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

For tubes with a longitudinal butt joint—

$$p=9{,}614{,}000\frac{t^{2\cdot21}}{l^{0\cdot9}d^{1\cdot16}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

For tubes with longitudinal and cross joints like ordinary boiler flues—

$$p=15{,}547{,}000\frac{t^{2\cdot35}}{l^{0\cdot9}d^{1\cdot16}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

[1] 'Proc. Inst. of Civil Engineers,' vol. xlvi.

The working pressure should not exceed one-fourth to one-sixth of the collapsing pressure given by these formulæ.

The last formula is the one most often applicable in practice. As it is not in a very convenient form for calculation the following may be adopted:

$$p = 15{,}547{,}000 \frac{t^2}{l d} \frac{\alpha}{\beta \gamma}$$

Then if the following values are given to the variable co-efficients α, β and γ, the collapsing pressure is found approximately by a simple calculation, and logarithms may be dispensed with.

When t lies between	0·061 and 0·087	0·087 and 0·119	0·119 and 0·159	0·159 and 0·206	0·206 and 0·261	0·261 and 0·325	0·325 and 0·399	...
α = . .	0·40	0·45	0·50	0·55	0·60	0·65	0·70	...
When t lies between	0·399 and 0·483	0·483 and 0·577	0·577 and 0·682	0·682 and 0·800	0·800 and 0·931	0·931 and 1·07	...	...
α = . .	0·75	0·80	0·85	0·90	0·95	1·00	...	...
When l lies between	13 and 25	25 and 51	51 and 110	110 and 253	253 and 628	...	...	...
β = . .	0·75	0·70	0·65	0·60	0·55	...	...	...
When d lies between	2·4 and 4·0	4·0 and 6·5	6·5 and 10·2	10·2 and 15·5	15·5 and 22·9	22·9 and 33·0	33·0 and 47·0	47 and 65
γ = . .	1·2	1·3	1·4	1·5	1·6	1·7	1·8	1·9

It is of very great importance to determine the limits within which the formula for collapsing resistance is applicable, for it is impossible to believe that it is correct for very long or very short tubes. Consideration of the way in which a tube breaks up in collapsing leads the author to fix, provisionally, the following limits, within which only the formula should be trusted.

(*a*) *Maximum limit of length.*—When the length exceeds $6·7\ d^2$ inches in length, the strength will probably become independent of the length. Then the strength should be calculated as if the length were $6·7\ d^2$ only. Thus for tubes with longitudinal and cross-joints which exceed this limit of length (from eq. 3) we get—

$$p=2,810,000\frac{t^{2·85}}{d^3} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The following short table indicates the maximum limit of length for ordinary cases.

Diameter . .	4	8	12	18	24	36 inches.
Maximum limit of length .	107	429	965	2170	3860	8680 ,,
	9	35	80	180	320	720 feet.

(*b*) *Minimum limit of length.*—The formula for collapse will probably cease to apply if the length is less than

$$l_{\text{min.}}=4468\frac{t^{2·22}}{d^{0·18}}. \quad . \quad . \quad . \quad . \quad . \quad (4a)$$

The following Table gives the values of the minimum length to which the formula of collapse is applicable.

Values of $l_{\text{min.}}$ in inches when

The thickness being	The diameters are					
	4	8	12	18	24	36
$\frac{1}{8}$	34	30	28	26	25	23
$\frac{1}{4}$	—	142	131	122	116	108
$\frac{3}{8}$	—	348	324	301	286	265
$\frac{1}{2}$	—	—	614	570	542	504
$\frac{5}{8}$	—	—	1007	935	889	826
$\frac{3}{4}$	—	—	—	1403	1334	1239

It will be seen that, for moderately thick plates, the least length for which the formula of collapse can be trusted, is greater than the ordinary length of boiler flues. It would seem, therefore, that collapse rings in such cases are not necessary. They may, however, be very judiciously em-

ployed as a provision against deterioration of form, or overheating, which seriously weaken the flue.

(*c*) *Limit of thickness.*—The formula also ceases to be applicable if t is greater than $\frac{d}{19}$, and if this limit is exceeded formula (5) should be used.

41. *Resistance of tubes to a crushing pressure.*—The strength of tubes, the dimensions of which fall outside the limits thus assigned, is not known experimentally; but it seems rational in such cases to fix a limit to the crushing stress per sq. in. in the material of the flue or tube. Hence in such cases, using equation (2), p. 37,

$$p=\frac{2ft}{d} \quad . \quad . \quad . \quad . \quad (5).$$

where p is the working pressure, if f is the safe working limit of stress. For riveted furnace tubes the Board of Trade allow a working stress $f=4000$ lbs. per sq. in. on the gross section of the tube, not deducting the rivet holes. Then the working pressure is

$$p=8000\,\frac{t}{d} \quad . \quad . \quad . \quad (5a)$$

Ordinary boiler flues provided with collapse rings are almost always outside the limits within which the formulæ of collapse are applicable. Their strength may therefore be calculated, first by formula (5) or (5*a*), and, afterwards, it may be examined whether the formulæ (1), (2), or (3), require a less working pressure.

42. *Collapse rings.*—When the limit of length of a flue exceeds the values in equation 4*a* above, it may be greatly strengthened by rigid rings riveted to it at intervals. Fig. 16*a* above shows three forms of such rings; at *a* is the T-iron ring first used by Sir W. Fairbairn: this is spaced off the flue by ferrules to prevent the overheating of the plates; at *c* is another mode of applying a T iron ring; at *b* is the collapse ring introduced by Mr. Adamson. Mr.

Adamson's ring is a solid forged ring $\frac{3}{8}$ to $\frac{1}{2}$ in. thick. The flue plates are flanged out and riveted to this on each side. A forged T-iron ring not riveted to the flue has sometimes been used. It serves to prevent deformation. If a flue is more than double the minimum limit of length it will be doubled in strength by a collapse ring at its centre. If more than three times the minimum limit in length, it is tripled in strength by two equidistant collapse rings. But there is no reason to believe that a length shorter than the minimum length is increased in strength by a collapse ring

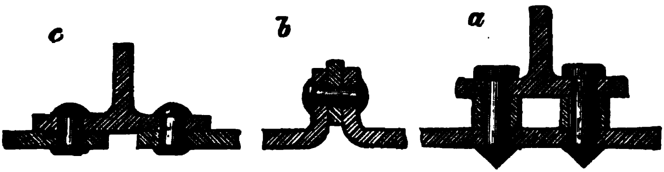


Fig. 16*a*.

except so far as it may prevent accidental flattening of the flue during work. When collapse rings are applied at short intervals of about 8 or 10 feet, the effective length of the flue may be taken to be the length between two collapse rings. Then the strength is to be taken as that given by the collapse formula or by the crushing formula, whichever gives the smaller value.

Compound Stress.

43. I. *Tension or pressure, combined with bending.*—When a force P, fig. 17, acts in a plane passing through the axis of the bar, and parallel to that axis, the stress on transverse sections of the bar is equivalent to that due to a direct tension or pressure P, and a bending moment P*r*. The greatest stress is then

$$f = \mathrm{P}\left(\frac{1}{\mathrm{A}} + \frac{r}{\mathrm{Z}}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

where A is the transverse sectional area, and Z the modulus

of the section. The greatest stress will be a tension or pressure, according as P tends to extend or to compress the bar.

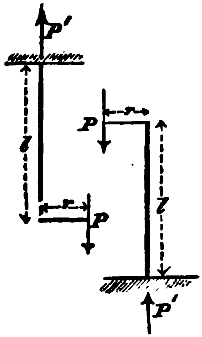


Fig. 17.

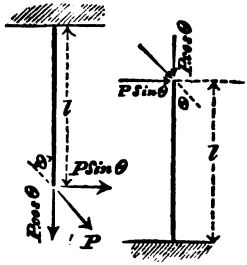


Fig. 18.

II. When the force P acts as above, but not parallel to the axis of the bar, fig. 18, its direction will intersect that axis, at some distance l from the point of support. At that point, resolve the force P into its components. The component P Cos θ produces a simple tension, or compression; the other component P Sin θ produces bending, the greatest bending moment being P l Sin θ. Then the greatest stress at the section nearest the point of support is

$$f = \text{P}\left(\frac{\text{Cos}\ \theta}{\text{A}} + \frac{l\ \text{Sin}\ \theta}{\text{Z}}\right) \quad . \quad . \quad . \ (26)$$

44. III. *Combined twisting and bending.*—Let the force P act in a plane perpendicular to the axis of the bar, at a distance r from the axis, and at a distance l from the point of support. The force P will give rise to a parallel reaction P_1 at the point of support, and the bar will be subjected to a wrenching moment $\text{P}\sqrt{(r^2+l^2)}$. It will not affect the conditions of equilibrium, if we introduce two opposite forces P', P'_1, each equal to P or P_1. Then the wrenching moment will be seen to be equivalent to a simple

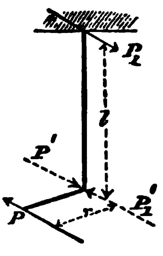


Fig. 19.

twisting moment, due to P and P′, and a bending action, due to P_1 and P'_1. The twisting moment is $T=P\,r$, and the greatest bending moment is $M=P\,l$.

Let M_e be a simple bending moment, which would produce an effect on the bar, equivalent to that due to the combined bending and twisting action. Then the theory of elasticity furnishes the two following values of M_e, according as we have regard to the greatest stress or the greatest strain induced in the bar

$$M_e=\tfrac{1}{2}M+\tfrac{1}{2}\sqrt{(M^2+T^2)} \quad . \quad . \quad . \quad . \quad (27)$$
$$=\tfrac{3}{8}M+\tfrac{5}{8}\sqrt{(M^2+T^2)} \quad . \quad . \quad . \quad . \quad (27a)$$

The former value will be used in this Treatise. It can be put in the simpler approximate forms—

$$\left.\begin{aligned} M_e&=0{\cdot}98\,M+0{\cdot}2\,T \quad \text{if } l>r \quad . \quad . \quad . \quad . \\ &=0{\cdot}7\,M+0{\cdot}48\,T \quad \text{if } r>l \quad . \quad . \quad . \quad . \\ &=0{\cdot}914\,M+0{\cdot}414\,T \text{ if } r \text{ and } l \text{ are unknown} \; . \end{aligned}\right\}(28)$$

The greatest safe load in the above case is

$$P=\frac{2fZ}{l+\sqrt{(l^2+r^2)}}=\frac{fZ}{a\,l+b\,r} \text{ nearly} \quad . \quad . \quad . \quad (29)$$

where a and b are the numerical values of the constants in the approximate formulæ given above.

If an equivalent twisting moment is required instead of an equivalent bending moment, let T_e be that moment. Then

$$T_e=2M_e=M+\sqrt{(M^2+T^2)} \quad . \quad . \quad . \quad . \quad (29a)$$

45. *Stability of a shaft subjected to twisting and thrust.*—A long shaft like the screw shaft of a steamer becomes unstable in form, if the length between the bearings is too great, the action being similar to that which occurs with long columns as described above. Let l be the length of the shaft between the bearings; P the greatest load consistent

with stability as in § 38; T the twisting couple on the shaft, which may be obtained from the H.P. as in § 36. Then, if the twisting moment is neglected, we know already that

$$\frac{\pi^2}{l^2}=\frac{P}{E I} \quad \text{and hence} \quad P=\pi^2\frac{E I}{l^2}.$$

But if the twisting moment is taken into account,

$$\frac{\pi^2}{l^2}=\frac{P}{E I}+\frac{T^2}{4 E^2 I^2} \quad \text{and} \quad P=\pi^2\frac{E I}{l^2}-\frac{T^2}{4 E I},$$

which can be obtained in a similar manner.

STRENGTH OF FLAT PLATES.

46. I. A flat plate of thickness t, is supported, but not fixed, on a circular support of radius r, and is uniformly loaded with p lbs. per sq. in. (fig. 20). Then the greatest stress is

$$f=\frac{5}{6}\frac{r^2}{t^2}p \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

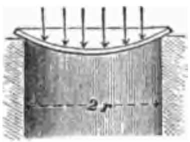


Fig. 20.

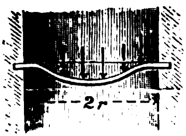


Fig. 21.

II. A circular flat plate, of radius r and thickness t, is encastré at the edge, and is uniformly loaded with p lbs. per sq. in. (fig. 21). Then the greatest intensity of stress is

$$f=\frac{2}{3}\frac{r^2}{t^2}p \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

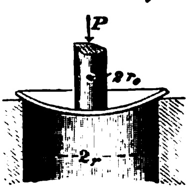


Fig. 22.

III. A circular plate, of radius r and thickness t, is supported at the edge, and loaded with a concentrated load P, applied at a circumference, the radius of which is r_0 (fig. 22). The greatest stress is

$$f=\left(\tfrac{4}{3}\log.\frac{r}{r_0}+1\right)\frac{P}{\pi t^2}. \quad . \quad . \quad . \quad (32)$$

$\frac{r}{r_0}=$	10	20	30	40	50
$\frac{4}{3}\log.\frac{r}{r_0}+1=$	4·07	5·00	5·53	5·92	6·22

The above rules are due to Grashof.

IV. *Strength of stayed surfaces.*—A flat plate, of thickness t, is supported uniformly by stays arranged in lines (fig. 23). Distance of stays from centre to centre $=a$, uniform load $=p$ lbs. per sq. in. The greatest stress in the plate is

$$f=\tfrac{2}{9}\frac{a^2}{t^2}p \quad . \quad . \quad . \quad (33)$$

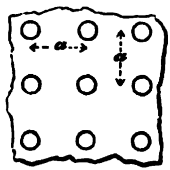


Fig. 23.

Each stay supports $p\,a^2$ lbs.

V. A rectangular plate, of thickness t, length l, and breadth b, is encastré at the edge, and loaded uniformly with p lbs. per sq. in. The greatest stress is

$$f=\tfrac{1}{2}\frac{l^4}{l^4+b^4}\frac{b^2}{t^2}p \quad . \quad . \quad . \quad . \quad . \quad (34)$$

VI. A square plate, s inches in length of side, is similarly supported and loaded. The greatest stress is

$$f=\tfrac{1}{4}\frac{s^2}{t^2}p \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

Some experiments by Mr. R. Wilson ('Engineering,' vol. xxiv. p. 239) appear to show that the ultimate resistance of flat plates is considerably greater than that obtained by putting $f=$ the breaking stress in the above formulæ. These formulæ are strictly applicable within the elastic limit only, and Mr. Wilson's plates may have been dangerously strained long before giving way. They did take a large set.

The Board of Trade rule for the flat stayed surfaces of marine boilers is as follows :—

Let p be the safe working pressure in lbs. per sq. in. ;

t the thickness of the plate in inches ; s the area of surface supported by one stay in sq. ins. ;

$$p=\frac{c\,(16t+1)^2}{s-6}.$$

The constant c has the following values :—

	Plates not exposed to fire	Plates with steam on one side and fire on the other	Plates with water on one side and fire on the other
Stays with nuts and washers three times the diameter of the stay, and $\frac{2}{3}$rds the plate thickness.	100	60	—
Stays with nuts only .	90	54	—
Stays screwed into plate and nutted . . .	—	—	80
Stays screwed into plate and riveted . . .	—	36	60

The strength of stays will be considered in the next chapter.

CHAPTER IV.

ON FASTENINGS.

Riveted Joints.

47. The simplest fastening is the rivet, employed to unite wrought iron, soft steel or copper plates. A rivet is virtually a bolt, with the head, body and nut in one piece. It is a permanent fastening, only removable by chipping off the head. Bolts are most often used with the straining force parallel to the axis, so that the bolt is in tension; but rivets are almost always placed at right angles to the straining force, so as to be in shear. They are not reliable in tension.

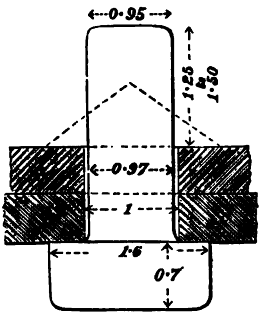


Fig. 24.

A rivet is formed of round bar, and, when ready for use, has the form shown in fig. 24. It is parallel for about half its length, and very slightly tapers for the remainder. The head is cup-shaped, or more often, pan-shaped, as shown. For iron plates, the rivets are of very soft uniform iron, and for steel plates they are of either iron or soft steel. They are made in rivet-making machines of various kinds, being pressed, while red hot, in suitable dies. When used, the rivets are again heated to red heat, placed in the rivet hole in the plates to be connected, and then the second head is formed by hand, or by

machine. In hand riveting, the tail of the rivet is held up, while the head is formed by two riveters working with hammers, and the head is either made conical by the hammers alone, or finished by the aid of a cup-shaped die, called a snap. In machine riveting, the rivet is pressed between two dies, actuated by a lever, or by steam or hydraulic pressure. Machine riveting causes the rivet to fill up the holes more perfectly than hand riveting, but is more liable to form the head eccentrically to the rivet. Steel rivets at first gave trouble from the injury to the quality of the steel in reheating the rivets. Hence for a time iron rivets were used with steel plates, although this involved a sacrifice of part of the advantage of using steel plates. Now steel rivets are generally used with steel plates. The rivet steel, however, is of a very soft quality, and its shearing resistance is not very much greater than that of rivet iron. Very good rivet iron or rivet steel may be riveted up cold. This process is often used when the plates are less than $\frac{3}{8}$ inch thick, as, for instance, in constructing gasometers.

Rivet holes are most commonly made by punching. This somewhat rough process is objectionable, on two grounds. The spacing of the rivet holes is not perfectly accurate, and the metal round the hole is more or less injured by the compression and lateral flow of the material, at the moment of punching. With very soft, ductile plates, it is believed that the injury done in punching is comparatively small, if the punch is sharp. But, with rigid plates, the injury is apparently serious, the plates being weakened 15 to 30 per cent. The injury is due, not to cracks formed in the plate, but to the pressure straining it beyond its limit of elasticity, and thus altering its homogeneous character and power of equal elongation under strain. The metal near the hole becomes more rigid than that farther off, and hence, when the joint is subjected to strain, the stress is not uniformly distributed on the metal between the rivet holes. Hence, steel plates should be

annealed after punching, or the holes should be punched smaller than the size of the rivet, and then enlarged by a cutting tool (such as a rymer) to the full size.[1]

The hole made by punching is slightly conical because the diameter of the hole in the die block is slightly larger than the punch, a clearance space of about $\frac{1}{32}$nd of the rivet diameter being left all round. Sometimes this conicity is entirely removed by rymering out the holes before riveting. If this rymering is done after the plates to be riveted are brought together it ensures the perfect agreement of the corresponding holes. The old plan of driving a conical drift into the rivet holes is an objectionable method of ensuring agreement, as it very imperfectly smoothes the rivet holes and to a certain extent injures the plates. In boiler and other work required to be staunch, the conicity of the holes is rather advantageous. The two plates are so punched that when brought together the smaller ends of the conical holes are inside the joint and the larger ends outside. The rivet then takes a form which holds the plates together. In boilers where one end of a plate is over and the other under the adjoining plate, some edges must be punched from one side of the plate, and the other edges from the other side. Sometimes the arris at the edge of rivet holes is removed by a countersink tool, and occasionally the countersink is of sensible depth, as in fig. 27. Riveting of this kind is sometimes used in shipbuilding, and it has the advantage that the rivet head is less likely to break off.

To obviate the objections to punching, the holes are sometimes drilled. The process of drilling is, in most cases, more expensive than punching, but the holes are more accurate in size and spacing. On the other hand, the sharp, square edge of a drilled hole appears to be unfavour-

[1] The practice at the present time appears to be this. Steel plates less than $\frac{1}{2}$ inch thick are punched, and are not generally annealed after punching. Plates $\frac{1}{2}$ inch to $\frac{3}{4}$ inch thick should be annealed after punching, or the holes rymered after punching. Plates more than $\frac{3}{4}$ inch thick should be drilled.

able to the resistance of the rivet. Sir W. Fairbairn showed that the resistance of the rivet was increased by slightly rounding the edges of the hole.

When the riveting is done at red heat, the contraction of the rivet, in cooling, nips the plates powerfully, and causes considerable tension on the rivet. In very long rivets, this may cause fracture of the rivet, and to prevent this the tail end is cooled before placing it in the rivet hole. In ordinary riveting, the contraction is advantageous in securing staunchness of the joint. Further, the contraction creates frictional resistance to slipping between the plates, which enables the joint to sustain a considerable force, even when the rivets do not fit the holes. The tension in the rivet may be estimated at 21,000 lbs. per square inch of its section, and the friction due to this would be about 7,000 lbs. per inch of rivet section. Experiments show a still greater friction, but if the tension in the rivet exceeds the elastic limit, its permanence cannot be relied on. English engineers entirely neglect the friction, in estimating the strength of the joint, the reasons assigned being that the amount of tension in the rivet is not ascertainable, and that vibrations and other causes, tending to slightly elongate the rivet, may, in course of time, destroy it altogether.

If the thickness of the plates, through which a rivet passes, is 6 inches or more, it is better to use bolts instead of rivets.

The staunchness of the joint, or its power of resisting the tendency to leak, when subjected to steam or water pressure, depends on the nearness of the rivets to the edge of the plate, and their nearness together. The metal between two rivets is in the position of a beam subjected to uniform pressure, and tending to deflect. If the joint is not naturally staunch, it may be rendered so by caulking, that is, burring down a narrow strip at the edge of the plate by a chisel (fig. 25). In fig. 25 the condition of the plates before caulking is shown at *a*. At *b* is a fullering tool used to close up the plates; at *c*, a caulking tool used to burr

down the edge of the plate. Mr. Webb does not use a caulking tool at all, but only a fullering tool having a small

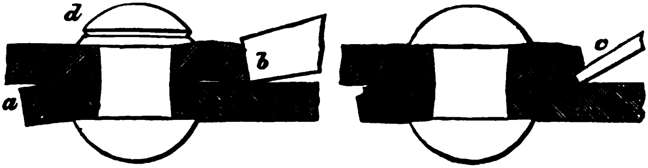


Fig. 25.

projection, which indents the plate at the middle of its thickness. In the best boiler work the plates are planed on the edges with a slight bevel, before riveting, and this much facilitates the closing of the joints by fullering or caulking. It is a point of importance in boiler work to arrange the joints so that they can be caulked. The rivet heads are sometimes caulked also, as shown in fig. 25, *d.*

Mr. Webb has found that the magnetic oxide on the surface of plates as they come from the rolling mill diminishes the staunchness of the joint. He has adopted the plan of sponging the surfaces of the joint with a solution of sal ammoniac before riveting. The plates then adhere more closely and require less caulking. He afterwards

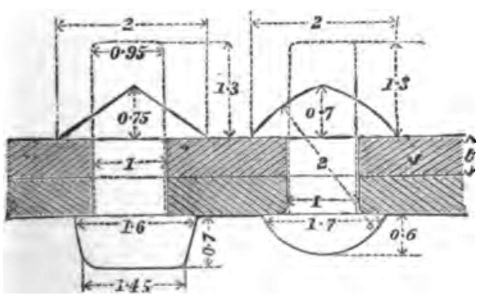


Fig. 26.

sponges the whole interior of the boiler with sal ammoniac to prevent irregular corrosion in working.

The experiments of the Manchester Steam Users' Association have shown that machine-riveted work is somewhat stronger than hand-riveted work.

48. *Forms of rivets.*—Fig. 26 gives the proportions of the rivets commonly used in hand-riveting, the heads formed by the riveter being of conical form. Fig. 27 gives proportions for the rivets generally used in machine riveting.

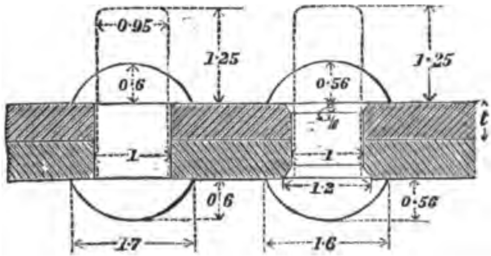


Fig. 27.

In boiler work the rivet head is rather larger than for girder work. Fig. 28 shows a countersunk rivet, which is only used when the surface of the plate must be fair and without projections. Countersunk rivets weaken the plate more, and are less reliable than ordinary rivets. The proportions of the head vary from 1·5 in diameter and 0·5 in height to 1·6 in diameter and 0·4 in height. The conical heads shown in fig. 26 are formed entirely by hand hammers, and are not finished with a snap. They are most used where there is restricted space for hammering, and are less reliable than cup or spherical heads. The cup-shaped head may be formed in hand riveting by a die or snap, which requires the use of a sledge hammer.

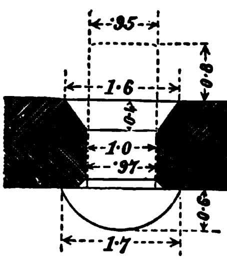


Fig. 28.

The numbers on the figures are proportional to the diameter of the rivet, and give good ordinary proportions, although it must be remembered that the sizes used by

different engineers vary more or less. To fill the rivet hole and form the head, a length equal to about $\frac{3}{4}$ of the diameter is required in countersunk riveting, and 1·3 to 1·7 times the diameter in ordinary riveting.

Lap and butt riveting.—When one plate is made to overlap the other, and one or more lines of rivets are put through the two, the riveting is lap riveting (fig. 31). When the plates are kept in the same plane, and a cover plate, or butt strap, is put over the joint and riveted to each, the riveting is butt riveting (fig. 32).

Single and double riveting.—If there is one line of rivets in lap riveting, or one line on each side of the joint in butt riveting, the joint is single riveted (figs. 31, 32). If there are two lines in lap, or two lines on each side of the joint in butt riveting, the joint is double riveted (fig. 33).

Single and double shear joints.—When the plates are so arranged that they tend to shear the rivets in a single plane (fig. 31), the joint is a single shear joint. If the plates tend to shear the rivet in two planes, the joint is a double shear joint; such a joint is shown in the lower section in fig. 32.

Combined lap and butt joint.—A form of joint intermediate between a lap and a butt joint has recently come

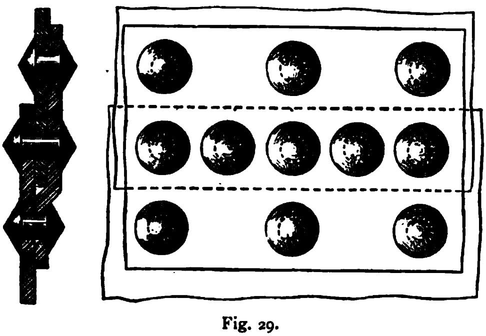


Fig. 29.

into use for locomotive boilers. It is shown in fig. 29. It consists of a lap joint with a cover plate outside the joint.

There are three rows of rivets, the middle row having twice as many rivets as the outside rows. The ordinary proportions for $\frac{3}{8}$-inch plates are: rivets $\frac{13}{16}$-inch diameter; pitch of middle row 2 inches; pitch of outside rows 4 inches.

49. *Size of rivets for plates of different thickness.*—Let t=thickness of plate, d=diameter of rivet, f_s=resistance of plate to shearing, f_c=resistance of punch to crushing. The area sheared by the punch is $\pi\, d\, t$, and the resistance to shearing is $\pi\, d\, t\, f_s$. The strength of the punch is $\frac{\pi}{4} d^2 f_c$. Hence, if

$$\pi\, d t f_s \text{ is greater than } \frac{\pi}{4} d^2 f_c,$$

$$\text{or if } d \text{ is less than } 4\, t \frac{f_s}{f_c},$$

the punch will crush before the plate shears. If $f_c = 4 f_s$, d must not be less than t, or the plate cannot be punched. To allow a margin of safety for the punch, the rivet diameter is rarely less than one and a half times the thickness of the plate, except when the plates are drilled. The diameter of rivets in practice ranges from

$$d = \tfrac{3}{4}\, t + \tfrac{3}{8} \text{ to } \tfrac{7}{8}\, t + \tfrac{3}{8}$$

and a very simple and convenient rule is

$$d = 1 \cdot 2 \sqrt{t} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

This rule will be adopted in the following calculations.

In practice the real diameter of the rivet in the joint is somewhat greater than its nominal diameter. The rivet hole is made about 4 per cent. larger in diameter than the rivet. In riveting up, the rivet is compressed so as to fill the rivet hole. Hence, in calculations on the strength of the joint the rivets should be taken about 4 per cent. larger than their nominal diameters. The following table gives the diameters of rivets calculated by this rule, the size to

the nearest sixteenth of an inch, and the probable real diameter of the rivet after riveting.

Diameters of Rivets for different thicknesses of Plates.

Thickness of Plates t.	Diameter of Rivets d.		Diameter of rivets after riveting $1{\cdot}04\,d$.
$\frac{1}{4}$	0·60	$\frac{9}{16}$	0·59
$\frac{5}{16}$	0·67	$\frac{11}{16}$	0·72
$\frac{3}{8}$	0·73	$\frac{3}{4}$	0·78
$\frac{7}{16}$	0·79	$\frac{13}{16}$	0·85
$\frac{1}{2}$	0·85	$\frac{7}{8}$	0·91
$\frac{9}{16}$	0·90	$\frac{7}{8}$	0·91
$\frac{5}{8}$	0·95	$\frac{15}{16}$	0·97
$\frac{3}{4}$	1·04	$1\frac{1}{16}$	1·10
$\frac{7}{8}$	1·12	$1\frac{1}{8}$	1·17
1	1·20	$1\frac{3}{16}$	1·24

50. *Overlap of plates and pitch of rivets.*—If the rivet hole is too near the edge of the plate, the latter is liable to be burst through in punching; and if the pitch of the rivets, or distance from centre to centre, is too small, a crack extends between them. A practical limit is thus fixed for the minimum overlap and pitch. A good rule is, that the distance from the edge of the rivet hole to the edge of the plate, or to the edge of the next rivet hole, should not be less than the diameter of the rivet. We thus get the proportions shown in fig. 30, as the *minimum* proportions of overlap and pitch. The unit is the diameter of the rivet.

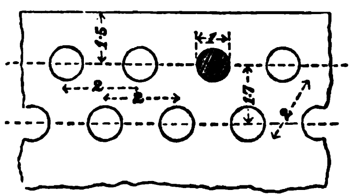


Fig. 30.

51. *Proportions of riveted joints.*—Fig. 31 shows an ordinary single-riveted lap joint. Fig. 32 a similar butt joint. The pitch p is determined by the rules given below. The other dimensions are given by the proportional numbers, the unit for which is the diameter of the rivet. The

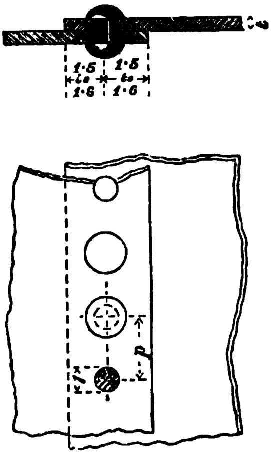


Fig. 31.
Unit=*d*.

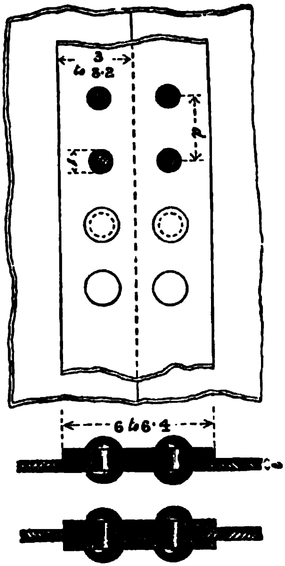


Fig. 32.

objection to a lap joint is, that the straining force in one plate

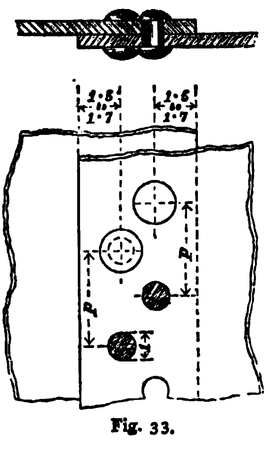


Fig. 33.

is not directly opposed to that in the other, but forms with it a couple tending to bend the joint. Grooving of boiler plates is, indirectly, due to this bending action. The objection also applies to the butt joint, with a single butt strap or cover, when in some positions. The butt joint, with two cover plates, is free from bending action. Butt joints are preferable to lap joints, for the longitudinal joints of boilers, and the cross joints are also sometimes made with a cover strip, welded into a ring and shrunk on.

It would appear sufficient when one butt strap is used, to make it equal in thickness to the plates connected; and when two are used, to make each of them half the thickness of the plates connected. In some experiments, butt straps of these proportions proved to be weaker than the plates. It seems therefore more prudent to make the single butt strap $1\frac{1}{8}$ times the plate thickness, and the double butt straps each $\frac{5}{8}$ the plate thickness. Butt straps should be cut from plates, and not made of rolled bars, which are weak when strained at right angles to the direction of rolling.

Fig. 33 gives the proportions of a double-riveted joint, the unit being as before the diameter of the rivet. The oblique distance from the centre of rivet in one row to the centre of rivet in the next row, may be equal to the pitch in single riveting, but in any case must not be less than $2\,d$. A butt joint would be similar, but would have four rows of rivets.

Resistance of Riveted Joints.

52. Let d=diameter of rivets after riveting.

p=pitch of rivets.

t=thickness of plates.

l=semi-overlap, or distance from centre of rivet to edge of plate.

f_t=tenacity of material of plates.

f_c=resistance to crushing of plates or rivets.

f_s=shearing resistance of rivets.

T=resistance of a strip of the joint of width p.

Modes of fracture of riveted joints.—Consider, for simplicity, a simple, single-riveted lap joint, subjected to tension. Since each rivet supports a strip of plate, whose width is p, we may consider such a strip,

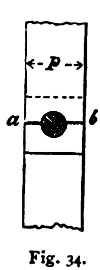


Fig. 34.

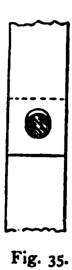


Fig. 35.

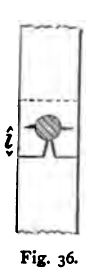


Fig. 36.

Fig. 37.

independently of the rest. Such a strip, subjected to tension, might fracture in four ways.

(1.) The plate may tear across, along the line of minimum section $a\,b$ (fig. 34). The area of either plate at $a\,b$ is $(p-d)\,t$, and the resistance to tension is $f_t\,(p-d)\,t$.

(2.) The plate and rivet may be crushed, as shown in fig. 35, and this will render the joint loose and insecure. The area of the plate or rivet supporting the pressure, estimated normally to the pressure, is $d\,t$, and this is called the bearing area. The resistance to crushing is $f_c d\,t$.

(3.) The plate may break across in front of the rivet (fig. 36), the action being similar to the transverse fracture of a bar, fixed at the ends, and loaded at the centre. The bending moment is about $\frac{1}{8}\,\mathrm{T}\,d$. Equating this to the moment of resistance of the section of the plate

$$\mathrm{T}=\frac{1}{3}\frac{(2\,l-d)^2\,t f_t}{d}.$$

(4.) The rivet may shear across (fig. 37). The area resisting shear is $\frac{\pi}{4}d^2$, and the resistance to shearing is $\frac{\pi}{4}d^2 f_s$.

53. *Condition that there may be the greatest economy of material in the joint.*—The condition which ensures the greatest economy of material is that the resistances of the strip to tearing, crushing, breaking or shearing, should be equal. Hence

$$\begin{aligned}
\mathrm{T}&=(p-d)\,t f_t \quad . \quad . \quad . \quad . \quad . \quad (2)\\
&=d\,t f_c \quad . \quad . \quad . \quad . \quad . \quad (3)\\
&=\tfrac{1}{3}\,(2\,l-d)^2 t f_t \quad . \quad . \quad . \quad . \quad (4)\\
&=\frac{\pi}{4}\,d^2 f_s \quad . \quad . \quad . \quad . \quad . \quad (5)
\end{aligned}$$

For a multiple riveted joint with n rows of rivets, there are n rivets to a strip of the width p. Consequently $\frac{\mathrm{T}}{n}$ must be

substituted for T in equations (3), (4), and (5). If the rivets are in double shear, equation (5) becomes—

$$T = \frac{\pi}{2} d^2 f_s \quad . \quad . \quad . \quad . \quad . \quad (5a)$$

54. *The tenacity of iron and steel used for riveted work.*—Specimens of iron and steel suitable for riveted work of a length of 4 to 10 inches (exclusive of the ends held in the machine) break in the testing machine with the tensions given in the following table :—

	Tenacity in lbs. per sq. inch.
Iron plates	42,000 to 50,000
Steel plates	58,000 to 66,000
Rivet iron	62,000
Rivet steel	66,000

These tenacities will be termed the original tenacities to distinguish them from the tenacities after the plates have been drilled, punched, or riveted.

Tenacity of drilled plates.—When a row of holes is drilled in a plate and it is then tested, it is forced to break along the line of holes where the section is least. This line will be termed the line of fracture, and is marked *ff* in some of the woodcuts. At first sight it would appear that the tenacity (or stress per unit of section of metal along the line of fracture) should be identical with the original tenacity of the undrilled plate. The balance of experimental evidence is, however, that the tenacity of the drilled plate is 10 or 12 per cent. greater than this. In breaking an ordinary test bar the load causes fracture at the weakest section of a more or less considerable length of bar. In the case of a drilled bar no such selection of a weak section is possible. Besides this, it is probable that the form of the short portions of metal between the holes is such, that the flow of the material during the last stages of testing is more hindered than in an ordinary test bar, and that hence there may be a diminished contraction of area during fracture, and conse-

quently a real gain of strength to resist fracture in the drilled plate.

Tenacity of punched plates.—The experiments on the tenacity of punched plates are extremely discordant. A number of experiments show a loss of tenacity after punching varying from 5 per cent. up to 20 per cent. in iron plates, and from 8 per cent. to 35 per cent. in steel plates. With steel plates the loss increases with the thickness of the plate; with both steel and iron it is diminished by making the hole in the die block one-fourth greater in diameter than the punch; and the loss completely disappears, and the original tenacity is restored, if the plate is annealed after punching; or if a small ring, 0·04 to 0·08 inch in thickness, is rymered out of the punched hole.

There can be no doubt that, in punching, a portion of metal is squeezed laterally into the plate, and a condition of permanent stress is induced in the metal immediately surrounding the hole. It is this which is got rid of by annealing or rymering.

On the other hand, many of the experiments on the effect of punching have been made in an imperfect way. A narrow strip has been prepared for testing, and the hole then punched in it. In that case the material may be greatly injured by the lateral expansion of the unsupported material on each side of the hole. Some experiments on steel plates, in which a wide plate was punched and afterwards slotted into testing strips, showed a tenacity 5 per cent. less than that of similar drilled strips, but still 5 per cent. greater than the original tenacity of the plate.

There are a number of experiments on riveted joints which give so low a tenacity for the net section of the joint, that it is impossible not to suspect some error in the experiments. The Manchester Steam Users Association made some experiments on the bursting of an actual boiler by hydraulic pressure, and afterwards broke some test specimens of the joints in a testing machine. The stress on the boiler joints at the moment of bursting was 18 or 19 tons per sq.

inch, or very nearly the original tenacity of the material. The test specimens, on the other hand, broke with about 13 tons. There is no known reason why there should be so great a difference in the breaking weight in these two cases.

55. *Tenacity of the metal in riveted joints.*—When a portion of riveted joint is tested, further conditions affect the tenacity estimated on the section of fracture. The friction of the plates, if it exists, resists the breaking force and adds to the strength of the joint. But since a considerable relative movement of the plates occurs before fracture, and the rivets are greatly distorted, it is probable that the influence of friction is small or zero. Putting this aside, in most forms of joints, the resultant of the load does not pass through the middle of the plates, and hence the joint bends as well as stretches. The real stress is then greater than the quantity load divided by area of section, and the strength of the joint is diminished.

Let P be the breaking load of a riveted joint which has ω sq. ins. of section, through the weakest section or line of fracture. Then $\frac{P}{\omega}$ is the *apparent tenacity* of the joint, being less than the real tenacity because the stress due to bending is neglected. In general $\frac{P}{\omega}$ will be less than the original tenacity (§ 54) of the material of the joint f_t, although in some cases it may be greater, for the reasons mentioned in discussing the tenacity of plates with drilled holes. Let $\frac{P}{\omega} = kf_t$. Then k is a coefficient depending on the way in which the holes are made and the form of the joint. The following are values of k from various experiments on joints.

IRON PLATES.

	Values of k.
Single-riveted joints, drilled	·88
,, ,, punched	·77
Double-riveted joints, drilled	·95
,, ,, punched	·85

STEEL PLATES.

Single-riveted, drilled	1·00
,, ,, punched	·90
Double-riveted, drilled	1·06
,, ,, punched	1·00
Treble-riveted, drilled	1·08

These numbers must be considered as approximations only, experiments giving more or less discordant values. It is to be understood, also, that a plate rymered or annealed after punching is equal in strength to a drilled plate.

Apparent tenacity of iron and steel in different kinds of joints.—Taking the average original tenacity of iron plates at 46,000 lbs. per sq. in., and that of steel plates at 62,000 lbs. per sq. in., the probable apparent tenacity of different joints will be as follows :—

	Apparent tenacity in lbs. per sq. in. f_t Iron Plates.	Steel Plates.
Single-riveted, drilled	40,500	62,000
,, ,, punched	35,400	55,800
Doubled-riveted, drilled	43,700	65,700
,, ,, punched	39,000	62,000
Treble-riveted, drilled	45,000	67,000

56. *Shearing resistance of iron and steel.*—The shearing resistance of iron and steel bars, when sheared in a testing machine, is found to be very constantly $\frac{4}{5}$ths of their tenacity.

	Shearing resistance in lbs. per sq. in.
Rivet iron	49,600
Rivet steel	52,800

57. *Apparent shearing resistance of rivets in riveted joints.*—When a riveted joint gives way by shearing, the apparent shearing stress (load divided by sectional area of rivets) is different from the shearing resistance of the rivets in consequence of the friction and bending, and perhaps from the obliquity of the section at which the rivet shears. It also appears that the sharp edge of a drilled hole causes the rivet to shear with a rather less stress than the blunter edge of a punched hole. The shearing resistance of iron rivets is

given pretty definitely in experiments on riveted joints; but in experiments on steel joints with steel rivets, the shearing resistance varies very greatly. The following table gives values selected from the experiments which appear most reliable.

	Apparent shearing resistance of rivets in riveted joints f_s
Iron rivets, in punched holes . . .	46,000
,, ,, drilled holes . . .	43,000
Steel rivets, in punched holes . . .	53,000
,, ,, drilled holes . . .	49,000

58. *Ratio of apparent tenacity of plates to shearing resistance of rivets.*—Using the values of the apparent tenacity and shearing resistance given above, the following values of the ratio of resistance to tearing and shearing in different kinds of joints are obtained.

Ratio of Tearing and Shearing Resistance $\frac{f_t}{f_s}$, *in riveted joints.*

	Iron plates, Iron rivets		Steel plates, Steel rivets	
	Drilled, or punched and annealed or rymered	Punched	Drilled, or punched and annealed or rymered	Punched
Single-riveted .	0·94	0·77	1·26	1·05
Double-riveted .	1·02	0·85	1·34	1·17
Treble-riveted .	1·05	—	1·36	—

59. *Ratio of section of plates to section of rivets.*—In most tables of the pitch of rivets which have been published, a fixed ratio has been assumed for the tearing and shearing resistance in all forms of joints. In many of these, for instance, it has been assumed that the shearing and tearing resistances are equal. Any examination of experiments on riveted joints will, however, show that the ratio of the tearing to the shearing resistance varies considerably. Now a joint will be strongest when the areas of

plate in the line of fracture and of the rivets in the plane of shearing are such that the joint will give way by shearing or tearing indifferently. Putting ω_s ω_t for the shearing and tearing areas,

$$\omega_s f_s = \omega_t f_t$$

$$\frac{\omega_s}{\omega_t} = \frac{f_t}{f_s}.$$

For if the shearing section is in excess of the amount necessary to balance the tearing section, the rivets are larger than necessary, and the section of fracture is diminished; and if the tearing section is in excess, the rivets will give way before the whole strength of the plate is called into action. Consequently, the shearing and tearing sections should be in the inverse ratio of the apparent shearing and tearing resistances; and if these latter vary for different kinds of joints, so also should the ratios of the sections.

Experiment shows that the shearing and tearing resistances vary a good deal with the quality of the iron and steel plates, the care taken in punching or drilling the rivet holes, and the quality of the rivet iron and rivet steel. Especially, the ratio $\frac{f_t}{f_s}$ is greater for steel than iron, and sensibly greater with drilled than with punched holes. Judgment must be used in determining what that ratio is in given cases, but the values given above may be taken as probable average values. Where special experiments on the materials used cannot be made, the values of $\frac{f_t}{f_s}$ given above may be taken as the best values of the ratio $\frac{\omega_s}{\omega_t}$ in designing a joint.

60. *Theoretical overlap.*—Taking the diameter of a rivet at the value given in (1) (§ 49), and eliminating t in the value for the breaking resistance of the portion of plate in front of a rivet in (§ 52), we get

$$\mathrm{T} = 0{\cdot}23(2l-d)^2 \, d f_t.$$

If the portion of plate is to be at least as strong as the rivet,

$$0{\cdot}23(2l-d)^2\, d f_t \overline{\overline{>}}\, 0{\cdot}785\, d^2 f_s$$

$$l \overline{\overline{>}}\, 0{\cdot}925 \sqrt{d} \sqrt{\frac{f_s}{f_t}} + \frac{d}{2}.$$

Now the largest value of $\frac{f_s}{f_t}$, from the table above, is for iron 1·3, and for steel 0·85. Inserting these values,

$$l \overline{\overline{>}}\, 1{\cdot}05\sqrt{d}+\frac{d}{2} \text{ for iron}$$

$$\overline{\overline{>}}\, 0{\cdot}85\sqrt{d}+\frac{d}{2} \text{ for steel}$$

$d=$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{4}$
For iron $l=$	1·00	1·14	1·29	1·41	1·55	1·67	1·80
For steel $l=$	0·86	0·98	1·12	1·22	1·35	1·46	1·57
$1{\cdot}5d$ $=$	0·75	0·94	1·12	1·31	1·50	1·69	1·88

The ordinary practical rule (§ 50) is to make $l=1{\cdot}5d$, and values of $1{\cdot}5d$ are given for comparison. It will be seen that for the smaller diameters of rivets it would be desirable to increase a little the overlap assigned by the ordinary practical rule. For the larger diameters of rivets an overlap of $1{\cdot}5d$ is amply large enough.

61. *Theoretical rivet diameter for equal crushing and shearing resistance.*—The value of the crushing pressure f_c at which a joint begins to yield is entirely unknown. With all joints broken in the testing machine, and especially with steel joints, the rivet holes are found to have become elliptical, and the rivets to be deformed before fracture. In some experiments, it appeared that this crushing action had the effect of diminishing the apparent tenacity of the plates or the apparent resistance of the rivets. By making the rivets small and numerous the crushing stress can be reduced. Hence, it has been proposed as desirable that the rivet diameter should be so determined that the crushing stress should be below the limit found to be in-

jurious. If f_c is that limit, then from equations (3) and (5) for rivets in single shear—

$$f_c\, d\, t \overline{\overline{>}} \frac{\pi}{4} d^2 f_s$$

$$d \overline{\overline{<}} 1{\cdot}27 \frac{f_c}{f_s} t.$$

Similarly for rivets in double shear—

$$d \overline{\overline{<}} 0{\cdot}635 \frac{f_c}{f_s} t.$$

The value of the crushing stress f_c, which produces injury to the tenacity or shearing resistance of the joint, is very uncertain. In the case of steel joints there is no indication of injury with crushing pressures of 50 tons per sq. inch.

If $\frac{f_c}{f_s} = 2$, then

$d \overline{\overline{<}} 2{\cdot}54\, t$ for rivets in single shear

$\overline{\overline{<}} 1{\cdot}27\, t$ for rivets in double shear.

Then, with the ordinary proportions of rivets, the crushing action will in no case need to be considered in single-shear joints, and only in double-shear joints when the plates are less than $\frac{7}{8}$ inch thick. On the whole the author is inclined to believe that the importance of crushing action has been exaggerated, and it will be assumed in the following calculations that crushing action is sufficiently provided for with the ordinary proportions of diameter of rivet to thickness of plate, and that no special attention needs to be paid to it.

62. *Pitch of single-riveted joints.*—Equating the tearing and shearing resistance (§ 52),

$$T = (p-d)\, t f_t = 0{\cdot}785\, d^2 f_s$$

$$p = 0{\cdot}785 \frac{d^2}{t} \cdot \frac{f_s}{f_t} + d. \qquad (6).$$

From this formula values of the pitch for single riveted joints have been calculated, for values of $\frac{f_t}{f_s}$ ranging from 0·75 to 1·35. The table above (§ 58) will be a guide as to the most probable value of $\frac{f_t}{f_s}$ in differently made joints, and the headings of the following tables have been made to agree with that table.

Single Riveting.

Thickness of plates	Nominal diam. of rivets	Real diam. of rivets	Iron Rivets: Iron punched plates		Iron Rivets: Iron drilled or punched and annealed or rymered plates		Steel Rivets: Steel punched plates		Steel Rivets: Steel punched and annealed or rymered plates	
			Pitch of rivets for values of $\frac{ft}{fs}=$							
			·75	·85	·95	1·0	1·05	1·15	1·25	1·35
$\frac{5}{16}$	$\frac{11}{16}$	·72	2·46	2·25	2·09	2·02	1·96	1·85	1·77	1·69
$\frac{3}{8}$	$\frac{3}{4}$	·78	2·48	2·28	2·12	2·06	1·99	1·89	1·81	1·72
$\frac{7}{16}$	$\frac{13}{16}$	·85	2·58	2·38	2·22	2·15	2·09	1·98	1·90	1·81
$\frac{1}{2}$	$\frac{7}{8}$	·92	2·69	2·48	2·32	2·25	2·19	2·08	2·00	1·90
$\frac{5}{8}$	$\frac{15}{16}$	·98	2·59	2·40	2·25	2·19	2·13	2·03	1·95	1·87
$\frac{3}{4}$	$1\frac{1}{16}$	1·10	2·79	2·59	2·43	2·37	2·31	2·20	2·12	2·04
$\frac{7}{8}$	$1\frac{1}{8}$	1·17	2·81	2·62	2·46	2·40	2·34	2·24	2·16	2·08
1	$1\frac{1}{4}$	1·30	3·07	2·86	2·70	2·63	2·56	2·45	2·36	2·28

63. *Pitch of double-riveted joints.*—For double-riveted joints, there are two rivets to each strip of a width equal to the pitch. Hence

$$T=(p-d)\,t f_t=\frac{\pi}{2}\,d^2 f_s.$$

$$p=1{\cdot}57\,\frac{d^2 f_s}{t\,f_t}+d. \quad . \quad . \quad . \quad (7)$$

From this formula the following table has been computed, for values of $\frac{f_t}{f_s}$ ranging from 0·85 to 1·35.

Double Riveting.

Thickness of plates	Nominal diam. of rivets	Real diam. of rivets	Iron Rivets: Iron punched plates	Iron Rivets: Iron drilled or punched and rymered plates		Steel Rivets: Steel punched plates	Steel Rivets: Steel drilled or punched and rymered plates
			Pitch of rivets for value $\frac{f_t}{f_s}=$				
			·85	1·00	1·10	1·20	1·35
$\frac{5}{16}$	$\frac{11}{16}$	·72	3·78	3·33	3·12	2·91	2·66
$\frac{3}{8}$	$\frac{3}{4}$	·78	3·78	3·33	3·12	2·91	2·66
$\frac{7}{16}$	$\frac{13}{16}$	·85	3·91	3·45	3·24	3·03	2·77
$\frac{1}{2}$	$\frac{7}{8}$	·92	4·05	3·58	3·37	3·16	2·88
$\frac{5}{8}$	$\frac{15}{16}$	·98	3·82	3·39	3·18	3·00	2·76
$\frac{3}{4}$	$1\frac{1}{16}$	1·10	4·08	3·63	3·42	3·22	2·98
$\frac{7}{8}$	$1\frac{1}{8}$	1·17	4·06	3·63	3·42	3·23	2·99
1	$1\frac{1}{4}$	1·30	4·42	3·95	3·74	3·52	3·26

64. *Graphic method of designing joints.*—Schwedler has introduced a mode of designing joints which is particularly useful for irregular or complicated joints. The width w of a strip of plate of a strength equivalent to that of one rivet is given by the equation

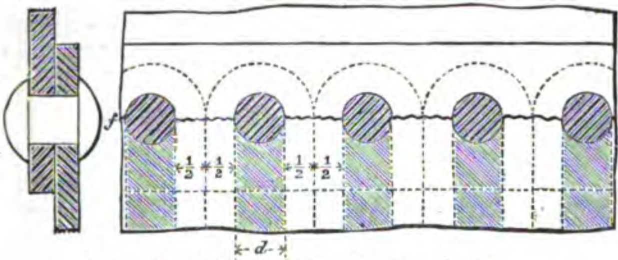
$$w\,t f_t = \frac{\pi}{4} d^2 f_s$$

the rivets being in single shear. Then

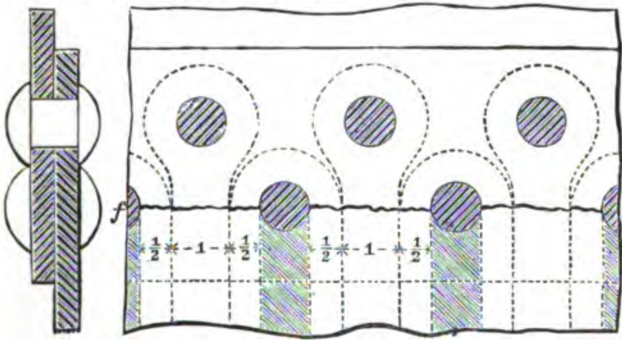
$$w = 0{\cdot}785 \frac{d^2}{t} \cdot \frac{f_s}{f_t} \quad . \quad . \quad . \quad (8)$$

Suppose d and t given, and the value of $\frac{f_t}{f_s}$ also given or taken from the table above. Then w can be calculated. If round each rivet a circle is drawn of diameter $d+w$, and from these circles lines are drawn cutting up the plate into strips of width w, a portion of plate of sufficient strength will have been allotted to each rivet, and any redundant portions will indicate useless material in the joint. The following figures show the application of this method to some forms of joint,

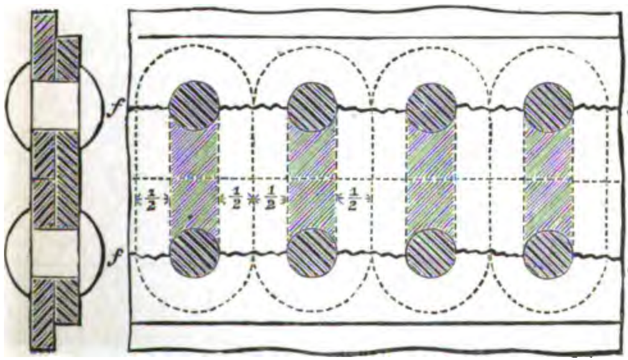
A. Single Riveted Lap Joint.



B. Double Riveted Lap Joint.



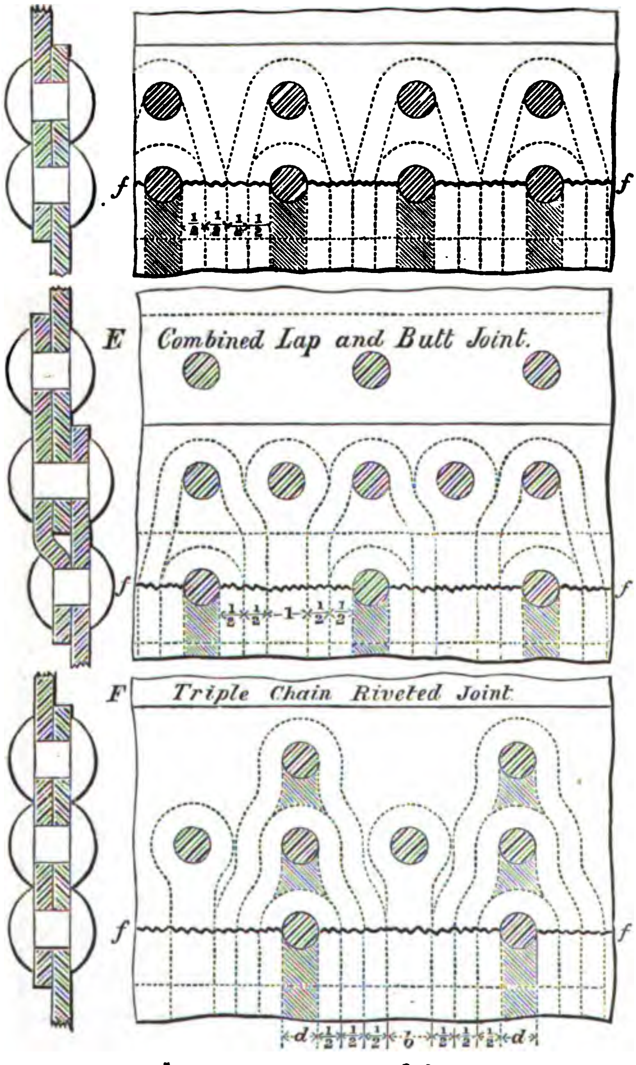
C. Cover Plate Single Riveted Butt Joint.



Unit w. ff, Line of fracture.

Fig. 38.

D. Double Chain Riveted Lap Joint.



Unit w. ***ff Line of fracture.***

Fig. 39.

the shaded portions being parts of the plate which do not add to the strength of the joint.

In fig. 38, A is a single-riveted and B a double-riveted joint so arranged that the shearing and tearing resistances are equal. The shaded portions represent metal unavoidably wasted, because the joint will give way at the line of fracture *ff*. C is the cover strip of a butt joint similarly designed.

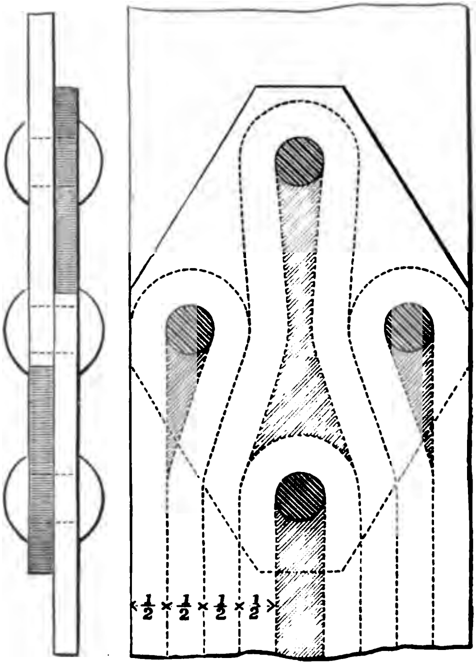


Fig. 40. Unit = *w*.

Fig. 39 shows joints of a more complicated construction. Fig. 40 shows the same method applied in designing a joint in a tie bar. As to the distance apart of the rows of rivets, no specific rule can be given, but it is rational to suppose that the strips carrying the load due to each rivet should not be too sharply bent.

65. *Efficiency of riveted joints.*—The efficiency of a joint is the ratio of the strength of the joint to the strength of an equal width of solid plate. Suppose the joint is properly designed, so that the shearing strength of the rivets is not less than the tearing resistance of the plates. Then, if there were no alteration of the tenacity of the plates by punching or drilling in making the joint, or by bending or crushing action under the load, the efficiency would be simply

$$\eta = \frac{p-d}{p}.$$

But since the apparent tenacity of the joint differs from the original tenacity of the plate in most cases, the true efficiency is

$$\eta = k\frac{p-d}{p}, \quad . \quad . \quad . \quad (9)$$

where k has the values given in the table above.

The values of η for a series of single and double riveted joints are given in the following table. They exactly correspond to the tables of pitches of joints given above, and the values of k are those already given for iron and steel plates. The efficiencies here given are slightly greater than those found in the most reliable experiments. But then in most experiments the joints have not had precisely the best proportion of shearing and tearing area.

Single Riveting.

Thickness of plates	Nominal diam. of rivets	Real diam. of rivets	Iron punched plates		Iron drilled or punched and annealed or rymered plates		Steel punched plates		Steel punched and annealed or rymered plates	
			Efficiency η of joints for values of k=							
			·77		·88		0·9		1·0	
$\frac{5}{16}$	$\frac{11}{16}$	·72	·55	·52	·58	·56	·57	·55	·59	·57
$\frac{3}{8}$	$\frac{3}{4}$	·78	·53	·51	·55	·54	·55	·53	·57	·55
$\frac{7}{16}$	$\frac{13}{16}$	·85	·52	·49	·55	·54	·53	·51	·55	·53
$\frac{1}{2}$	$\frac{7}{8}$	·92	·51	·49	·52	·52	·52	·50	·54	·52
$\frac{5}{8}$	$\frac{15}{16}$	·98	·48	·45	·49	·48	·49	·47	·50	·48
$\frac{3}{4}$	$1\frac{1}{16}$	1·10	·47	·44	·48	·47	·47	·45	·48	·46
$\frac{7}{8}$	$1\frac{1}{8}$	1·17	·45	·42	·46	·45	·45	·43	·46	·44
1	$1\frac{1}{4}$	1·30	·42	·40	·46	·45	·45	·43	·45	·43

Double Riveting.

Thickness of plates	Nominal diam. of rivets	Real diam. of rivets	Iron punched plates	Iron drilled or punched and rymered plates		Steel punched plates	Steel drilled or punched and rymered plates
			Efficiency η of joints for values of $k=$				
			·85	·95	1·00	1·00	1·06
$\frac{5}{16}$	$\frac{11}{16}$	·72	·69	·74	·77	·75	·77
$\frac{3}{8}$	$\frac{3}{4}$	·78	·68	·73	·75	·73	·75
$\frac{7}{16}$	$\frac{13}{16}$	·85	·66	·71	·74	·72	·74
$\frac{1}{2}$	$\frac{7}{8}$	·92	·65	·70	·73	·71	·73
$\frac{5}{8}$	$\frac{15}{16}$	·98	·63	·67	·69	·67	·69
$\frac{3}{4}$	$1\frac{1}{16}$	1·10	·62	·66	·68	·66	·68
$\frac{7}{8}$	$1\frac{1}{8}$	1·17	·60	·65	·66	·64	·65
1	$1\frac{1}{4}$	1·30	·60	·63	·65	·63	·64

66. *Breaking strength of a riveted joint.*—The breaking strength of any riveted joint designed with the proportions given above is easily calculated from the efficiency. Let f_t be the breaking stress of the solid plate per sq. in. of gross area. Then the strength of a structure with riveted joints is the same as if it were formed of solid plate of a tenacity ηf_t. Or if t is the thickness of the plates, then the riveted structure is as strong as if it were composed of solid plate of a thickness ηt.

Working strength of a riveted structure.—In bridge work and boiler work, the working stress is usually $\frac{1}{4}$th to $\frac{1}{6}$th of the breaking stress.

67. *Cold riveting for thin plates.*—For thin plates the rivets are hammered up cold. With plates $\frac{1}{8}$ inch to $\frac{3}{16}$ inch, the rivets are $\frac{1}{4}$ inch to $\frac{5}{16}$ inch diameter and $\frac{7}{8}$ inch pitch. The half width of overlap is about $\frac{1}{2}$ inch, and the staunchness of the joint is secured by laying between the plates, in a zigzag direction round the rivets, a string smeared with red lead.

68. *Plates with thickened edges.*—To obviate the loss of strength at the riveted joints of boilers, plates with thickened

edges have been used. Let t=thickness of plate, t_1=thickness of edge. Then if $t_1 = t \frac{p}{p-d}$ the joint will be as strong as the solid plate. The joint must be designed, as if the plate were t_1 inches thick. Only two edges of the plate are usually thickened, and these are placed, so as to form the longitudinal joint, which is subjected to the greatest strain.

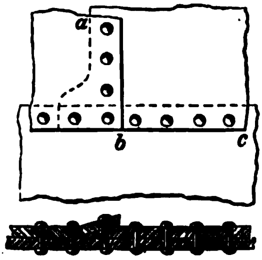


Fig. 41.

69. *Junctions of more than two plates.*—In boiler work, and other cases where the seams are to be watertight, a difficulty arises, where the cross joints intersect the longitudinal joints, because there three plates overlap. At those joints the edges of one or more plates are thinned out by forging, so that the joint may be solid throughout. Fig. 41 shows a joint, at which three plates overlap. The middle plate, $a\,b\,c$, is thinned out. Fig. 42 shows a four-plate connection, where each of the two interior plates

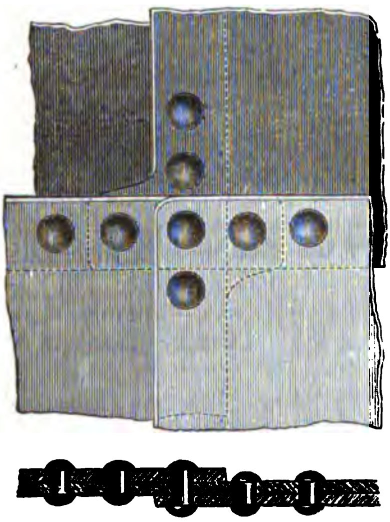


Fig. 42.

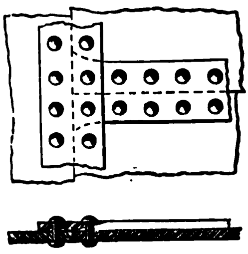
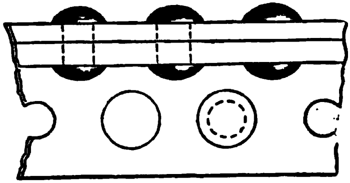


Fig. 43.

thins out at the corner. It will be seen that the forged end is so lengthened as to be supported by an additional rivet in the thin part.

A somewhat similar case arises, when boilers are butt riveted, in dealing with the covering strips, at the points where the cross and longitudinal joints meet. In the best boiler work, the plates are planed at the edges, and fitted together accurately. The longitudinal joints are then riveted up, the covering strip being thinned out at the end of the joint. Lastly, the circumferential covering strip is welded into a hoop of the exact size, and shrunk over the cylinder formed by the plates. Fig. 43 shows the junction of the covering strips.



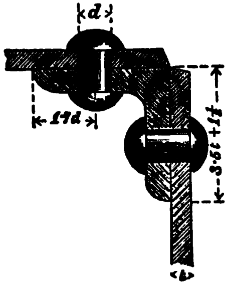


Fig. 44.

70. *Connection of plates not in one plane.*—This is commonly effected by the use of a kind of angular joint strip, called an *angle iron*. These angle irons are rolled of a great variety of sizes, and are of very great service in all descriptions of wrought-iron work. Fig. 44 shows an angle iron joint. No very definite rule can be given for the size of angle iron to be used, but generally the mean thickness of the angle iron is about equal to, or a little greater than, that of the plates to be connected. The width of each flange of the angle iron may be about four times the diameter of the rivets used, or may be $3{\cdot}25\,t + 1{\cdot}5$, where t is the thickness of the plates. The angle iron usually tapers, so that it is rather

thicker at the root than at the point. In bridge work, where the angle irons are used to confer stiffness, as well as strength, they are often heavier.

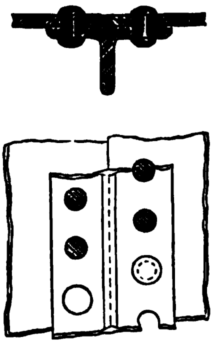


Fig. 45.

Fig. 45 shows a T iron joint, the object being to stiffen the plates against flexure.

Fig. 46 shows methods of connecting plates by flanging the plates themselves, instead of using angle irons. This is more expensive, and is impracticable when the plates are not of good quality. The curvature should not be too sharp. The inside radius may be, at least, four times the thickness of the plates. The width of overlap must be, at least, three times the diameter of the rivet.

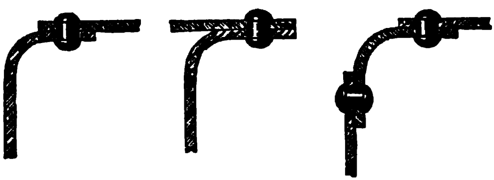


Fig. 46.

Figs. 47, 48, show joints used at the junction of the

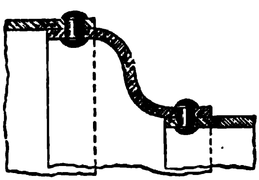


Fig. 47.

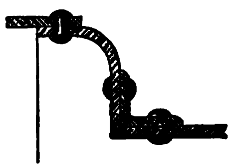


Fig. 48.

cylindrical barrel of locomotive boilers with the external fire-box.

71. *Connection of parallel plates.*—A case which frequently occurs, is where two plates, near together, require to be connected. For instance, at the bottom of the fire-box of locomotives, a connection has to be made between the inner and outer fire-box. The following sketches show how this may be effected.

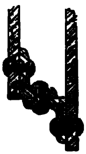
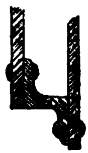
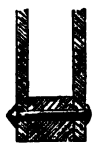
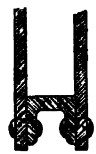
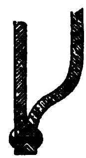






Fig. 49. Fig. 50. Fig. 51. Fig. 52. Fig. 53.

In fig. 49 there are two angle irons. · This is rather complicated, and there are inside joints, which cannot be caulked. Fig. 50 is simpler, but has an inside joint, which cannot be caulked. Fig. 52 is an admirable joint, and is formed by what is termed a channel iron. But it is difficult to bend the channel iron round the corners of the fire-box. Fig. 53 is simple, but forms a corner for the lodgment of sediment. Fig. 51 is the form most commonly used.

Elliptical rivets.—Since the efficiency of the joint is the ratio $\frac{p-d}{p}$ of the distance between the holes to the pitch, we may increase the efficiency, by using rivets of elliptical section. With such rivets, placed with their least breadth in the line of fracture of the plates, the quantity $p-d$ would be greater, while the shearing section remained the same. Such rivets have been used by Mr. Webb. By adopting the elliptical form, two variables, the axes of the ellipse, take the place of the single variable d, in the equations. It would thus be possible to satisfy the conditions of equal bearing, tearing, and shearing resistance, for rivets of any desired section.

72. *Position of rivets in tie bars and struts.*—When a bar, subjected to a longitudinal straining force, is attached at each end by a single rivet or pin, the rivets should be

placed on the centre line of the bar. It is a fair assumption, and must be nearly true, that the straining force acts through the centre of the rivet. Hence, if the rivets are in the centre line of the bar, the resultant straining force passes through the axis of the bar, and the stress on each transverse section is uniform. If the rivets are not so placed, one side of the bar is more strained than the other, and gives way before the other has fully exerted its powers of resistance. When there are several rivets at each end of a bar, they should, for the same reason, be placed symmetrically on either side of the axis, and as uniformly distributed as possible over the area in which they are placed. If they cannot be placed symmetrically, an approximation is made to the best conditions, by arranging them, so that their common centre of gravity falls on the axis of the bar. In that case, if each rivet supports the same fraction of the load, the resultant force will still pass through the axis of the bar.

73. *Cylindrical riveted structures.*—A cylindrical vessel made of numerous plates may be formed of a series of cylindrical rings alternately larger and smaller, so that each alternate ring can be slipped inside the others. Then if D is the diameter of the smaller rings, $D+2t$ is that of the larger ones. A second plan is to make the rings of equal diameter, and to use a butt strap over the joints. A third plan, common in boiler flues, is to make the rings conical, the diameters being D at one end, and $D+2t$ at the other. The rings are then slipped into each other, and the joints should be so placed that the flame does not directly strike the edges of the plate.

The strength of a cylindrical vessel to resist bursting has been given in eq. 2, § 26. Let D be the diameter of the boiler shell, t the thickness of the plates, f the tenacity of the plates, η the efficiency of the riveted joints. Then the pressure required to burst the boiler is

$$p=\frac{2\eta t f}{D} \quad . \quad . \quad . \quad (10)$$

and the working pressure should not exceed one-fifth of the bursting pressure. The rules for cylinders resisting an external collapsing pressure are given in §§ 40, 41.

Working strength of solid wrought-iron cylinders to resist internal pressure.

Thickness of cylinders 1 inch. Working stress taken at 7,600 lbs. per sq. in. for wrought iron, and 10,600 lbs. per sq. in. for steel.

Diameter Inches	Working Pressure in lbs. per sq. in. (Excess of internal over external pressure)	
	Iron	Steel
12	1267	1767
18	845	1177
24	633	884
30	507	707
36	422	589
42	362	505
48	317	463
54	282	393
60	253	354
66	230	321
72	211	294
78	195	272
84	181	252
90	169	235
96	158	221
102	149	208
108	141	196

For any other thickness of plate multiply the pressures in this table by that thickness. If the cylinder is riveted, multiply also by the value of η in the tables above, corresponding to the kind of joint used.

Thus suppose a boiler is 6 feet 6 ins. in diameter and constructed of $\frac{3}{8}''$ iron plates. From the table, a solid or welded iron shell of that thickness should carry $195 \times \frac{3}{8} = 73$ lbs. per sq. in. pressure. If, however, the shell is single-riveted, with punched rivet holes, $\eta = \cdot 51$ to $\cdot 53$, say $\cdot 52$. Then the safe working pressure is $0 \cdot 52 \times 73 = 38$ lbs. per sq. in. With steel plates and drilled holes, single-riveted $\eta = \cdot 55$ to $\cdot 57$, say $\cdot 56$. Then the safe working pressure would be

$272 \times \frac{3}{8} \times 0{\cdot}56 = 57$ lbs. per sq. in. With double-riveted joints, the working pressure would be $195 \times \frac{3}{8} \times 0{\cdot}68 = 50$ lbs. for iron punched plates; and $272 \times \frac{3}{8} \times {\cdot}75 = 76$ lbs. per sq. in. with steel drilled plates.

74. *Taper and curvature of boiler plates.*—When a boiler, boiler flue, or other cylindrical structure, is made up of

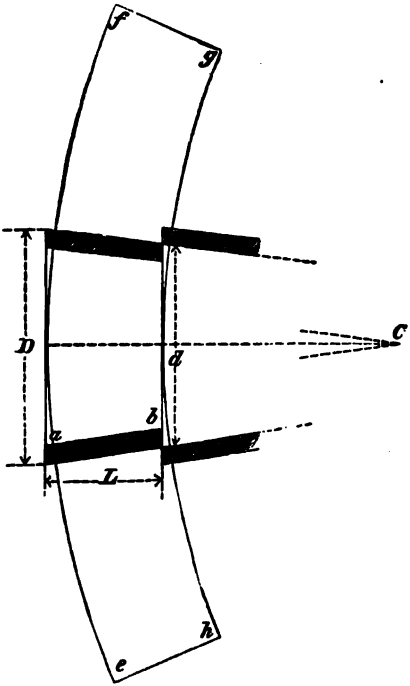


Fig. 54.

slightly conical rings, which are slipped over each other to form the overlap, fig. 54, the joints being what are technically termed 'following' joints, the plates, instead of being rectangular, must be portions of the development of a cone.

Let D be the greater, and d the less, diameter of the conical frustum, and L its length ; t, the thickness of the plates. Then $d = D - 2t$ very nearly. The development of the frustum is an annular segment $efgh$, drawn with radii, $R = Ca = Ce$, and $r = Cb = Ch$, and whose lengths, measured along the arcs ef and hg, are πD and πd. Since the incli-

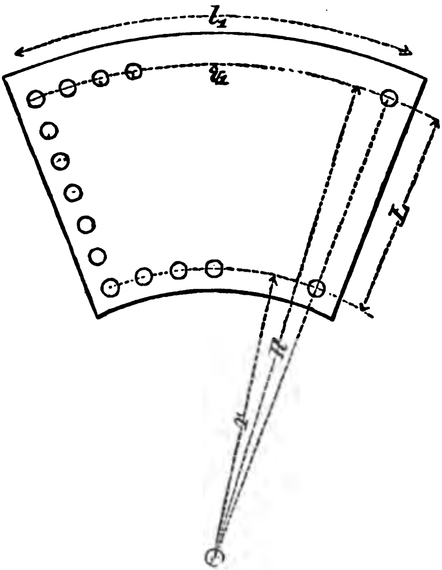


Fig. 55.

nation of the cone's sides is small, $R = \frac{D\,L}{D - d} = \frac{D\,L}{2\,t}$ nearly, and

$r = \frac{d\,L}{D - d} = \frac{d\,L}{2\,t}$ nearly.

For a boiler plate, let l_1, fig. 55, be the distance between longitudinal seams, measured at the larger end of cone, so that, if there are n plates in each ring, $l_1 = \frac{\pi D}{n}$. Let L be the

distance between the cross seams; v_1 and v_2, the versed sines of the arcs, formed by the rivets when developed.

$$R=\frac{D\,L}{2\,t} \qquad r=\frac{D\,L}{2\,t}-L$$

$$v_1=\frac{l_1^2 t}{4\,D\,L}$$

$$v_2=\frac{l_1^2(D-2\,t)t}{4\,D^2 L}$$

With these dimensions the centre lines of the rivets can be set out, and if, then, the width of overlap is added all round, the size of the plate is determined.

75. *Boiler stays* are fastenings analogous to rivets, which support flat surfaces. Those most commonly used, are of the form shown in fig. 56, which is a drawing of a copper stay for a locomotive fire-box. The stay is screwed through the two plates, which are connected, and then riveted over. Such stays are of wrought iron, of copper, or of steel. Copper has been most used, but iron is now often substituted for it. Such stays are rather liable to break across at the screw thread, inside the plates. To give warning of such a fracture, a small hole, $\frac{1}{8}$ in. or $\frac{1}{16}$ in. diameter, is sometimes drilled into the stay. If fracture occurs, warning is given by the leakage which ensues.

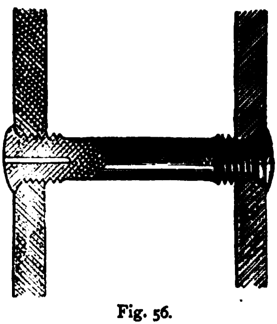


Fig. 56.

In some cases the stays have nuts instead of being riveted over, and they then afford a better support to the plate, especially if a large washer is used under the nut. In locomotives, the stays are very commonly $\frac{7}{8}$ in. in net diameter, and spaced 4 to $4\frac{3}{4}$ ins. apart. The fire-box plates

are $\frac{5}{16}$ to $\frac{3}{8}$ in. when of steel, and $\frac{3}{8}$ in. to $\frac{1}{2}$ in. when of copper.

The stays are arranged equidistant over the whole surface supported. If therefore D is the distance of the stays, centre to centre, and p the excess of internal over external pressure in lbs. per sq. in. (that is the steam pressure as shown by the steam guage), each stay supports D^2 sq. ins., and the load on it is $p\,D^2$. The stays may give way by tearing across at the screw thread, or more commonly the plate bulging round them widens the hole, and strips without much injuring the screw thread. The stays are to some extent strained by the relative movement of the plates in their own plane, due to the unequal expansion and contraction of the two plates.

The strength of stayed plates is given in § 46.

The following rules give ordinary proportions for stays. Let D be the distance apart of the stays, d their gross diameter, d_1 diameter at bottom of screw thread, t=thickness of plate, p=steam pressure.

$$\left.\begin{aligned} t &= 0{\cdot}011\ D\sqrt{p} \quad \text{copper plates} \\ &= 0{\cdot}009\ D\sqrt{p} \quad \text{iron or steel plates} \end{aligned}\right\} \quad (11)$$

$$\left.\begin{aligned} D &= \frac{91\,t}{\sqrt{p}} \quad \text{copper plates} \\ &= \frac{111\,t}{\sqrt{p}} \quad \text{iron or steel plates} \end{aligned}\right\} \quad (12)$$

For the stays the limiting stress may be taken at 9,000 lbs. per sq. in. for iron, or 5,000 lbs. per sq. in. for copper. Then

$$\begin{aligned} \frac{\pi}{4} d_1^2 &= \frac{D^2 p}{9000} \text{ for iron stays} \\ &= \frac{D^2 p}{5000} \text{ for copper stays.} \end{aligned}$$

Hence

$$\left.\begin{aligned} d_1 &= 0{\cdot}0119\ D\sqrt{p} \text{ for iron stays} \\ &= 0{\cdot}016\ \ D\sqrt{p} \text{ for copper stays} \end{aligned}\right\} \quad (13)$$

and the gross diameter outside the screw thread should be

$$\left.\begin{aligned} d &= 0{\cdot}0132\ \mathrm{D}\sqrt{p} + 0{\cdot}055 \text{ for iron} \\ &= 0{\cdot}0177\ \mathrm{D}\sqrt{p} + 0{\cdot}055 \text{ for copper} \end{aligned}\right\} \quad (14)$$

For Marine boilers the Board of Trade adopt a somewhat larger factor of safety. They require that the stress on iron stays should not exceed 5,000 lbs. per sq. in. of net section, so that the dimensions given above for copper stays would be those used by the Board of Trade for iron stays.

Diagonal stays.—Sometimes a stay is not normal to the surface supported. Let l be the length of the diagonal stay, and l_1 its projected length perpendicular to the surface supported. Then its net section should be $\frac{l}{l_1}$ times the section of an ordinary stay supporting the same area of plate.

CHAPTER V.

ON FASTENINGS.

BOLTS, NUTS, KEYS, AND COTTERS.

76. A *screw* is a cylindrical bar on which has been formed a helical projection or *thread*. The screw fits accurately into a hollow corresponding form, termed its *nut*. Pairs of elements thus formed are used in machinery (*a*) as fastenings, in which case they are commonly termed bolts; (*b*) for adjusting the relative position of two pieces; (*c*) for transmitting energy. It is chiefly as fastenings that they will be treated in the present chapter.

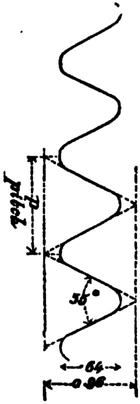


Fig. 57.

Bolts or fastening screws are chiefly used to resist straining forces which act parallel to the axis of the bolt and normal to the surfaces connected together. The bolt is then in tension. When the straining force acts perpendicular to the axis of the bolt and parallel to the surfaces connected, the bolt is in shear and is then equivalent to a rivet and may be proportioned by the same rules. A bolt differs from a rivet in this, that it permits the connected pieces to be easily disconnected again when necessary.

For manufacturing reasons, it is important that some common agreement should be arrived at, as to the form and dimensions of screws. Sir J. Whitworth first proposed a uniform system of screw threads, which is adopted univer-

sally in this country for all the more important parts of machines. For wrought-iron gas tubes, and for the cheaper kinds of metal-work, a fine pitched screw thread, known as the gas thread, is used. In America, Mr. Sellers has introduced a uniform system, very similar to Whitworth's.

77. *Pitch and form of screw threads.*—Most commonly, screw threads are triangular in section, as shown in fig. 57, which represents the standard Whitworth thread. About $\frac{1}{6}$th of the depth of the thread is rounded off at both top and bottom, to facilitate the cutting of the screw, and to render it less liable to injury.

Fig. 58 shows other forms of screw threads. That shown at *c* is rectangular in section, and is often called the square

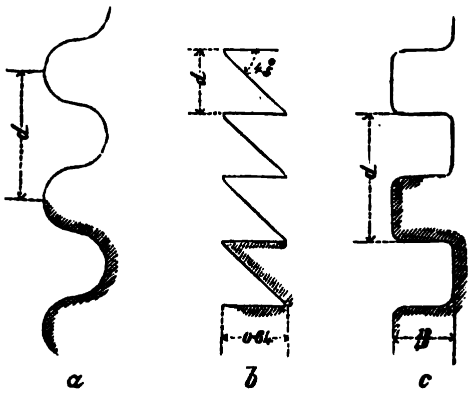


Fig. 58.

thread. The surface of the thread is nearly normal to the axial force against which the screw acts, and hence there is no oblique or bursting pressure on the nut. There is also less friction and less wear with threads of this form, but they are much more expensive to cut than triangular threads. Square-threaded screws are chiefly used to transmit motion. When a screw has to resist a force acting always in one

sense, the modified triangular screw thread, shown at *b*, may be used. This is termed the buttress thread. It has one surface normal to the axis of the screw like the square thread, and is as cheaply cut as the triangular thread. The breech screw of large guns is made of this form. Fig. 58*a* is a modified square thread which is used where the screw is liable to rough usage. It may be noted that the shearing section of a triangular thread for a given length of nut is twice as great as that of a square thread. The triangular thread also permits a finer pitch.

The pitch of screws is fixed by practical experience, so as to be suitable for cast and wrought iron. The pitch and number of threads per inch, as arranged by Whitworth for the different diameters of screws, are given in a table below.

The following formulæ give values nearly the same as those in the tables :—

Whitworth Threads.

	Triangular Threads	Square Threads	
Pitch $=p=$	$0{\cdot}08d+0{\cdot}04$	$0{\cdot}16d+0{\cdot}08$	(1)

$$\text{Number of threads per inch} = \frac{1}{p}$$

Diameter at bottom of thread

$$\left.\begin{aligned} &= d_1 = d - \frac{1{\cdot}28}{n} = 0{\cdot}9d - 0{\cdot}05, \text{ for triangular threads} \quad . \\ &= d - \frac{3{\cdot}8}{n} = 0{\cdot}85d - 0{\cdot}075, \text{ for square threads} \quad . \end{aligned}\right\} (2)$$

A square-threaded bolt is, therefore, slightly weaker, in tension, than a triangular-threaded bolt.

Fig. 57 shows the method of designing a Whitworth thread. Two parallel lines are drawn, $0{\cdot}96\,p$ apart. These are intersected by lines, inclined at 55°. Lastly, $\frac{1}{6}$th of the depth of the triangular spaces so obtained, is rounded off, both at top and at bottom. The square thread has usually twice the pitch of a triangular thread of the same diameter, and the depth of the thread is $\frac{18}{40}$ of the pitch.

Whitworth Screws.

Diam. of Screw	Number of threads per in.	Pitch in inches	Diam. at bottom of thread	Section at bottom of thread sq. ins.	Diam. of Screw	Number of threads per in.	Pitch in inches	Diam. at bottom of thread	Section at bottom of thread sq. ins.
d.	n.	p.	d_1.		d.	n.	p.	d_1.	
3/16	24	·041	·136	·015	2	4½	·222	1·716	2·31
¼	20	·050	·186	·027	2¼	4	·250	1·966	3·02
5/16	18	·056	·241	·046	2½	4	·250	2·180	3·73
⅜	16	·063	·295	·068	2¾	3½	·286	2·430	4·64
7/16	14	·071	·347	·095	3	3½	·286	2·634	5·43
½	12	·083	·394	·122	3¼	3¼	·308	2·884	6·51
⅝	11	·091	·509	·204	3½	3¼	·308	3·106	7·59
¾	10	·100	·622	·304	3¾	3	·333	3·356	8·86
⅞	9	·111	·733	·422	4	3	·333	3·574	10·01
1″	8	·125	·840	·554	4¼	2⅞	·348	3·824	11·46
1⅛	7	·143	·942	·697	4½	2⅞	·348	4·055	12·88
1¼	7	·143	1·067	·894	4¾	2¾	·364	4·305	14·52
1⅜	6	·167	1·162	1·060	5	2¾	·364	4·534	15·12
1½	6	·167	1·286	1·30	5¼	2⅝	·381	4·764	17·79
1⅝	5	·200	1·411	1·47	5½	2⅝	·381	5·014	19·71
1¾	5	·200	1·494	1·75	5¾	2½	·400	5·238	21·57
1⅞	4½	·222	1·591	1·987	6	2½	·400	5·488	23·64

A table of the number of threads for screws of intermediate decimal sizes will be found in Shelley's 'Workshop Appliances,' p. 103.

Gas threads.—For the wrought-iron tubes used for conveying gas, the Whitworth screw thread is not suitable. For these a special system of threads has been adopted, finer in pitch, and cutting less deeply into the metal of the tube, than the Whitworth thread would do.

Diameter in inches	⅛	¼	⅜	½	¾	1	1½	2
Pitch in inches	·036	·053	·053	·071	·071	·091	·091	·091
No. of threads per inch	28	19	19	14	14	11	11	11

Sellers's screw threads.—In the American, or Sellers's, system of screw threads, the sides of the thread are inclined at 60°, and the angles at top and bottom are truncated to form a flat ⅛th of the pitch in width. The pitch is

$$p = 0{\cdot}07\,d + 0{\cdot}06 \text{ nearly} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1\,a)$$

$$d_1 = 0{\cdot}91\,d - 0{\cdot}08 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2\,a)$$

78. *Strength of screw bolts.*—Putting P for the axial straining force acting on the bolt; d_1=diameter at bottom of thread; f=working stress per unit of area,

$$P = \frac{\pi}{4} d_1^2 f \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and replacing d_1 by d, by means of equation (2),

$$\left.\begin{aligned} d &= {\cdot}055 + 1{\cdot}127 \sqrt{\left(\frac{P}{f}\right)}, \text{ for triangular threads} \quad . \\ &= {\cdot}085 + 1{\cdot}32 \sqrt{\left(\frac{P}{f}\right)}, \text{ for square threads} \quad . \end{aligned}\right\} (4)$$

This is a slight improvement of the formula commonly used in determining the size of bolts. In order that it may be strictly applicable, the bolt ought to be subjected to an axial force only; but, in consequence of the obliquity of the bearing surface of the thread, the pressure between the bolt and nut has a component tending to twist the bolt; hence the stress on it is really a combined torsion and tension, and not a simple tension. It is also probable, that at the section of the bolt at the bottom of a thread, the stress is not quite uniformly distributed. There is there a rapid change of section, and the stress is probably greater at the angle of the thread, and less in the interior of the bolt than the mean value. We may allow, roughly, for both the above causes of increased straining action, by taking for f a value less than that suitable for simple tension.

In many cases, however, there is an uncertainty in the determination of the load P. One part of the load, due to the forces acting on the machine, which may be termed the effective load, is ascertainable with tolerable accuracy. Another part, which is due to the force used in tightening the nut, and the amount of which depends on the skill and

care of the workman, is less easily determined. If the nut is not screwed up before the load comes upon it, or, if it is screwed up, provided the connected pieces are not in actual contact, the effective load alone needs to be considered. On the other hand, if the bolt is screwed up, so as to develop a reaction between the connected pieces, this additional load may be and often is greater than the effective load. Practical experience shows that we may *roughly* allow for the difference of the action in these cases thus :—For press screws, and other bolts, which do not require to be tightened before the load comes upon them, f may be taken at 6,000 lbs. per sq. in. For accurately fitted bolts, requiring to be tightened moderately, such as foundation bolts, $f = 4{,}000$.

In the case of cylinder cover bolts, the bolt must be tightened up to such a tension that the joint does not leak. The strain so put on the bolt is greater than that due to the steam pressure, and consequently it is rather for this straining action than for the steam pressure on the cover that the bolt must be calculated. We may in such cases assume the strain necessary to keep the joint tight to be proportional to the pressure on the cover, and allow for the strain put on by the spanner by taking a sufficiently high factor of safety. Thus the following values may be taken :—

	Diameter of Cylinder	f lbs. per sq. in.
Less than	10 ins.	2,000
„	15 „	3,000
„	20 „	4,000
„	60 „	5,000
Greater than	60 „	6,000

Flange bolts should not in any case be less than $\frac{5}{8}$ in. in diameter. The net diameter at the bottom of the thread and the effective section of the bolt are given in the preceding table.

Let D be the diameter of a cylinder, p the steam or

water pressure in the cylinder in lbs. per sq. in., d_1 the effective diameter, and n the number of bolts in the flanges of the cylinder. Then

$$\frac{\pi}{4} d_1^2 n f = \frac{\pi}{4} D^2 p$$

$$d_1 = \sqrt{\left(\frac{p}{nf}\right)} D$$

where f may have the values just given. In roughly made joints, as for instance in joints with a gasket ring, the value of f is sometimes as low as 1,500 lbs. per sq. in.

79. *Strength of bolts, taking torsion into account.*—Suppose that a load P is suspended from a square-threaded screw,

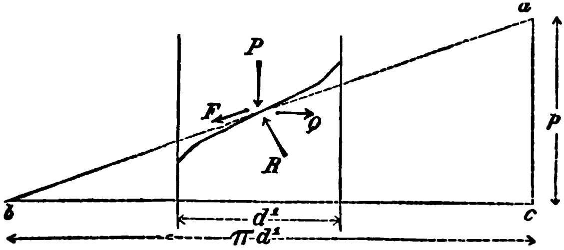


Fig. 59.

and that it is screwed up, without, however, bringing the parts connected by the screw into actual contact, so as to develop a reaction between them, additional to the load. In that case, the friction of the nut on its support does not affect the stress in the bolt, and there is a definite relation between the load P, and the twisting force Q, applied to the bolt. Let fig. 59 represent a screw thread at its mean diameter d^1. The forces, acting between any two elements of the bolt and nut, are a vertical force P, due to the load; a horizontal force Q, due to the pull on the spanner; a reaction R, normal to the thread; and a friction F, parallel to the thread.

The obliquity of the thread is the same as that of its

development $a\,b$ on a plane surface, which makes, with the horizontal, an angle whose tangent is $p \div \pi d^1$. Neglecting the small difference between the mean and outside diameters, d and d^1 of the thread, we get

$$Q = P\frac{p + \mu \pi d}{\pi d - \mu p} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where μ is the co-efficient of friction. If the thread is triangular instead of square, the normal reaction is greater, in the ratio of the slant length of a thread to its half thickness at the root. Hence, for triangular threads

$$Q = P\frac{p + 1{\cdot}15\,\pi \mu d}{\pi d - 1{\cdot}15\,\mu p} \quad . \quad . \quad . \quad . \quad (6)$$

The twisting moment of the force Q, acting at nearly $\frac{d}{2}$ from the axis, is $Q\frac{d}{2}$. Putting $\mu = 0{\cdot}15$, and using the previously found values for the pitch, we get

Twisting moment $= M = 0{\cdot}2\,P d$, nearly, in each case.

The greatest stress due to combined torsion and tension, is then—

$$f = \frac{4\,P}{\pi d_1^2}\left\{\frac{1}{3} + \frac{2}{3}\sqrt{\left(\frac{2\,M}{P d}\right)^2 + 1}\right\}$$

$$= 1{\cdot}339\frac{P}{d_1^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Putting for f, the safe working stress, and replacing d_1 by d, we get

$$\left.\begin{aligned} d &= {\cdot}055 + 1{\cdot}285\sqrt{\frac{P}{f}}, \text{ for triangular threads} \\ &= {\cdot}085 + 1{\cdot}361\sqrt{\frac{P}{f}}, \text{ for square threads} \quad . \end{aligned}\right\}(8)$$

Comparing these with equations (4), in obtaining which the twisting moment was neglected, we see that the twisting moment adds about 15 per cent. to the diameter necessary for the bolt.

80. *Strength of bolts when the initial tension, due to screwing up, is taken into account.*—A nut is screwed up by means of a spanner, whose leverage is, on the average, 15 d. Suppose that a nut is screwed up by a force Q, applied to the spanner at radius R; the tension P_1, produced in the bolt, being expended in compressing the pieces connected by the bolt. The friction of the nut, on its seat, balances part of the force Q. That friction acts approximately at a radius $\frac{2}{3}d$, and its magnitude is $\mu_1 P_1$ lbs. Hence

$$P_1 = \frac{Q}{\frac{2}{3}\mu_1 + \frac{1}{2}\frac{p+\mu\pi d}{\pi d - \mu p}} \cdot \frac{R}{d} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Taking $\mu_1 = \mu = 0{\cdot}15$, for a turned nut on a turned washer; $R = 15\,d$; $p = 0{\cdot}16\,d$, we get

$$P_1 = 82\,Q \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9\,a)$$

The force applied by a workman in screwing up a bolt, will vary with the size of the bolt, and his experience teaches him in what case a heavy pressure may be applied with safety. Taking the heaviest ordinary pull of the workman to be $Q = 30$ lbs.:

$$P_1 = 82 \times 30 = 2{,}460 \text{ lbs.}$$

a force sufficient to break a $\frac{3}{8}$-in. bolt, and to seriously injure a $\frac{1}{2}$-in. bolt. Hence, bolts of less than $\frac{3}{4}$-in. diameter are not used for joints requiring to be tightly screwed up.

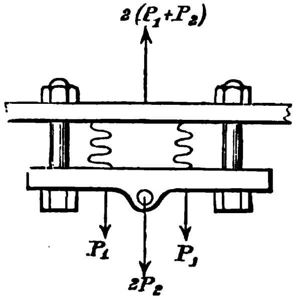


Fig. 60.

Suppose that bolts are used to connect together two parts of a machine, and that these are screwed up tightly before the effective load comes on the connected parts. Let P_1 be the initial tension on a bolt due to screwing up, and P_2 the additional load afterwards added. Then it can be shown that the greatest load on the

bolt may either be little different from P_1 or P_2, according as the former or the latter is greater, or it may approach the value $P_1 + P_2$; and to determine what its value is, the relative rigidity of the bolts and the parts they connect must be known. Thus, suppose the pieces A and B, fig. 60, are connected by two bolts. To render the action of the elasticity of the connected parts more evident, suppose that springs are interposed between A and B. After screwing up till a tension of P_1 lbs. is created on each bolt, by the elastic resistance of the springs to compression, let an additional load of $2P_2$ lbs. be hung on the lower plate. This load may in part relieve the original tension if the extension of the bolts diminishes the compression of the springs, and in that case the ultimate tension on each bolt may be little greater than P_1 or P_2, according as P_2 or P_1 is greater. But if the extension of the bolts is small compared with the compression of the springs, then the ultimate load will approach the value $P_1 + P_2$, because the elastic thrust of the springs is little affected by the additional load. In ordinary cases the elasticity of the parts bolted together takes the place of the elasticity of the springs. Where rigid flanges are bolted together metal to metal, it is probable that the extension of the bolts with any additional load relieves the initial tension, and the condition is similar to that in the former case described above. But in other cases, where the flanges are deflected by the bolts, or where elastic packing, such as india-rubber, is interposed, the extension of the bolts may very little affect the initial tension, and the condition is that of the latter of the two cases above. Since the latter assumption is more unfavourable to the resistance of the bolt [illegible] sumed that this contingency must be pro-vi[illegible]

[illegible] joint is made tight by screwing up, the [illegible] e the effective or useful load P_2 + the [illegible] o screwing up. Then

$$[illegible] = P_1 + P_2 = P_2 + 82\,Q.$$

$$\left.\begin{aligned} d &= 0{\cdot}055 + 1{\cdot}28\sqrt{\left(\frac{82\,Q + P_2}{f}\right)}, \text{ for triangular threads} \\ &= 0{\cdot}085 + 1{\cdot}35\sqrt{\left(\frac{82\,Q + P_2}{f}\right)}, \text{ for square threads} \end{aligned}\right\} \quad (10)$$

where for Q, is to be put the assumed value of the force used in screwing up.

Strength of Bolts.

Diameter of Bolt d	Strength when there is no stress due to screwing up	Pull on Spanner Q	Stress due to screwing up 82 Q	Effective strength when screwed up against an elastic flange
ins.	lbs.	lbs.	lbs.	lbs.
$\frac{1}{2}$	1008	16	1312	—
$\frac{5}{8}$	1836	18	1476	360
$\frac{3}{4}$	2736	20	1640	1096
$\frac{7}{8}$	3798	23	1890	1908
1	4986	25	2050	2936
$1\frac{1}{8}$	6273	27	2214	4069
$1\frac{1}{4}$	8046	29	2380	5666
$1\frac{3}{8}$	10044	32	2624	7420
$1\frac{1}{2}$	11700	34	2790	8910
$1\frac{3}{4}$	15750	39	3200	12550
2	20790	43	3530	17260
$2\frac{1}{4}$	27180	47	3940	23240
$2\frac{1}{2}$	33570	52	4260	29310
$2\frac{3}{4}$	41760	57	4670	37090
3	48870	61	5000	43870
$3\frac{1}{4}$	58590	65	5350	53240
$3\frac{1}{2}$	68310	70	5740	62570
$3\frac{3}{4}$	79740	74	6100	73640
4	90090	79	6500	9359
5	136080	97	7950	1281
6	212760	115	9450	2033

The preceding Table gives the strength of a bolt in two different conditions; (1) when subject only to the external load, the strength being calculated for a stress of 9,000 lbs. per sq. in. (2) When the bolt is used to compress an elastic flange, so that the effective strength to resist an additional load is the difference between the strength as

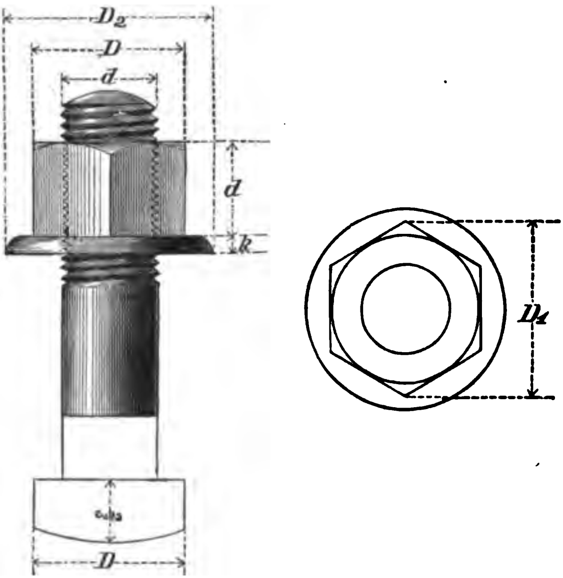


Fig. 61.

previously calculated and the stress due to screwing up. The allowance for the stress due to screwing up is necessarily empirical, and it has been assumed that the pull on the spanner is

$$Q = 7 + 18\,d \text{ lbs.}$$

The Table is not intended to supersede the exercise of judgment in particular cases.

81. *Proportions of bolts and nuts.*—Fig. 61 shows the most ordinary type of bolt, nut, and washer. The bolt has a square head, and a square neck, to prevent the rotation of the bolt, while the nut is being screwed up. The nut is hexagonal, and the washer circular. The washer is used when the bolt connects rough castings, and then forms a smooth seating, on which the nut turns. It is sometimes used for appearance only. The following rules give good proportions :—

Hexagon Nuts.

Diameter across flats $= D = 1.5\ d + 0.18$ to $1.5\ d + 0.44$ (rough),

$1.5\ d + 0.06$ to $1.5\ d + 0.18$ (bright).

Diameter across angles $= D_1 = 1.75\ d + 0.16$ to $1.75\ d + 0.4$ (rough),

$1.75\ d + 0.07$ to $1.75\ d + 0.2$ (bright).

$D = 1.5\ d + 0.18$, and $D_1 = 1.75\ d + 0.16$, are very nearly Whitworth's standard sizes for finished nuts. In drawings, on a small scale, it is accurate enough to take $D_1 = 2\ d$.

Height of nut $= d$.

Height of lock nut $= \frac{d}{2}$.

Square Nuts.

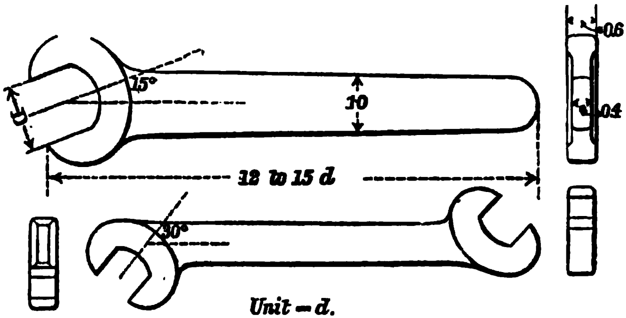
Diameter across flats $= 1.5\ d + 0.18$ to $1.5\ d + 0.44$ (rough),

$= 1.5\ d + 0.06$ to $1.5\ d + 0.18$ (bright).

Diameter across angles $=2{\cdot}12\,d+0{\cdot}25$ to $2{\cdot}12\,d+0{\cdot}6$ (rough),
$=2{\cdot}12\,d+0{\cdot}08$ to $2{\cdot}12\,d+0{\cdot}25$ (bright).

The head of the bolt may be square, hexagonal, or circular. Its height is $\frac{2}{3}\,d$ to d.

Length of spanner$=15\,d$ to $18\,d$. The usual form of spanners is shown in fig. 62.



Unit = d.

Fig. 62.

Washers.

Thickness $0{\cdot}15\,d$; diameter $\frac{9}{8}\,D_1$.

Small washers are usually 14 B.W.G., or 0·083 in. thick.

Washers, for wood, may be $3\,d$ in diameter, and $0{\cdot}3\,d$ in thickness.

	Diameter of Bolt											
	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2
Size of hole before tapping	$\frac{3}{16}$	$\frac{21}{64}$	$\frac{29}{64}$	$\frac{17}{32}$	$\frac{43}{64}$	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{31}{32}$	$1\frac{7}{64}$	$1\frac{5}{16}$	$1\frac{17}{32}$	$1\frac{3}{4}$
D	$\frac{17}{32}$	$\frac{11}{16}$	$\frac{29}{32}$	$1\frac{1}{8}$	$1\frac{5}{16}$	$1\frac{17}{32}$	$1\frac{23}{32}$	$1\frac{5}{16}$	$2\frac{3}{32}$	$2\frac{7}{16}$	$2\frac{25}{32}$	$3\frac{9}{64}$
D_1	$\frac{5}{8}$	$\frac{25}{32}$	$1\frac{1}{32}$	$1\frac{9}{32}$	$1\frac{1}{2}$	$1\frac{3}{4}$	$1\frac{15}{16}$	$2\frac{1}{4}$	$2\frac{3}{8}$	$2\frac{13}{16}$	$3\frac{5}{32}$	$3\frac{5}{8}$

Weight of Bolts and Nuts in Pounds.

	Diameter of Bolt											
	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2
Weight per inch of bolt .	·017	·034	·057	·087	·124	·17	·22	·28	·34	·49	·68	·88
Weight of square head .	·017	·044	·088	·126	·260	·397	·544	·694	·943	1·667	2·535	3·571
Weight of nut and part of bolt included . .	·022	·0530	·110	·172	·324	·494	·681	·873	1·188	2·088	2·954	4·387
Weight of washer and part of bolt included . .	·011	·0154	·039	·048	·096	·132	·161	·178	·273	·489	·723	1·014

SIZE OF WHITWORTH NUTS AND BOLTS.

The following are the sizes of bolts and nuts according to the Whitworth Standard, as revised some two years since. The exact sizes are given in decimals, and the nearest approximate sizes in sixty-fourths of an inch.

Diameter of bolts	Width of nuts across flats		Height of boltheads	
$\frac{1}{8}$	·338	$\frac{21}{64}$ *f*	·1093	$\frac{7}{64}$
$\frac{3}{16}$	·448	$\frac{29}{64}$ *b*	·1640	$\frac{5}{32}$
$\frac{1}{4}$	·525	$\frac{33}{64}$ *f*	·2187	$\frac{7}{32}$
$\frac{5}{16}$	·6014	$\frac{19}{32}$ *f*	·2734	$\frac{17}{64}$
$\frac{3}{8}$	·7094	$\frac{45}{64}$ *f*	·3281	$\frac{21}{64}$
$\frac{7}{16}$	·8204	$\frac{53}{64}$ *b*	·3828	$\frac{3}{8}$ *f*
$\frac{1}{2}$	·9191	$\frac{29}{32}$ *b*	·4375	$\frac{7}{16}$
$\frac{9}{16}$	1·011	$1\frac{1}{64}$ *b*	·4921	$\frac{31}{64}$ *f*
$\frac{5}{8}$	1·101	$1\frac{3}{32}$ *f*	·5468	$\frac{35}{64}$
$\frac{11}{16}$	1·2011	$1\frac{13}{64}$ *b*	·6015	$\frac{19}{32}$ *f*
$\frac{3}{4}$	1·3012	$1\frac{19}{64}$ *f*	·6562	$\frac{21}{32}$
$\frac{13}{16}$	1·39	$1\frac{25}{64}$ *b*	·7109	$\frac{45}{64}$ *f*
$\frac{7}{8}$	1·4788	$1\frac{31}{64}$ *b*	·7656	$\frac{49}{64}$
$\frac{15}{16}$	1·5745	$1\frac{37}{64}$ *b*	·8203	$\frac{13}{16}$ *f*
1	1·6701	$1\frac{43}{64}$ *b*	·875	$\frac{7}{8}$
$1\frac{1}{8}$	1·8605	$1\frac{55}{64}$ *f*	·9843	$\frac{63}{64}$
$1\frac{1}{4}$	2·0483	$2\frac{3}{64}$ *f*	1·0937	$1\frac{3}{32}$
$1\frac{3}{8}$	2·2146	$2\frac{7}{32}$ *b*	1·2031	$1\frac{13}{64}$
$1\frac{1}{2}$	2·4134	$2\frac{13}{32}$ *f*	1·3125	$1\frac{5}{16}$
$1\frac{5}{8}$	2·5763	$2\frac{37}{64}$ *b*	1·4128	$1\frac{27}{64}$
$1\frac{3}{4}$	2·7578	$2\frac{3}{4}$ *f*	1·5312	$1\frac{17}{32}$
$1\frac{7}{8}$	3·0183	$3\frac{1}{16}$ *f*	1·6406	$1\frac{41}{64}$
2	3·1491	$3\frac{5}{32}$ *b*	1·75	$1\frac{3}{4}$
$2\frac{1}{8}$	3·337	$3\frac{11}{32}$ *b*	1·8523	$1\frac{55}{64}$
$2\frac{1}{4}$	3·546	$3\frac{35}{64}$ *b*	1·9687	$1\frac{31}{32}$
$2\frac{3}{8}$	3·75	$3\frac{3}{4}$	2·0781	$2\frac{5}{64}$
$2\frac{1}{2}$	3·894	$3\frac{57}{64}$ *f*	2·1875	$2\frac{3}{16}$
$2\frac{5}{8}$	4·049	$4\frac{3}{64}$ *f*	2·2968	$2\frac{19}{64}$
$2\frac{3}{4}$	4·181	$4\frac{3}{16}$ *b*	2·4062	$2\frac{13}{32}$
$2\frac{7}{8}$	4·3456	$4\frac{11}{32}$ *f*	2·5156	$2\frac{33}{64}$
3	4·531	$4\frac{17}{32}$ *b*	2·625	$2\frac{5}{8}$

The thickness of the nuts is in every case the same as the diameter of the bolts : *f* = full, *b* = bare.

82. *Different forms of nuts.*—Ordinary nuts are chamfered off at an angle of 30° to 45°, as shown at *a*, fig. 63 ; or they are finished with a spherical bevil, struck with a radius of about 2 *d*, as shown at *b*. *Flange nuts*, *c*, are used when the hole, in which the bolt is placed, is considerably larger than the bolt itself. The flange covers and hides the hole. *Cap nuts*, *d*, are used where leakage along the screw thread

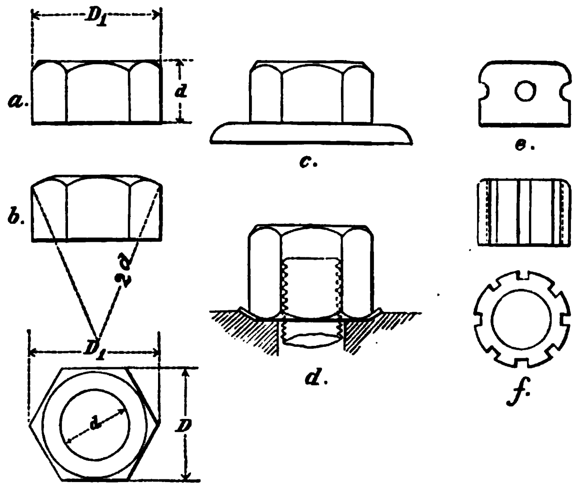


Fig. 63.

is feared. In the figure, a thin, soft copper washer is shown, which prevents leakage under the nut. *Circular nuts*, *e*, are occasionally used. They have holes, in which a bar, termed a 'Tommy,' is placed, for screwing them up. Sometimes grooves are cut, as shown at *f*. Steel nuts may be used, if great durability is required.

83. *Different forms of bolt heads.*—In fig. 64, *a* is a cup-shaped, *b* a countersunk, and *c* a square bolt-head. Rotation of the bolt is prevented in *a* by a square neck, in *b* by a set screw, in *c* by a snug forged on the bolt. Fig. 65 shows

a T-headed bolt in front and side elevation. Fig. 66 shows an eye bolt. Fig. 67 shows a spherical-headed bolt used,

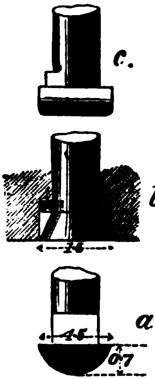


Fig. 64.

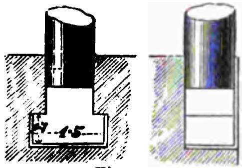


Fig. 65.

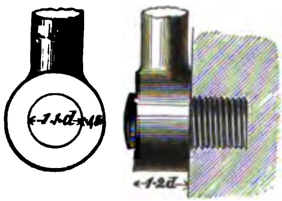


Fig. 66.

sometimes, for railway fastenings, with a square neck. The spherical head allows the bolt to take a fair bearing on the rail. The other figure shows a cup-head, with a snug forged

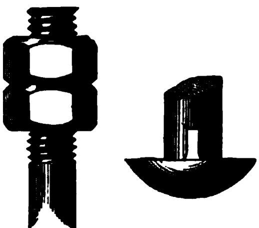


Fig. 67

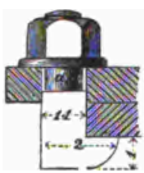


Fig. 68

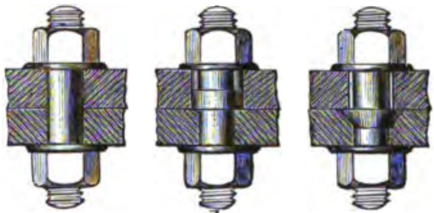
Fig. 69.

on the bolt, to prevent rotation when the bolt is screwed up. Proportional unit$=d$, in all these figures.

Fig. 68 is a *hook bolt*, which is used when one piece is too small to have a bolt hole through it, or when it is objectionable to weaken the piece by a bolt hole. Fig. 69

is a *stud*, which is screwed into one of the connected pieces, and remains in position when the nut is removed. Fig. 70 is a *set screw*, or bolt not requiring a nut. Fig. 71 shows;

Fig. 70.



a *b* *c*

Fig. 71.

at *a*, a nut-headed bolt, or bolt having two loose nuts, instead of a nut and head; at *b* and *c* similar bolts, with an intermediate head or flange. These bolts remain in place when the top nut is removed.

Fig. 72 is a bolt leaded into stone work. The tail of the bolt is rectangular, with jagged edges.

Fig. 73 is a fang bolt used for attaching ironwork to wood, and especially for attaching rails to sleepers. The fangs of the broad triangular plate, which forms the nut, bite into the wood, while the bolt is rotated by the

Fig. 72.

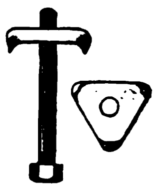
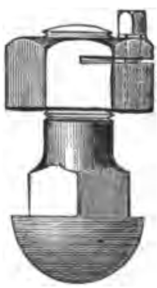


Fig. 73.

head, which bears on the ironwork. The large area of the nut prevents crushing of the wood.

84. *Locking arrangements for nuts* are intended to prevent the gradual unscrewing of nuts, subjected to vibration and frequent changes of load. No nut accurately fits its bolt; a certain amount of play, however minute, always exists. When a nut, having play, is subjected to vibration, it gradually slacks back. This is, to a great extent, prevented by double nuts, shown in fig. 74. One of the nuts is termed a lock nut, and is usually half as thick as the ordinary nut. When there are two nuts, the whole load may be thrown on the outer nut. The outer nut ought, therefore, to be the thicker nut. It is common in practice to put the thinner nut outside, the reason being, that ordinary spanners are sometimes too thick to hold the thin nut, when screwed home first. The more correct arrangement is that shown in the figure.

Fig. 74.



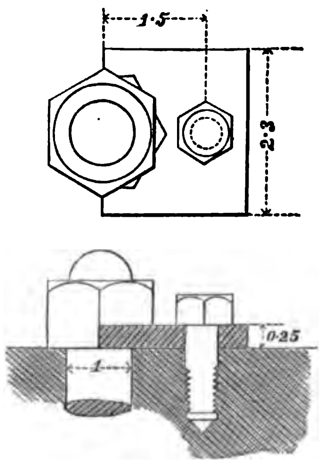


Fig. 75. Unit=*d*. Fig. 76.

Fig. 75 shows the form of Wiles's lock nut, which is now often used on quick-running machine parts subjected to a good deal of vibration. It is very simple and can be locked on any part of the bolt. The nut is half cut through

by a saw cut, and a small set screw is used to close slightly the jaws thus formed, after the nut is screwed home. The nut then grips the screw thread tightly. For nuts under one inch in diameter the set screw is omitted, and the jaws of the saw cut are slightly closed by a hammer blow, before the nut is put in place.

Another plan is to drill a hole through the top of the bolt above the nut, and drive a split pin or cotter through. The nut must always be in the same place when screwed up. A better plan is shown in fig. 76, a stop plate being used, fixed on one side of the nut. The set screw in the stop plate may have its diameter$=\frac{1}{4}\,d+\frac{1}{8}$.

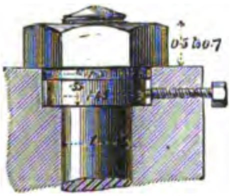


Fig. 77.

Fig. 78.

A very neat arrangement is shown in fig. 77; the lower part of the nut is turned circular, and fits in a recess in the piece connected by the bolt. A set screw is tapped through, and bears on the side of the nut. The diameter of the set screw may be $\frac{1}{8}\,d+\frac{1}{8}$. A stop ring is sometimes used (fig. 78), with a set screw tapped through it. The stop ring is of brass, or wrought iron, and it is prevented from turning by a stop pin of the same size as the set screw.

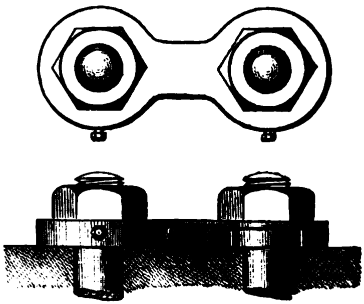


Fig. 79.

Fig. 79 shows a neat way of applying this last mode of locking to a pair of nuts. The locking plate embraces the turned part of both nuts, and the stop pin is dispensed with.

Elastic washers have been used as substitutes for lock nuts. Fig. 80 shows Grover's spring steel washer. When the nut is tightened up, the washer becomes nearly, but not quite, flat, and its elasticity neutralises the play of the nut on the bolt.

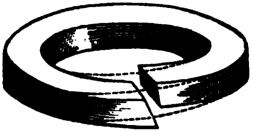


Fig. 80.

85. *Bolting of cast-iron plates.*—Cast-iron plates are united by bolts ; flanges, to receive the bolts, are cast on the plates, and these may be external or internal. The flanges are of the same thickness as the plates, or a little thicker. The bolts are never less than $\frac{3}{4}$ in. in diameter, and the

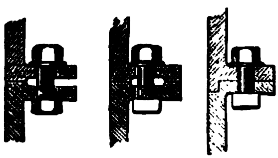


Fig. 81.

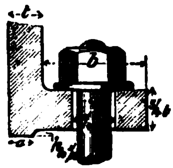


Fig. 82.

bolt diameter may be equal to the flange thickness. The fitting part of the flanges is often a narrow 'chipping strip,' which is faced by hand, or in the planing machine. Fig. 81 shows three arrangements of the flanges and bolts. Fig. 82 gives the ordinary proportions of the bolt and flange.

Bolt diameter$=d=\frac{5}{4}t+\frac{1}{8}$ (but not less than $\frac{3}{4}$ in.).

Pitch of bolts about 6 d, or less, if necessary for strength.

Width of chipping strip$=a=\frac{5}{4}t$.

Width of flange$=b=2d+\frac{3}{4}$.

The open space between the flanges is sometimes filled with rust cement.

85*a*. *Stay bolts* for tanks. The flat surfaces of tanks are supported by stay bolts connecting opposite sides of the tank, and thus directly resisting the water pressure. Fig. 83 shows the form of such bolts for cast and wrought iron tanks.[1] The end of the bolt is enlarged in fig. 83*a*, so that the body of the bolt is of the same strength as the screw at the bottom of the thread. Then $d_1 = 0{\cdot}9d - 0{\cdot}05$. The cast-iron plate through which the bolt passes is

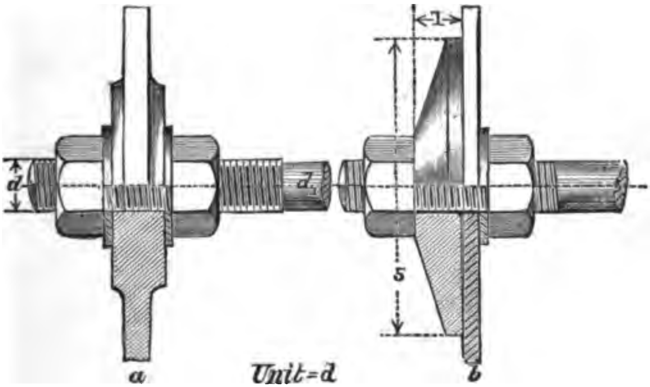


Fig. 83.

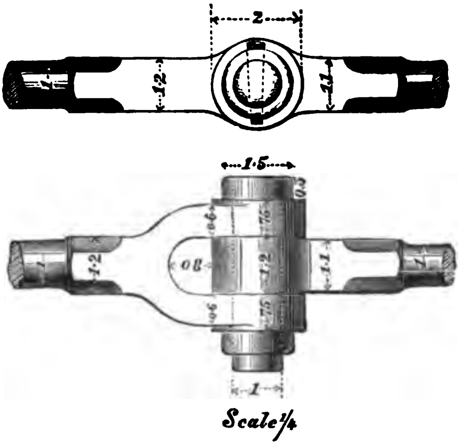
thickened to spread the pressure of the bolt. In fig. 83*b*, the wrought-iron tank plate cannot easily be thickened, but a large washer plate of cast iron is used to spread the bolt pressure.

JOINT PINS. KNUCKLE JOINT.

86. A joint pin is a kind of bolt, so placed as to be in shear. Fig. 84 shows an arrangement known as a knuckle joint. The proportions are empirical. If the joint pin were subjected to simple shear at two sections, it would be strong enough, when its diameter was equal to 0·7 of the diameter of the rods. But the pin wears, and is then subjected to

[1] Hydraulic Machinery, by W. Anderson.

bending, as well as shearing. When there is much motion at the joint, the width of the eyes of the rods, and the length of the pin, may be increased.



Unit=diam. of pin.
Fig. 84.

Keys.

87. Keys are small wedge-shaped pieces used to fix wheels, pulleys, cranks, and other pieces on shafts. The

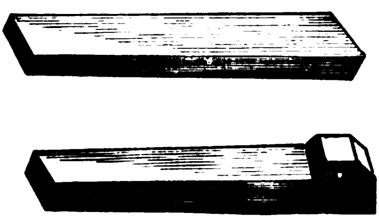


Fig. 85.

function of a key is to prevent the piece rotating relatively to the shaft; but, from the friction of the key in the keyways, it offers also some resistance to sliding along the shaft.

Fig. 85 shows the usual form of keys. The lateral sides are parallel, but the thickness tapers. The lower figure shows a key with a head which is necessary for slacking back the key in cases where the other end is inaccessible.

Keys may be saddle keys (fig. 86*A*); keys on flats (fig. 86*B*); or sunk keys (fig. 86*C*). Saddle keys are used for fixing light pulleys and other pieces. The rotation of

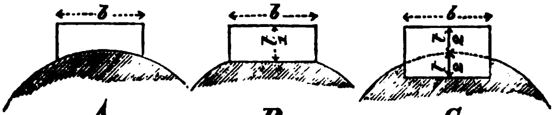


Fig. 86.

the pulley or other piece on the shaft is prevented only by the friction of the key. The taper of a key is about 1 in 64 to 1 in 100. Keys on flats are more secure than saddle keys. The flat is parallel to the axis of the shaft, and rather wider than the key. The taper may be the same as for a saddle key.

Sunk keys are much more satisfactory because slipping is entirely prevented unless the key shears. Keyways are slotted in the shaft and in the piece fixed on it, and the key is accurately fitted in the key ways. It should fit at the sides even more accurately than on the top and bottom. A wood pattern of the key is usually first fitted in the key ways, and this serves as a guide in forging and planing the key. A sunk key with a saddle key placed at right angles to it, fig. 87, is a very good arrangement. If a wheel nave is accidentally bored out slightly larger than the boss on which it is fitted, it rocks if held by a sunk key only. This rocking is entirely prevented by the additional saddle key, which insures a bearing between the eye and shaft boss at three points in the circumference.

88. *Staking on.*—In the case of large pieces four keys are sometimes used, fig. 88, or, when the shaft boss is

square, eight keys, fig. 89. In the latter case the labour of planing the whole of the shaft boss and slotting out the square eye of the piece to be fixed to it is generally saved, only the key seats being faced. When four or eight keys

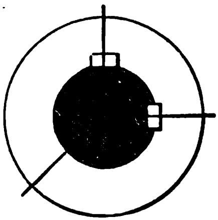


Fig. 87.

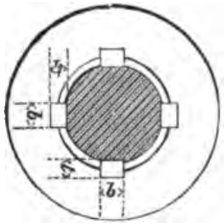


Fig. 88.

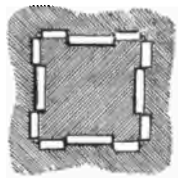


Fig. 89.

are used, there is a limited power of adjusting the piece keyed to the shaft so as to be coaxial with the shaft.

89. *Cone keys.*—When a wheel has to be bored out, to pass over a shaft boss, cone keys may be used to fix it on the shaft, fig. 90. These are of cast iron and are cast in a single piece, with three parting plates, nearly but not quite dividing it into three pieces. The casting is bored and turned, and afterwards split and the rough edges chipped away. There are thus obtained three cast-iron slightly tapering conical or saddle keys, of the thickness necessary to fill the space between the eye of the pulley and the shaft. They resist the tendency to rotation of the piece keyed on by friction alone. The eye of the pulley must be bored slightly conical.

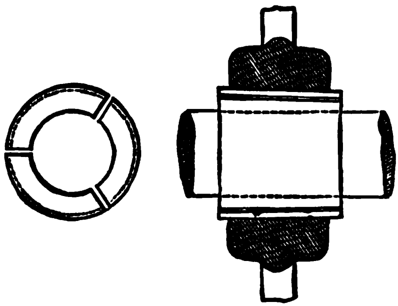


Fig. 90.

Pins.—A taper pin (fig. 91) is sometimes used in place of a key. It is sunk half in the shaft, half in the piece keyed on.

Where very fast connection between the pieces is required, as in the case of cranks keyed on crank shafts, the crank is usually bored but slightly smaller than the crank shaft, expanded by heat and shrunk on. Rotation is then prevented by a key or pin.

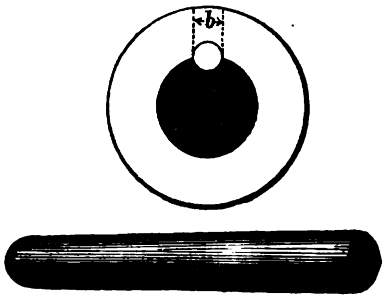


Fig. 91.

90. *Taper of keys.*—Let P be the pressure between the pieces acting through the key in the plane of rotation, μ the coefficient of friction, θ the angle of repose for the surfaces of contact so that $\mu = \tan\theta$.

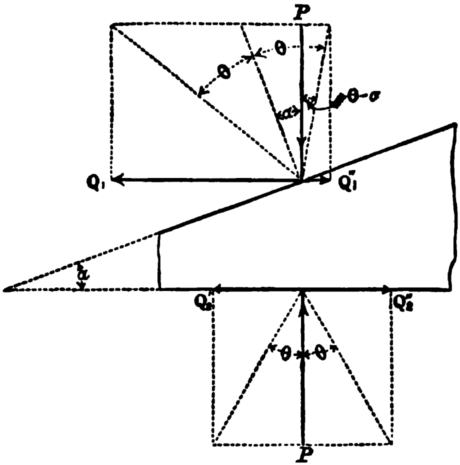


Fig. 92.

Then when the key is being driven home the force required is

$$Q_1' + Q_2' = P\{\tan(\theta + \alpha) + \tan\theta\}$$

and when the key is driven back the force required is

$$Q_1'' + Q_2'' = P\{\tan(\theta - \alpha) + \tan\theta\}$$

If in the latter case

$$\theta - \alpha = -\theta$$

$$\text{or,} \qquad \alpha = 2\theta$$

no additional force is required to slack the key, beyond the horizontal component of the pressure on the sloping face. Hence $\alpha = 2\theta$ is a limiting value for the inclination of the key, and if this is exceeded the key will not remain in position after driving home. For metal on metal, somewhat oily, $\mu = 0{\cdot}08$ and $\theta = 4\frac{1}{2}°$, consequently the limiting angle for a key is about 9°, corresponding to a taper of about 1 in 7. The actual taper of keys is much less than this, being seldom greater than 1 in 64, and generally 1 in 100 to 1 in 150. The taper is smallest in the most accurate work.

91. *Strength of keys.*—For saddle keys no very exact rules can be given. They are only used where the stress between the connected pieces is small. For sunk keys, let b=width, l=length of key, t=mean thickness of key, f_s, f_c shearing and crushing resistance of material. The resistance to shearing is $b\,l\,f_s$. The bearing surface of the key on the sides of the key way is $\frac{1}{2}\,t\,l$, and the crushing resistance is $\frac{1}{2}\,t\,l\,f_c$.

If $f_c = 2f_s$, then $t = b$, or the key should be square in section. For practical reasons the key is generally wider than this, and then its shearing resistance is usually in excess.

92. *Common proportions of keys.*—The following rules are empirical, and in general give an excess of strength to the key:

Diameter of eye of wheel, or boss of shaft$=d$

Width of key$=b=\frac{1}{4}d+\frac{1}{8}$ . . . . (11)

Mean thickness of sunk key $= t = \frac{1}{8}d + \frac{1}{8}$ . . . (12)

" key on flat $= t_1 = \frac{1}{10}d + \frac{1}{16}$ . . . (12*a*)

When wheels or pulleys transmitting only a small amount of power are keyed on large shafts, the dimensions above are excessive. In that case let H.P. be the horses-power transmitted by the wheel or pulley, N the number of rotations per minute; or let P be the force acting at its circumference in lbs., and R its radius in inches. Then it is sufficient to take in the expressions above,

$$d = \sqrt[3]{\frac{100 \text{ H.P.}}{\text{N}}} \text{ or } \sqrt[3]{\frac{\text{P R}}{630}}. \quad . \quad . \quad (13)$$

COTTERS.

93. A cotter is a tapered bar driven through two pieces which are to be connected, and prevents their separation by resistance to shearing at two cross sections. The cotter should be so designed as to diminish as little as possible the strength of the connected pieces. When the cotter is long it serves to adjust the length of the pieces connected. When driven home it shortens the total length, and *vice versâ.*

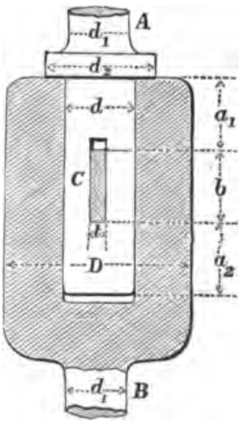


Fig. 93.

The simplest form of cotter is shown in fig. 93. In fig. 94 *a*, *b*, the fixing resists equally a thrust or tension. Fig. 94 *c* is an arrangement for resisting tension only, often used for foundation bolts. The enlarged or gib ends of the cotter prevent its displacement.

Fig. 94 *e*, *f*, shows another arrangement for resisting only a tension, the cotter being divided into two parts

termed respectively the gib and cotter. The cotter way is here exactly parallel. The rod is diminished beyond the connected pieces to a section equivalent to that through the cotter way. Fig. 94*g* shows an arrangement for both thrust and tension, often used for pistons and piston rods.

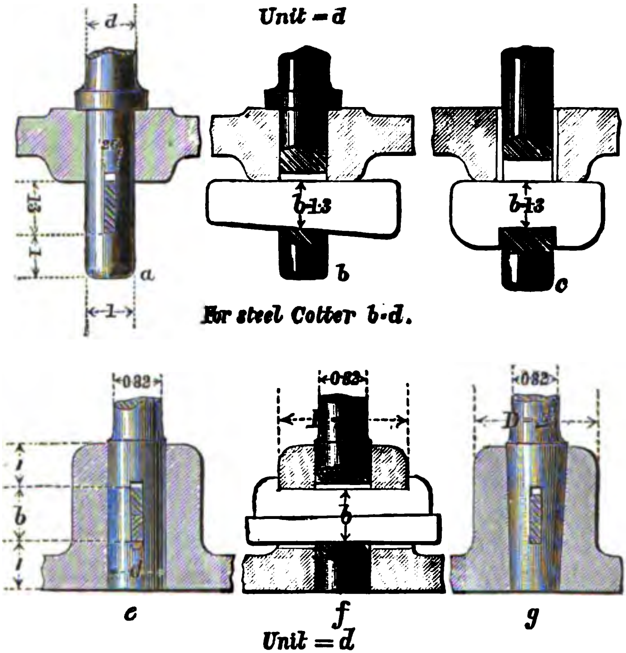


Fig. 94.

The diameter of the rod through the cotter hole should be 1¼ times the net diameter in the untapered part. The sides of the tapered part slope at an angle of 1 in 30 to 1 in 60 with the axis of the rod. The total taper is therefore 1 in 30 to 1 in 15.

94. *Taper of cotter.*—It is easy to show for cotters as

for keys that the horizontal force necessary to drive the cotter home is

$$Q_1'+Q_2'=P\{\tan(\theta+\alpha_1)+\tan(\theta+\alpha_2)\}$$

where θ is the angle of repose of the materials. Similarly the force necessary to drive back the cotter is

$$Q_1''+Q_2''=P\{\tan(\theta-\alpha_1)+\tan(\theta-\alpha_2)\}$$

and the cotter will slip back without any additional force if

$$Q_1''+Q_2''=0, \text{ and then } \alpha_1+\alpha_2=2\theta.$$

Taking θ for slightly greasy metal at $4\frac{1}{2}°$,

$$\alpha_1+\alpha_2 \text{ must not exceed } 9°$$

corresponding to a total taper of 1 in 7. The taper in practice is less than this for safety. For simple cotters a taper

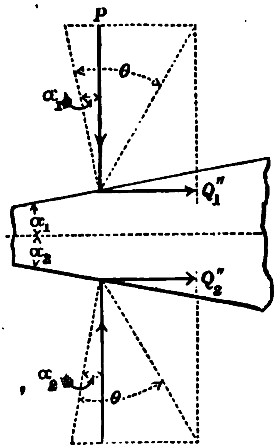


Fig. 95.

of 1 in 24 to 1 in 48 is usual. When a set screw or other means of preventing the slacking of the cotter is added, the taper may be 1 in 8 or 1 in 6.

95. *Strength and proportions of cotters.*—Let P be the longitudinal stress transmitted along the cottered rods. f_t, f_s, f_c the safe stress on the material for tension, shearing

and compression. Then the stresses on the different sections at which the cottered parts (fig. 93) may give way are :—

For the small parts of the rods A and B, $P=\frac{\pi}{4}d_1^2 f_t$ . . (1)

For the section of A through cotter holes

$$P=\left(\frac{\pi}{4}d^2-dt\right)f_t \quad . \quad . \quad . \quad (2)$$

For the section of B through the cotter holes

$$P=\left\{\frac{\pi}{4}(D^2-d^2)-(D-d)t\right\}f_t \quad . \quad . \quad (3)$$

For the two sections at which the cotter shears

$$P=2\,b\,t\,f_s \quad . \quad . \quad . \quad . \quad (4)$$

For the bearing surface of the cotter on A

$$P=d\,t\,f_c \quad . \quad . \quad . \quad . \quad . \quad (5)$$

For the bearing surface of the cotter on B

$$P=(D-d)\,t\,f_c \quad . \quad . \quad . \quad . \quad (6)$$

For the bearing surface of the collar on A on the end of B

$$P=\frac{\pi}{4}(d_2^2-d^2)f_c \quad . \quad . \quad . \quad . \quad (7)$$

The values of the limiting stresses may be as follows :—

	Wrought iron	Cast iron	Steel
Tearing resistance f_t	10,400	3,600	15,000
Shearing resistance f_s	8,320	2,700	12,000
Crushing resistance f_c	20,800	20,800	30,000

Using these values, we get the following table :—

Proportions of Cotters.

	Thickness of Cotter t From (2) and (5)	Width of Cotter b From (4) and (2)	Diameter d_1 From (1) and (2)	Diameter D From (2) & (6)	Diameter D From (2) & (3)	Diameter d_2
A, B and C all wrought iron .	0·26d	1·26d	0·82d	2d	1·53d	1·61d
A, B wrought iron, C steel . .	0·26d	1·11d	0·82d	2d	1·53d	1·61d
A wrought, B cast iron, C steel	0·26d	1·11d	0·82d	2d	2·1 d	1·61d

There are two values of D of which the larger is to be taken. Cases will be found in practice where the proportions of the cotter seem less than the above, but in such cases the rod A is, for stiffness or some other reason, of excessive dimensions. The cotter is often made of steel because it is harder and not so easily set up by hammering. The dimensions a_1 a_2 might be calculated for shearing, but as the shearing resistance, longitudinally, in wrought iron is less than transversely, it is better to take

$$a_1 = a_2 = b$$

which allows a considerable margin of safety.

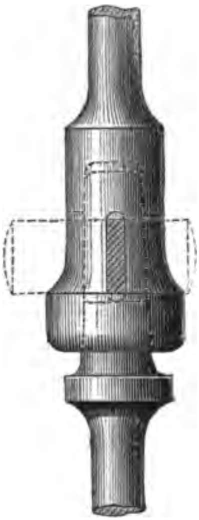


Fig. 96.

96. *Various arrangements of cotters.*—Fig. 96 shows the mode of cottering pump rods. The socket is conical. The collar is provided merely to facilitate the disengagement of the parts after they have been for some time in use. The cotter having been driven out, wedges can be driven between the collar and socket so as to force them apart.

Split pins are virtually very small cotters, driven home like ordinary cotters and the split end opened out to prevent slacking back (fig. 97).

Fig. 98 shows a bolt cottered into a casting. The effective diameter of the bolt is the diameter at the bottom

Fig. 97.

Fig. 98.

of the screw thread, and that is the value to be taken for

d in proportioning the cotter. The proportions marked on the figure have been so modified that the unit is the gross diameter *d* of the bolt.

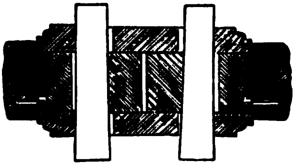


Fig. 99.

Fig. 99 shows a cottered coupling, suitable for rods transmitting a longitudinal force.

If excessive taper must be given, to obtain sufficient draught, the end of the cotter is screwed, as shown in fig. 100,

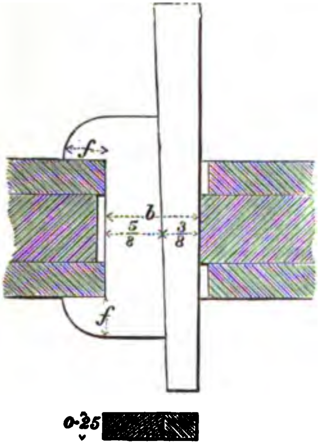
Fig. 100.

and a nut, bearing on a recessed washer or short tube, holds it in place.

Gib and Cotter.

97. When a cotter is used to connect strap-shaped parts to a more rigid rod, the cotter is divided into two parts, one acting as an ordinary cotter, the other having hooked ends, intended to prevent the spreading of the strap. It is convenient to make the outside of the gib parallel to the outside of the cotter, and to obtain the necessary draught, by inclining the division plane between them. The taper is usually 1 in 24 to 1 in 48 for simple cotters, and 1 in 8 to 1 in 16, when the slacking of the cotter is prevented by a

screw. The total breadth *b* and thickness *t* of the gib and cotter, are the same as for a simple cotter.



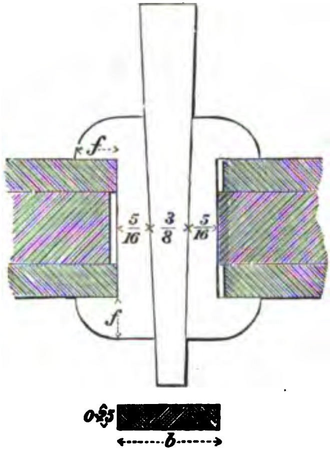


Fig. 101. Unit=*b*. Fig. 102.

Figs. 101, 102, show ordinary proportions of gibs and cotters. The unit for the proportions is the breadth *b*.

Fig. 103.

Fig. 104.

The simplest way of securing a cotter is by a screwed prolongation of the gib, as shown in fig. 103. A set screw,

passed through one of the connected pieces, is sometimes used.

Fig. 104 shows another arrangement of gibs and cotter. In this case the space is restricted, and the draught required is very small. The cotter is secured by a screwed end, nut and washer.

CHAPTER VI.

ON PIPES AND CYLINDERS.

98. PIPES and cylinders, subjected to internal pressure, form parts of many machines. The proper proportions of these, and the modes of jointing them, will form the subject of the present chapter.

CAST-IRON PIPES AND CYLINDERS.

Thickness of cast-iron pipes used for water mains.—The ordinary rule for the thickness of a cylindrical vessel, subjected to an internal or bursting pressure, is given in § 26, eq. 2.

Let t = thickness of cylinder, in ins.

d = diameter of cylinder, in ins.

p = excess of internal over external pressure, in lbs. per sq. in.

f = safe limit of stress, in lbs. per sq. in.

$$t=\frac{p\,d}{2\,f} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The average tenacity of the cast iron used for pipes, may be taken at 18,500 lbs. per sq. in. Taking the factor of safety at $3\frac{1}{3}$, the highest safe tension is 5,500 lbs. per sq. in. Allowance must, however, be made,—(*a*) for the irregular thickness of cast-iron pipes, which are often slightly thinner on one side than on the other; (*b*) for stresses due to hydraulic shock in the pipe, and to bending in consequence

of pressure of the earth above, or settlement of the earth beneath the pipe. A sufficient allowance will be made, if the pipe is calculated for three times the actual working pressure, or, what amounts to the same thing, if the limit of stress is taken at one-third the value given above. Hence, the apparent factor of safety for pipes is $3 \times 3\frac{1}{3} = 10$, and the greatest safe stress, due to the actual pressure in the pipe, is 1,850 lbs. per sq. in.

In the mains used for the conveyance of water, the external pressure is 1 atmosphere, or 33 ft. of water pressure, and the greatest internal pressure is generally less than 7 atmospheres, or 231 ft. of water. Hence, the excess of internal over external pressure may be taken at 6 atmospheres, or 90 lbs. per sq. in. Putting this value in the formula above, we get

$$t = \frac{90\,d}{2 \times 1850} = \cdot 0231\,d. \qquad (2)$$

Internal diameter of pipe in ins.								
4	8	12	16	20	24	30	36	42
Thickness of pipe in ins.								
0·0924	0·185	0·277	0·370	0·462	0·554	0·693	0·832	0·970
Thickness to nearest sixteenth of an inch.								
$\frac{1}{8}$	$\frac{3}{16}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{11}{16}$	$\frac{13}{16}$	1

99. This table shows that some of the thicknesses given by the above rule, although ample margin of strength has been allowed, are so small that the pipes could not be cast with any certainty of success. Many years ago, the following rule for the thickness of water mains was given by Mr. Hawksley,

$$t = 0{\cdot}18\sqrt{d}$$

That rule represents, very fairly, the least thickness which it is desirable to attempt to cast. The following rule agrees still better with practical experience. Let $t_{\text{min.}}$ be the least thickness which should be adopted for a cylindrical pipe casting, of ordinary length and of diameter d, in order that there may be no special difficulty in getting it cast. Then

$$t_{\text{min.}} = 0{\cdot}11\sqrt{d} + 0{\cdot}1 \quad . \quad . \quad . \quad (3)$$

Diameter of pipe in ins.											
4	8	12	16	20	24	30	36	42	48	54	60
Least thickness of pipe in ins.											
·320	·411	·481	·540	·592	·639	·703	·760	·813	·862	·908	·953
Thickness to nearest sixteenth of an inch.											
$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$	$\frac{15}{16}$	1

Pipes of the thicknesses here given, will in general be safe for pressures not exceeding 6 atmospheres, or 90 lbs. per sq. in., when under 20 ins. diameter, and for 5 atmospheres, or 75 lbs. per sq. in., when under 60 ins. diameter. When pipes are subjected to greater pressure, it is desirable to use the more exact formula given in § 26 (eq. 3), in calculating the thickness. Putting in that formula $f = 1850$, it becomes

$$\frac{t}{d} = \frac{1}{2}\left\{\sqrt{\frac{2775+p}{2775-p}} - 1\right\} \quad . \quad . \quad . \quad . \quad (4)$$

From this formula the following table has been calculated.

Excess of Internal over External Pressure		Ratio of thickness to diameter of pipe
In lbs. per square inch	In feet of head of water	$\frac{t}{d}$
75	173	·021
90	208	·026
105	242	·030
120	277	·035
135	311	·039
150	346	·044
165	381	·048
180	415	·053
195	450	·058
210	484	·063
225	519	·068
250	577	·077
275	634	·085
300	692	·095
350	808	·114
400	923	·134
450	1039	·156
500	1154	·179
750	1731	·332
1000	2308	·603

100. In the following table, the second and third columns give the least thickness of pipe which it is practicable to cast, which is the thickness to be adopted when equations (1) or (4) give a less value. The other columns give thicknesses calculated by equation (4). It should be remembered, that in obtaining these thicknesses an allowance has been made for bending stress, and hence somewhat less thicknesses may be adopted, in pipes so supported as to be protected from any bending. To convert feet of head of water into lbs. per sq. in., multiply by 0·4333. In water mains for towns, a thickness about 25 per cent. greater than that given in this table is often adopted in practice.

Internal diameter of pipe in ins.	Least thickness of pipe by eq. (3)		Thickness necessary for strength, by eq. (4), for working internal pressures in lbs. per sq. in. and ft. of head amounting to							
	Exact	Correct to nearest sixteenth	75 lbs. 173 ft.	90 lbs. 208 ft.	105 lbs. 242 ft.	120 lbs. 277 ft.	150 lbs. 346 ft.	180 lbs. 415 ft.	210 lbs. 484 ft.	250 lbs. 577 ft.
2	·256	$\frac{1}{4}$	...	...	...	...	...	...	...	...
3	·291	$\frac{5}{16}$	...	...	...	...	...	...	...	...
4	·320	,,	...	...	...	...	...	...	...	...
5	·346	$\frac{3}{8}$	...	...	...	...	...	...	...	·385
6	·369	,,	...	...	...	...	...	...	·378	·462
7	·391	,,	...	...	...	...	...	...	·441	·539
8	·411	$\frac{7}{16}$	...	...	...	...	...	·424	·504	·616
9	·430	,,	...	...	...	...	...	·477	·567	·693
10	·448	$\frac{1}{2}$	...	...	...	...	...	·530	·630	·770
12	·481	,,	...	...	...	...	·528	·636	·756	·924
14	·512	$\frac{9}{16}$	...	...	...	...	·616	·742	·882	1·08
16	·540	,,	...	...	...	·560	·704	·848	1·01	1·23
18	·567	,,	...	...	...	·630	·792	·954	1·13	1·39
20	·592	$\frac{5}{8}$	...	...	...	·700	·880	1·060	1·26	1·54
22	·616	,,	...	...	·66	·770	·968	1·166	1·39	1·69
24	·639	,,	...	...	·72	·840	1·056	1·272	1·51	1·85
30	·702	$\frac{11}{16}$	...	·780	·90	1·05	1·320	1·590	1·89	2·31
36	·760	$\frac{3}{4}$	...	·936	1·08	1·26	1·584	1·908	2·27	2·77
42	·813	$\frac{13}{16}$	·882	1·092	1·26	1·47	1·848	2·226	2·65	3·23
48	·862	$\frac{7}{8}$	1·008	1·248	1·44	1·68	2·112	2·544	3·02	3·70
54	·909	$\frac{15}{16}$	1·134	1·404	1·62	1·89	2·376	2·862	3·40	4·16
60	·952	,,	1·260	1·560	1·80	2·10	2·640	3·180	3·78	4·62
72	1·033	1	1·512	1·872	2·16	2·52	3·168	3·816	4·54	5·54
84	1·108	$1\frac{1}{8}$	1·764	2·184	2·52	2·94	3·696	4·452	5·29	6·47

101. *Thickness of steam pipes and steam cylinders of cast iron.*—This is determined in precisely the same way as the thickness of water mains. For steam pipes, the thicknesses given in the preceding table will answer. For steam cylinders an allowance has to be made for re-boring. The cylinder thickness may be obtained from the following equation:

$$t=\frac{p\,d}{3700}+c=\cdot 00027\,p\,d+c \quad . \quad . \quad . \quad . \quad (5)$$

where c ranges from 0·5 to 0·75 in carefully constructed engines, and is as much as 1·0 in some cases. The thickness of engine cylinders is determined almost entirely with respect to facility of casting, and is generally excessive as regards strength.

WROUGHT-IRON AND STEEL TUBES.

102. Wrought-iron and steel pipes or tubes are obtainable of the following descriptions:—

(1) Butt-welded pipes used for gas, water, and steam, of wrought iron. These are made of from $\frac{1}{8}$ inch to 4 in.

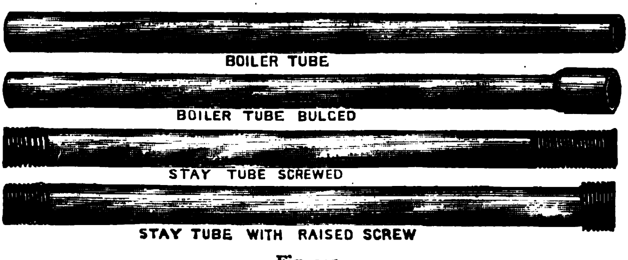


Fig. 105.

internal diameter, and in lengths usually not exceeding 14 feet. They can be obtained up to 20 feet in length, if necessary. The steam tubes are two gauges thicker than the gas tubes.

Wrought-Iron Tubes.

Internal diameter in ins.	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼	3½	3¾	4	4¼	4½	4¾	5	5¼
Thickness B. W. G.	9	9	9	9	8	8	8	8	7	7	7	7	7	6	6	6	6	5
Thickness in ins.	0·158	·158	·158	·158	166	·166	·166	·166	187	·187	·187	187	187	208	208	208	·208	·217
Greatest working pressure in lbs. per sq. in.	1720	1370	1145	980	930	825	745	675	745	690	640	595	560	615	580	550	520	530
Greatest length	Usual length 12 to 14 feet. Can be made to 20 feet.																	

Internal diameter in ins.	5½	5¾	6	6¼	6½	6¾	7	7½	8	8½	9	9½	10	10½	11	12
Thickness B. W. G.	5	5	5	4	4	4										
Thickness in ins.	·217	217	217	239	·239	239	0·25	0·25	0·25	0·3125	0·3125	0·3125	0·375	0·375	0·375	0·375
Greatest working pressure in lbs. per sq. in.	505	485	460	505	485	470	485	455	425	525	500	470	565	540	515	480
Greatest length	As above.		Length 16 or 17 feet.													

(2) Lap-welded wrought-iron steam tubes, which are stronger than butt-welded tubes. They are proved to 400 lbs. per sq. in. before being sent out. The preceding table gives the usual dimensions of these tubes.

The thickness of such tubes will not be exactly regular. Suppose that when t is the nominal thickness, $t-\frac{1}{16}$ is the effective thickness which can be relied on in calculating the strength. Then by equation (2) § 26,

$$f=\frac{p\,d}{2(t-\frac{1}{16})};$$

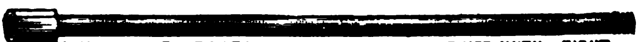
and taking the greatest safe stress at 4 tons or 8,960 lbs. per sq. in.

$$t=\cdot0000558\,p\,d+\tfrac{1}{16} \quad . \quad . \quad (6)$$

By this rule the working pressures p for different diameters d, given in the table above, have been calculated. Lap-welded steam tubes of the dimensions given above are proved to 400 lbs. per sq. in. before being sent out.

Figs. 105, 106, show the forms in which wrought-iron and steel tubes can be obtained.

HYDRAULIC TUBE (PLAIN)



HYDRAULIC TUBE WITH HEXAGON SOCKET SCREWED WITH RIGHT & LEFT HAND THREADS

Fig. 106.

(3) Weldless steel tubes are also manufactured, which are used for boiler tubes, hydraulic pipes, and occasionally for other purposes, such as hollow shafting and boring rods. The following tables give the usual dimensions of such tubes.

Weldless Steel Boiler Tubes.

External diameter in inches	$1\frac{1}{2}$	$1\frac{5}{8}$	$1\frac{3}{4}$	$1\frac{7}{8}$	2	$2\frac{1}{8}$	$2\frac{1}{4}$	$2\frac{3}{8}$	$2\frac{1}{2}$	$2\frac{5}{8}$	$2\frac{3}{4}$	$2\frac{7}{8}$	3	$3\frac{1}{8}$	$3\frac{1}{4}$	$3\frac{3}{8}$	$3\frac{1}{2}$
Thicknesses B. W. G.	16 to 10	16 to 10	16 to 10	16 to 10	16 to 10	15 to 10	15 to 10	14 to 10	14 to 10	14 to 10	14 to 10	13 to 10	13 to 10	13 to 10	13 to 10	13 to 10	13 to 10
Length in feet	13 feet.											10 feet.					

Weldless Steel Boiler and Hydraulic Tubes.

External diameter in inches		$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{1}{2}$	$1\frac{5}{8}$	$1\frac{3}{4}$	$1\frac{7}{8}$	2	$2\frac{1}{8}$	$2\frac{1}{4}$	$2\frac{3}{8}$	$2\frac{1}{2}$	$2\frac{5}{8}$	$2\frac{3}{4}$	$2\frac{7}{8}$	3	$3\frac{1}{8}$	$3\frac{1}{4}$	$3\frac{3}{8}$	$3\frac{1}{2}$
Thicknesses in inches	Least	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{3}{32}$
	Greatest	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
Maximum length 15 feet.																										

The steel of which these tubes are made has a tensile strength of about 30 tons per sq. in. For hydraulic tubes the working strength may be taken at 10 tons per sq. in. Putting this for the value of f in eq. 2, § 26, we get

$$t = 0{\cdot}00002232\,p\,d, \quad . \quad . \quad (7)$$

where p is the working pressure in lbs. per sq. in., and d the internal diameter of the tube.[1]

103. *Thickness of pipes of other materials:*

For lead pipes $t = {\cdot}0025\,p\,d + \frac{3}{16}$

copper pipes $t = {\cdot}00018\,p\,d + \frac{1}{16}$

wrought-iron pipes $t = {\cdot}00006\,p\,d + \frac{1}{16}$ (if welded)

$= {\cdot}00012\,p\,d + \frac{1}{16}$ (if riveted)[2]

Copper steam, blow-off, or water pipes for engines are usually $\frac{1}{4}$ in. thick; feed water pipes, $\frac{3}{16}$ in.; and exhaust steam pipes, $\frac{1}{8}$ in. thick.

Pipe Joints.

104. Cast-iron pipes are connected by flange-joints, or by spigot and faucet joints. The former are stronger,

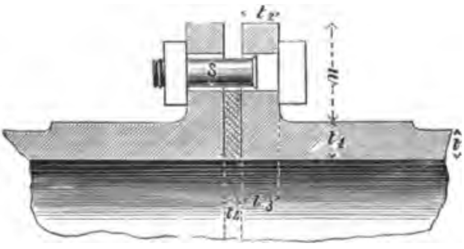


Fig. 107.

easier to connect or disconnect, and are always used when the pipes are placed vertically. The latter are less costly,

[1] Messrs. E. Lewis and Sons of Wolverhampton, and the Weldless Steel Tube Company of Birmingham, supplied the information as to the dimensions of tubes.

[2] See, however, the more exact rules for riveted boilers in § 73.

and are better for pipes laid in the ground, because they permit the pipes to adapt themselves to the inequalities of the ground while being laid, and the line of pipes retains a slight flexibility.

105. *Flanged Joints.*—The proportions of flanges have been to some extent given in § 85. Fig. 107 shows one form of flanged joint for pipes, for which the following proportions may be used:—

$$\text{Thickness } t_1 = t_2 = \tfrac{5}{4}\,t$$

$$t_3 = \tfrac{3}{2}\,t$$

$$t_4 = \tfrac{3}{8} \text{ in.}$$

$$\text{Width } = w = 2\,\delta + 1$$

$$\text{Diam. of bolts} = \delta = 0{\cdot}016\,d\sqrt{\frac{p}{n}} + 0{\cdot}4$$

$$\text{Number of bolts} = n = 2 + \frac{d}{2}$$

$$\text{Diam. of bolt-hole} = \delta + \tfrac{1}{8}$$

The joint shown is made with a lead ring. The joint may be made by facing the flanges, and bringing them together with a string smeared with red lead, or an india-rubber or gutta-percha ring interposed. A rough joint is made with a ring of wrought iron, covered with tarred rope, the space between the flanges being filled up with rust cement.

106. Mr. Robert Briggs laid down proportions for flanged pipes some years since, which have been very widely adopted in American practice. As these have been tested by a large experience, it will be useful to quote them here for comparison with the rules given above. These rules are intended to apply (1) to pipes under pressures up to 75 lbs. per sq. in. (or 170 feet of head), the flanges being faced all over, and having bolts with hexagon nuts and heads; (2) to pipes for pressures up to 100 lbs. pressure per sq. in. (230 feet of head), with flanges thick enough to permit the use of packing rings placed within the bolts as a substitute for surfacing and wide enough for bolts with

square nuts and heads. The form of Mr. Briggs' rules has been modified so as to render them applicable to pipes of thicknesses different from those given by him. Fig. 108 gives the reference symbols.

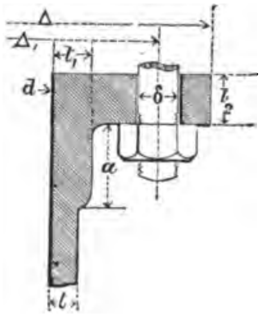


Fig. 108.

Mr. Briggs's Rules for Pipe Flanges.

	Pipes for 75 lbs. pressure.	Pipes for 100 lbs. pressure.
Thickness of pipes $=t$	$=0{\cdot}026d+0{\cdot}25$	$0{\cdot}0304d+0{\cdot}3$
Thickness of boss $=t_1$	$=1{\cdot}2t$	$1{\cdot}15t+0{\cdot}1$
Length of boss $=a$	$=1{\cdot}92t+0{\cdot}62$	$1{\cdot}64t+0{\cdot}61$
Thickness of flange (finished)	$=1{\cdot}28t+0{\cdot}08$	$1{\cdot}31t+0{\cdot}2$
,, ,, (rough)	$=1{\cdot}41t+0{\cdot}10$	$1{\cdot}48t+0{\cdot}42$
Radius of hollow at angle	none	$0{\cdot}26t+0{\cdot}08$
Diameter of flange $=\Delta$	$=\left\{\begin{matrix} d+2{\cdot}4t+ \\ 4{\cdot}28\delta+0{\cdot}6 \end{matrix}\right\}$	$\left\{\begin{matrix} d+2{\cdot}82t+ \\ 5\delta+0{\cdot}66 \end{matrix}\right.$
Diameter of bolt circle $=\Delta_1$	$=\left\{\begin{matrix} d+2{\cdot}4t+ \\ 2{\cdot}08\delta+0{\cdot}4 \end{matrix}\right\}$	$\left\{\begin{matrix} d+2{\cdot}82t+ \\ 2{\cdot}5\delta+0{\cdot}56 \end{matrix}\right.$
Diameter of holes in flange	$=1{\cdot}03\delta+0{\cdot}03$	$1{\cdot}03\delta+0{\cdot}03$
Number of bolts $=n$	$=0{\cdot}705d+2{\cdot}18$	$0{\cdot}705d+2{\cdot}18$

Diameter of bolts $=\delta=$

$$1{\cdot}182\sqrt{\left\{\frac{0{\cdot}01785\,\Delta^2{}_1+0{\cdot}2052}{n}\right\}}+0{\cdot}0492$$

in which equation we may take $\Delta_1=1{\cdot}1043d+2{\cdot}01$. The

following rule is much simpler, and gives nearly the same values :

$$\delta = \frac{0{\cdot}1674d + 0{\cdot}305}{\sqrt{n}} + 0{\cdot}05.$$

The following table gives dimensions calculated by Mr. Briggs' rules.

Cast-Iron Flanged Pipes.

	Internal diameter of pipe	Thickness of body	Thickness of boss	Length of boss	Thickness of flange finished	Thickness of flange rough	Diameter of bolt holes	Outside diameter of flange	Diameter of bolt circle	Number of bolts	Diameter of bolts
For 75 lbs. pressure or 170 feet of head	3	·328	·40	1·25	·50	·56	·55	6½	5¼	4	$\frac{1}{2}$
	3½	·341	·42	1·28	·51	·57	·61	7¼	$5\frac{9}{10}$	4	$\frac{9}{16}$
	4	·354	·43	1·30	·53	·59	·61	8	$6\frac{7}{16}$	5	$\frac{9}{16}$
	5	·380	·46	1·35	·56	·63	·61	9	$7\frac{1}{8}$	6	$\frac{9}{16}$
	6	·406	·49	1·40	·60	·67	·68	10¼	$8\frac{11}{16}$	6	$\frac{5}{8}$
	8	·458	·55	1·50	·66	·74	·68	12½	$10\frac{8}{10}$	8	$\frac{5}{8}$
	10	·510	·61	1·60	·73	·81	·81	15	$13\frac{3}{16}$	10	$\frac{3}{4}$
	12	·563	67	1·70	:80	·89	·93	17¾	$15\frac{9}{16}$	10	$\frac{7}{8}$
	16	·667	·79	1·90	·93	1·01	93	22	$19\frac{8}{10}$	14	$\frac{7}{8}$
For 100 lbs. pressure or 230 feet of head	3	·383	·55	1·25	·72	·8	·61	7½	6	4	$\frac{9}{16}$
	3½	·398	·56	1·28	·74	·82	·61	8	6½	5	$\frac{9}{16}$
	4	·414	·58	1·30	·76	·84	·68	9	7¼	5	$\frac{5}{8}$
	5	·444	·62	1·35	·80	·89	·68	10	8⅜	6	$\frac{5}{8}$
	6	·474	·65	1·40	·84	·93	·68	11	9⅜	6	$\frac{5}{8}$
	8	·535	·72	1·50	·92	1·02	·68	13½	11¼	8	$\frac{5}{8}$
	10	·596	·79	1 60	1·00	1·11	·81	16	14	10	$\frac{3}{4}$
	12	·657	·86	1·70	1·08	1·20	·93	19	16¾	10	$\frac{7}{8}$
	16	·778	1·00	1·90	1·24	1·38	·93	23½	21	14	$\frac{7}{8}$

Fig. 109 shows a pipe joint, where one flange is loose. The joint is made tight by a lead ring, well stemmed in.

Fig. 110 shows what is termed a lens joint, which is very easily made tight, and adapts itself to a slight flexure in the direction of the pipe. The joint is made by a gun-metal ring, turned to spherical surfaces.

Fig. 111 shows the joint used by Sir W. Armstrong, for

the pipes of his accumulator. These pipes are subjected to the enormous water-pressure of 800 lbs. per sq. in. The pipes are of the best remelted cast-iron, and are tested to

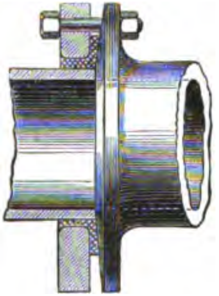


Fig. 109.

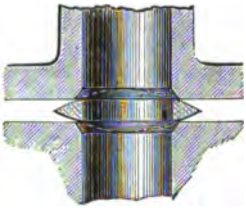


Fig. 110.

3,000 lbs. per sq. in.. When 5 ins. diameter they are 1 in. thick. Each end of the pipe has two strong elliptical flanges, with two bolts. One pipe slightly enters into the

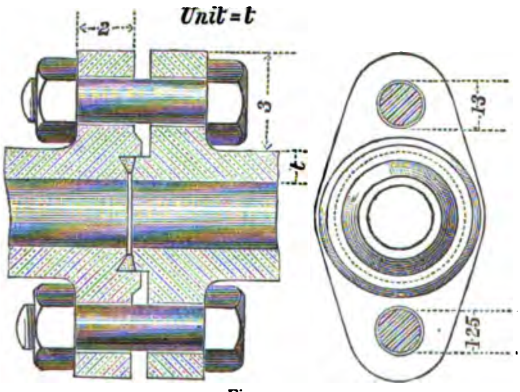


Fig. 111.

other, forming a dove-tailed recess, in which is placed a gutta-percha ring, $\frac{1}{4}$ in. diameter.

107. *Cylinders for great internal pressure.*—In the construction of vessels for great hydraulic or other pressure, special

difficulties arise. If made of boiler plate, the riveted joints diminish the strength of the vessel and limit the thickness of the plates which can be used. If cast iron is adopted, the vessel must be ponderous in consequence of the low tenacity of the material, and the fluid sometimes escapes through porous parts of the casting. Dr. Siemens has described the construction of an air reservoir, to sustain 1,000 lbs. pressure per sq. in., of steel rings. These rings (40 in. in diameter and 12 in. in depth) were rolled out of ingots in a tyre mill, and had a slight flange at their edges. The ends of the reservoir were made hemispherical, and beaten out of steel plate. The joints were made by turning a V-groove in the faces of the rings and placing in it a packing ring made of $\frac{5}{16}$ annealed copper wire. The rings and ends were held together by 20 steel longitudinal bolts, passing through two rings bearing on the hemispherical ends of the cylinder. These bolts were $1\frac{1}{4}$ in. in diameter with enlarged screwed ends.[1]

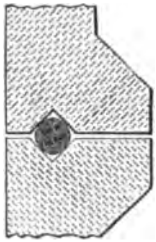


Fig. 112.

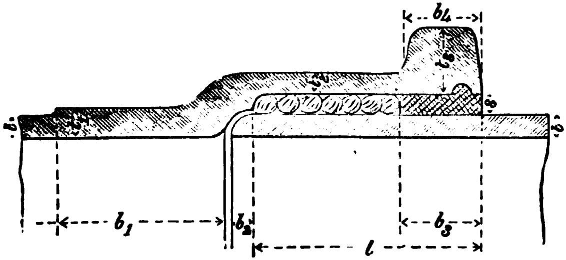


Fig. 113.

108. *Socket Joints.*—Socket pipes should be cast vertically with the socket at bottom. Socket pipes are jointed either with a gasket and lead joint, a rust joint, or a bored and turned joint. Fig. 113 shows an ordinary lead joint. When the pipes are in place, a few coils of gasket or tarred rope

[1] Proc. Inst. Mech. Eng. April 1878.

are driven into the socket. Clay is then put round the outside of the socket, and a lead ring is cast in it. The clay is removed and the lead stemmed tightly into the socket. The proportions may be as follows: Let t=thickness, and d=diameter of pipe;

$$t_1 = 1{\cdot}07\ t + \tfrac{1}{16}$$
$$t_2 = 0{\cdot}025\ d + \tfrac{1}{4} \text{ to } 0{\cdot}025\ d + 0{\cdot}6$$
$$t_3 = 0{\cdot}045\ d + 0{\cdot}8$$
$$s = 0{\cdot}01\ d + {\cdot}25 \text{ to } 0{\cdot}01\ d + {\cdot}375$$
$$b_1 = 0{\cdot}075\ d + 2\tfrac{1}{4}$$
$$b_2 = t_2$$
$$l = 0{\cdot}09\ d + 2\tfrac{3}{4} \text{ to } 0{\cdot}1\ d + 3$$
$$b_4 = b_3 = 0{\cdot}03\ d + 1$$

A rust joint is very similar to a lead joint except that iron cement is stemmed in with a cold chisel in place of the lead. The iron cement consists of cast-iron borings or turnings, which should be passed through a sieve of eight meshes to the inch. One ounce of sal ammoniac is added to each hundredweight of cast iron, and the mass is damped. When it has heated, it may be kept for some time in water.

Fig. 114 shows a different form of socket. The proportions may be the same as those just given.

Figs. 115, 116 show two forms of bored and turned socket and spigot joints. When the bored and turned part is long, the pipes are rigid, and are liable to be broken by the earth pressure. Hence, the fitting part is now often only $\tfrac{5}{8}$ in. in length,

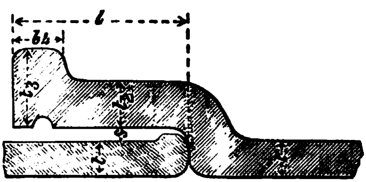


Fig. 114.

and has a very slight taper. The joint is made by painting over the faced part with red-lead, or with fresh and liquid Portland cement. The pipe is then put in place, and driven

home by a wooden mallet, or by swinging the next length of pipe. The socket is filled up with cement. Joints of this

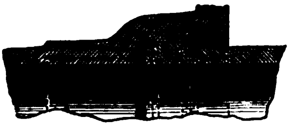


Fig. 115.

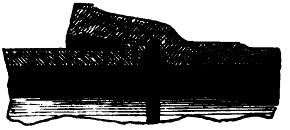


Fig. 116.

kind are more easily and quickly made than lead joints, but lead joints are preferable in passing round curves. Socket pipes should be cast with the socket downwards, and about a foot of length should be allowed at the spigot end, into which the scoriæ may rise, and which is broken off when the pipe is cast.

109. With socket pipes care must be taken to provide against the end thrust due to the water pressure at bends. At a right-angled bend, the total pressure in the direction of the tangents to the bend on a pipe of diameter d feet, through which water is passing under a pressure head of h feet, and with a velocity of v feet per second, is

$$\frac{\pi}{4} d^2 \text{G} \left(h + \frac{v^2}{2g} \right) \text{lbs.}$$

Fig. 117.

where G=62·4 lbs. Hence the resultant pressure bisecting the angle of the bend is

$$1{\cdot}414 \frac{\pi}{4} d^2 \text{G} \left(h + \frac{v^2}{2g} \right) \text{lbs.} = 69{\cdot}29 d^2 \left(h + \frac{v^2}{2g} \right)$$

This must be provided against by casting a foot on the pipe and abutting it against a block of masonry.

A good joint for pipes used for temporary purposes is

shown in fig. 117. A loose tapered double socket is fitted over the ends of the pipes. The joints are made by india-rubber rings driven into the sockets.

110. *Joints for lead pipes.*—Lead pipes are useful, because they are easily bent. They are manufactured by drawing them over a mandril by hydraulic pressure. They are sometimes lined with tin, when used to convey water which dissolves the lead. Joints may be made by flanging out the ends of the pipes, and compressing these flanges between two iron rings, with bolts. Commonly, the joint is made by soldering, and is termed a plumber's wiped joint, fig. 118.

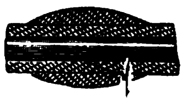


Fig. 118.

111. Fig. 119 shows two ways of making a socket joint for wrought-iron pipes. The sockets are of cast iron. Wrought iron is chiefly used for very large or very small pipes. From its thinness, it is liable to more injury from corrosion than cast iron, and hence it has been suggested that large wrought-iron mains should be lined with a thin coating of Portland cement.

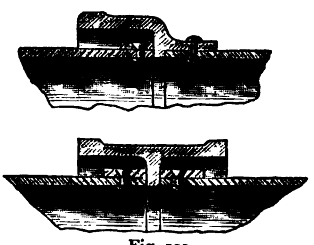


Fig. 119.

Joints for wrought-iron and steel tubes.—Fig. 120 shows the ordinary forms of joints. At *a* one pipe is bulged and the other screwed into it. At *b* an internal screwed ferrule or socket piece is inserted. At *c*, an external socket is shown screwed on both pipe ends, and at *d* a similar socket piece of lighter form.

A very good form of joint is shown in fig. 121, where security against leakage under great pressure is necessary. The two pipe ends are screwed with right and left handed threads. The socket or coupling piece then draws the ends together into metallic contact. A union joint shown in fig.

122, may also be used, with a packing ring of leather or india-rubber, if necessary.

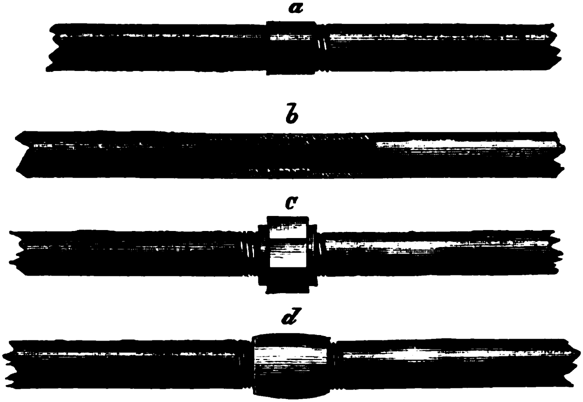


Fig 120.

Boiler tubes.—The modes of fixing boiler tubes in tube plates is shown in fig. 123. The lower figure shows an

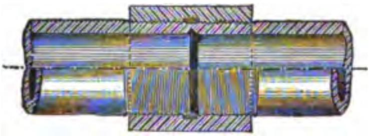


Fig. 121.

ordinary boiler tube, fixed at one end by slightly enlarging the tube and riveting over the end. At the other end a steel

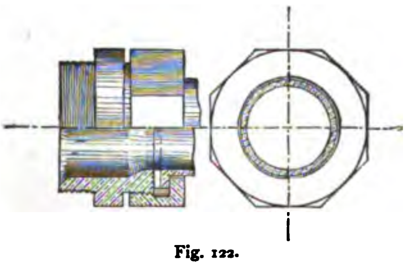


Fig. 122.

ferrule is driven in. The upper figure is a boiler tube adapted also to act as a longitudinal stay rod. For this purpose the ends are screwed and the tube fixed by a pair of thin nuts at each end. Sometimes one end of the stay tube is enlarged. This facilitates getting it into place.

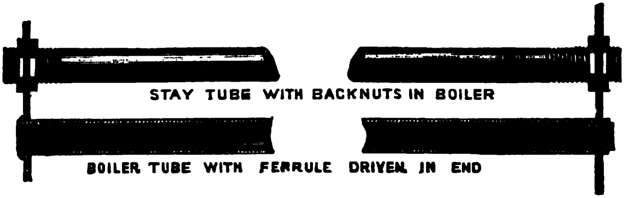


Fig. 123.

Boiler and stay tubes are subjected to external pressure and must be calculated by the rules for resistance to collapse.

Condenser tubes.—Condenser tubes are fixed in the tube plate in a different way. Generally some very simple form of stuffing box is used, which can be easily and cheaply made. Perhaps the simplest method is that shown in fig.

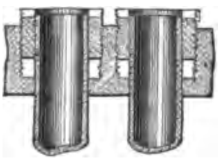


Fig. 124.

124. The tube plate is drilled with recesses round the tube ends which form stuffing boxes. The gland is simply a short piece of screwed tube cut off by a saw. The packing is a ring of tape. This method permits the free expansion of the condenser tube to occur without causing leakage.

CHAPTER VII.

JOURNALS, PIVOTS, AXLES AND SHAFTING.

JOURNALS.

112. JOURNALS are those parts of rotating pieces, which are supported by the frame of the machine. They are commonly cylindrical, but sometimes spherical or conical. Some journals run constantly, others support a piece which moves occasionally. In the latter case, the strength of the journal is chiefly to be considered; in the former, durability and freedom from liability to heat are as important as strength. Some journals are subjected to straining forces in the plane of their axis only, which produce bending and shearing stress. Others are subjected to bending and torsion, and are calculated by the rules for combined stress. Lastly, some journals are supported at one end only; others, which may be termed neck journals, are supported at both ends.

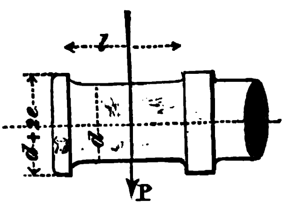


Fig. 125.

113. *Form of journals.*—The ordinary form of journals is shown in fig. 125. The bearing part of the journal is turned accurately cylindrical, and is terminated by raised parts or collars, which bear against the ends of the brass steps in which the journal revolves, and limit its end play.

The length of the brass step is, in some cases, 0·9 of the length of the journal. This permits a limited longitudinal motion, which ensures uniform wear of the step. In other cases, longitudinal play would interfere with the action of the machine, and it is made as small as possible. The proportional unit for fig. 125, is $e = \frac{d}{8} + \frac{1}{8}$ to $\frac{d}{10} + \frac{1}{8}$.

114. *Friction of Journals.*—It is not possible to estimate exactly the friction of journals, because the distribution of the pressure on the surface of the journal is dependent on the wear of the step. The frictional resistance to motion must be between the limits $\frac{\pi}{2} \mu \text{P}$ and $\frac{4}{\pi} \mu \text{P}$, where μ is the coefficient of friction, and P the load on the journal. It probably approaches the former limit when the journal is new, and the latter when it is worn. It is very commonly taken at μP simply, which it would be if the journal touched the step at a line only. Taking this value, which is accurate enough for journals in their ordinary condition, the value of μ, according to Morin's experiments, is 0·05 to 0·07 for good journals, well lubricated. Rühlmann and Waltjens' experiments give μ=0·02 to 0·03 for wrought iron on gun metal. Experience with railway journals appears to show that the coefficient of friction is often much less than this. Experiments by Kirchweger gave, for the coefficient of friction of railway axles, μ=0·009 to 0·01 for wrought iron on white metal, and μ=0·014 for wrought iron on gun metal, when the journals were lubricated with oil. Experiments by Bokelberg and Welkner gave coefficients ranging from 0·0028 for small loads and low velocities, to 0·013 for great loads and high velocities, and in these experiments the coefficients were less for gun metal than for white metal. The experiments are not accordant, but they show that the friction is sometimes much less than it would be if calculated from Morin's values. Generally the evidence is strong that for lubricated surfaces the friction increases with the velocity

of rubbing and the intensity of pressure. The work expended in friction at N revs. per min. is

$$=T=\mu P \times \frac{\pi}{12} d N \text{ ft. lbs. per min.} \quad . \quad . \quad (1)$$

115. *Length of journals.*—Common experience shows that for journals working at high speeds, a greater length is necessary than for journals running at low speeds. Journals running at 150 revs. per min. are often only one diameter long. Fan-shafts running at 1,500 revs. per min. have journals 6 or 8 diameters long.

If the journal works occasionally for short periods of time, it is desirable to make it short, because the less the length of the journal, the smaller its diameter may be for a given load, and consequently the less will be the friction. If the journal runs constantly, it requires a larger surface to ensure durability and coolness in working. The larger surface is better obtained by increasing the length than by increasing the diameter of the journal. In the former case, the friction remains the same, in the latter it is increased. Some increase of diameter is necessary to maintain equal strength, but the diameter should be increased only so much as is necessary for that purpose.

If a journal is strong enough, when of length l and diameter d, then if, for any reason, the length is increased to l_1, the diameter must be

$$d_1 = d \sqrt[3]{\frac{l_1}{l}} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

to obtain the same strength as before.

The length of journals, to ensure durability and cool working, depends not only on the pressure to which the journal is subjected, but also on the material of the journal and steps; on the kind of motion, whether continuous rotation or oscillation; on the perfection of the lubricating arrangements, and on the accuracy of workmanship. It is not surprising, therefore, that, in different cases, journals,

subjected to the same load and running at the same speed, should have different lengths.

The work T expended in friction produces a quantity of heat,

$$\text{H}=\frac{\text{T}}{\text{J}}=\mu\,\text{P}\;\frac{\pi\,d\,\text{N}}{12\,\text{J}}\quad . \quad . \quad . \quad . \quad (3)$$

where J is Joule's equivalent, or 772 778 ft. lbs. That heat should be dissipated as fast as it is generated, by conduction from the surface of the journal, and it appears to be a reasonable assumption that the rate of dissipation is proportional to the area of the surface through which the heat is conducted, or to the product $d\,l$. Let h units of heat be dissipated per minute, for each unit of bearing surface, $d\,l$. Then

$$h\,d\,l=\frac{\mu\,\text{P}\,\pi\,d\,\text{N}}{12\,\text{J}}$$

$$l=\frac{\mu\,\pi}{12\,\text{J}\,h}\times\text{P}\,\text{N}=\frac{\text{P}\,\text{N}}{\beta}\quad . \quad . \quad . \quad (4)$$

where β is a constant, to be ascertained by experience in different cases. This formula may be put in a more convenient form for crank-pins. Let H. P. be the indicated horses' power transmitted to the crank-pin, R the crank radius in inches; then the mean pressure on the crank-pin is

$$\text{P}=\frac{33000\times\text{H. P.}\times\cancel{12}}{\substack{4\,\text{R}\,\text{N}\\12}}\text{ nearly.}$$

Inserting this value in (4)

$$l=\gamma\;\frac{\text{H. P.}}{\text{R}}\quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where $$\gamma=\frac{99000}{\beta}\quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

For marine engine crank-pins, β ranges from 250,000 to 300,000, and γ from 0·4 to 0·33.[1]

[1] In three recent ships for the Navy with crank-shaft bearings 16″ diam. the value of γ is found to be 0·208, 0·32, and 0·345.

For stationary engine crank-pins, the length given is usually greater. The older engines, which worked at low pressures, give $\beta=66{,}000$; $\gamma=1{\cdot}5$. Some more modern engines give values of β ranging from 100,000 to 200,000, and of γ from 1·0 to 0·45.

The outside crank-pins of locomotives are necessarily contracted in length as much as possible. They are often of steel, and the workmanship is very good. Taking the average speed of the engine at 60 ft. per sec., and the steam pressure during the stroke at 100 lbs. per sq. in., we get $\beta=1{,}000{,}000$ to $1{,}500{,}000$, and $\gamma=0{\cdot}1$ to $0{\cdot}066$.

Railway-carriage axles are so largely used that they probably approach the minimum size consistent with durability. An axle $3\frac{1}{4}$ ins. in diameter, and 8 ins. long, would run cool under a load of $2\frac{1}{2}$ tons at 60 miles per hour. These data give $\beta=400{,}000$, and $\gamma=0{\cdot}25$.

116. If a journal of length l works satisfactorily at N revolutions under a load P, and l_1, N_1, P_1 are corresponding values for another journal, working in similar conditions, then, if the above theory is correct,

$$\frac{l_1}{l}=\frac{P_1 N_1}{P N} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

For railway journals, running at the same speeds, the length should be simply proportional to the load. For engines of the same kind, running at the same speed, the crank-pin length should be proportional to the load on the piston. For locomotive engines, working at the same pressure, the crank-pin length should be proportional to the piston area.

117. The following table is calculated for $\beta=250{,}000$ $\gamma=0{\cdot}4$. For any other values of β or γ, the length is the tabular length $\times \frac{250{,}000}{\beta}$, or tabular length $\times \frac{\gamma}{0{\cdot}4}$.

Theoretical Journal Length in Inches.

Load on Journal in lbs.	Revolutions of Journal per minute					
	50	100	200	300	500	1,000
1,000	·2	·4	·8	1·2	2·0	4·0
1,500	·3	·6	1·2	1·8	3·0	6·0
2,000	·4	·8	1·6	2·4	4·0	8·0
3,000	·6	1·2	2·4	3·6	6·0	12·0
4,000	·8	1·6	3·2	4·8	8·0	16·0
5,000	1·0	2·0	4·0	6·0	10·0	20·0
10,000	2·0	4·0	8·0	12·0	20·0	40·0
15,000	3·0	6·0	12·0	18·0	30·0	...
20,000	4·0	8·0	16·0	24·0	40·0	...
30,000	6·0	12·0	24·0	36·0	...	...
40,000	8·0	16·0	32·0	...	...	...
50,000	10·0	20·0	40·0	...	...	...

118. *Empirical formulæ for journal length.*—It is obvious that the constants β and γ vary greatly in different cases. Probably also the theory just given is imperfect. Hirn's experiments show that the friction of lubricated journals varies about as the square root of the velocity. If this is the case, the assumption above would have to be considerably modified. Possibly some empirical rule may be found to agree better with journals of widely different dimensions. The following rule is that of Von Reiche,

$$l = 0{\cdot}00102 \sqrt{P} \sqrt[4]{N^3} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

This makes the length increase more slowly with the pressure and speed than the rule above. Putting the equation in the form

$$l = c\sqrt{P}$$

N=	150	200	250	300	400	500	1000
c=	·0437	·0542	·0641	·0735	·0912	·1078	·1813

119. The ordinary empirical mode of proportioning the length of journals, is to make the length proportional to the

diameter, and to make the ratio of length to diameter increase with the speed. For wrought-iron journals,

$$\frac{l}{d} = {\cdot}004\,\text{N} + 1 \quad . \quad . \quad . \quad . \quad (9)$$

N=50	100	150	200	250	500	1000
$\frac{l}{d}$=1·2	1·4	1·6	1·8	2·0	3·0	5·0

Cast-iron journals may have $\frac{l}{d} = \frac{9}{10}$, and steel journals may have $\frac{l}{d} = 1\frac{1}{4}$ of the above values.

120. An examination of different locomotive journals led Schepp[1] to propose an empirical formula of the following form for the length of journals. Let d be the diameter, l the length of the journal, P the mean load on the journal during a revolution, n the number of revolutions *per second*, a the greatest pressure, per unit of journal bearing surface, consistent with security against heating. Then

$$a\,d\,l = \text{P} + n\sqrt{\text{P}}.$$

Schepp finds for steel locomotive crank-pins, $a = 1{,}400$; for the journals of the supporting axles $a = 350$ lbs. per sq. in. If P is known we get for the necessary bearing surface

$$d\,l = \frac{1}{a}\{\text{P} + n\sqrt{\text{P}}\}. \quad . \quad . \quad . \quad . \quad (10)$$

121. *Strength of journals.*—In the simplest cases, journals are subjected exclusively to bending action, which may be considered to be uniformly distributed over the length of the journal.

Case I. Crank-pins, or overhung journals supported at one end. Let P be the uniformly distributed load on the journal in lbs.; d the diameter, and l the length of the journal in ins.; f the greatest stress per sq. in. for the material of the journal. The greatest bending moment at

[1] Die Haupttheile der Locomotiv-Dampfmaschinen.

the fixed end of the journal is $\frac{1}{2}$ P l. Equating this to the moment of resistance of a circular section, § 28,

$$\frac{\text{P}\,l}{2}=f\,\frac{\pi}{32}\,d^3=\frac{f\,d^3}{10{\cdot}2}$$

$$d=\sqrt[3]{\frac{5{\cdot}1}{f}}\sqrt[3]{\text{P}\,l}=\sqrt{\frac{5{\cdot}1}{f}}\sqrt{\left(\text{P}.\frac{l}{d}\right)}\;. \qquad . \text{(11)}$$

For wrought iron, f may be taken at 6,000 to 9,000 lbs. per sq. in., the stress on each fibre being alternately tensile and compressive (§ 25). For steel, f=13,500 to 9,000 lbs.; for cast iron, f=3,000 to 4,500 lbs.

$\sqrt[3]{\frac{5{\cdot}1}{f}}$=0·0947 to 0·0827 for wrought iron.
0·0827 „ 0·0723 „ steel.
0·1193 „ 0·1042 „ cast iron.

$\sqrt{\frac{5{\cdot}1}{f}}$=0·0238 to 0·0291 for wrought iron.
0·0291 „ 0·0194 „ steel.
0·0413 „ 0·0337 „ cast iron.

Diameter of Wrought-iron Crank-pin Journals in inches.

Load on Journal in lbs.	Ratio of Length to diameter						
	1·0	1·25	1·5	1·75	2	3	4
1,000	·84	·94	1·03	1·11	1·19	1·45	1·68
1,500	1·03	1·15	1·26	1·36	1·45	1·78	2·05
2,000	1·19	1·33	1·45	1·57	1·68	2·05	2·37
3,000	1·45	1·62	1·78	1·92	2·05	2·52	2·90
4,000	1·68	1·88	2·05	2·22	2·37	2·90	3·35
5,000	1·87	2·10	2·30	2·48	2·65	3·25	3·75
10,000	2·65	2·96	3·25	3·50	3·75	4·59	5·30
15,000	3·25	3·63	3·98	4·29	4·59	5·62	6·49
20,000	3·75	4·19	4·59	4·96	5·30	6·49	7·50
30,000	4·59	5·13	5·62	6·07	6·49	7·95	9·18
40,000	5·30	5·93	6·49	7·01	7·50	9·18	10·60
50,000	5·93	6·63	7·26	7·84	8·38	10·27	11·85

In this Table the safe stress f is taken at 9,000 lbs. per sq. in.

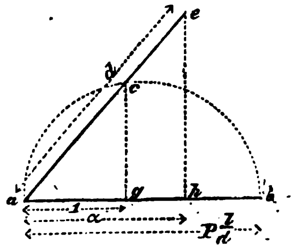


Fig. 126.

Equation 11 may be solved graphically. Take $a\,b = \text{P}\,\frac{l}{d}$, on any scale. Describe on $a\,b$ a semicircle. On the same scale take $a\,g$=unity, and draw a perpendicular. Then $a\,c = \sqrt{\text{P}\,\frac{l}{d}}$. Take $a\,h = \sqrt{\frac{5{\cdot}1}{f}}$, and draw a perpendicular to $a\,b$, cutting $a\,c$ produced in e. Then $a\,e = d$, the diameter required. It will usually be convenient to take $a\,b = \frac{\text{P}}{n^2}\cdot\frac{l}{d}$ and $a\,h = n\sqrt{\frac{5{\cdot}1}{f}}$, where n is some convenient multiplier. Then $a\,e = d$, as before. The multiplier may be chosen so as to make $a\,b$=about $1\frac{1}{2}$ to 3 times $a\,c$.

122. If in accordance with Von Reiche's rule (eq. 8), § 118, we take $l = 0{\cdot}00102\ \text{P}^{\frac{1}{2}}\,\text{N}^{\frac{3}{4}}$, then from equation (11) we get, for the diameter necessary for strength,

$$d = \text{P}^{\frac{1}{2}}\,\text{N}^{\frac{1}{4}}\sqrt[3]{\frac{0{\cdot}0052}{f}} \quad . \qquad (12)$$
$$= k\sqrt{\text{P}}$$

where k has the following values:

$f=$	$\sqrt[3]{\frac{0{\cdot}0052}{f}}=$	Values of k for N= 150	200	250	300	400	500
4,500	·001049	·00367	·00395	·00417	·00437	·00469	·00497
6,000	·000953	·00333	·00358	·00379	·00397	·00427	·00451
9,000	·000832	·00292	·00313	·00331	·00347	·00372	·00394
13,500	·000728	·00255	·00274	·00289	·00303	·00325	·00344

123. If Schepp's rule is taken (§ 120),

$$d\,l = \frac{1}{a}\left(\text{P} + n\sqrt{\text{P}}\right) \quad . \quad . \quad (13)$$

Now for strength

$$\frac{\text{P}\,l}{2} = \frac{f\,d^3}{10{\cdot}2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Eliminating l, we get,—

$$d=\sqrt[4]{\left\{\frac{5 \cdot 1}{f} \cdot \frac{P}{a}(P+n\sqrt{P})\right.} \quad . \quad . \quad . \quad (15)$$

where f=6,000 for wrought iron and 9,000 for steel.

124. *Journals in which the bearing pressure is limited.* —It is very common to limit the pressure on the bearing surface of a crank pin journal, as well as to limit the stress on the material. If d and l are the diameter and length of the journal, and P the steam pressure on the whole area of the piston, then the intensity of pressure on the bearing surface of the journal,

$$p=\frac{P}{d\,l},$$

ranges in different cases from 300 to 1,200 lbs. per sq. in. In some very quick-running engines the pressure is apparently greater, but in such cases the inertia of the piston and the expansive working of the steam diminish the load on the journal. Pressures of 1,200 lbs. are not uncommon in large marine engine crank-pins of over 8 inches in diameter. Suppose a value chosen for p. Then to determine the diameter and length of the journal there are two equations.

$$l=\frac{P}{p\,d}. \quad . \quad . \quad . \quad . \quad (16)$$

Inserting this in equation (11)—

$$d=\sqrt[4]{\frac{5 \cdot 1}{p\,f}}\;\sqrt[2]{P} \quad . \quad . \quad (17)$$

The following table facilitates the use of this equation.

Values of $\sqrt[4]{\frac{5 \cdot 1}{p\,f}}$ *for different values of* p *and* f.

	If f=	For p = 500	750	1,000	1,200
Cast iron	4,500	·0388	·0350	·0326	·0312
Wrought iron	6,000	·0361	·0326	·0304	·0290
Wrought iron or steel	9,000	·0326	·0295	·0274	·0262
Steel	13,500	·0295	·0266	·0248	·0237

125. *Case II. Neck journals.*—The cross-head pins of engines are supported at each end, and loaded uniformly. The bending moment is then $\frac{1}{8}$ P *l*.

$$d = \sqrt[3]{\frac{1·28}{f}} \sqrt[3]{\text{P}\, l} = \sqrt{\frac{1·28}{f}} \sqrt{\left(\text{P}\frac{l}{2}\right)} \quad . \quad . \quad (18)$$

The diameter of a neck journal may, therefore, be 0·63 of the diameter of the equivalent overhanging crank-pin journal, if it has the same length.

126. *Case III. Journals subjected to a transverse load and a twisting force.*—Crank-shaft journals. Let fig. 127

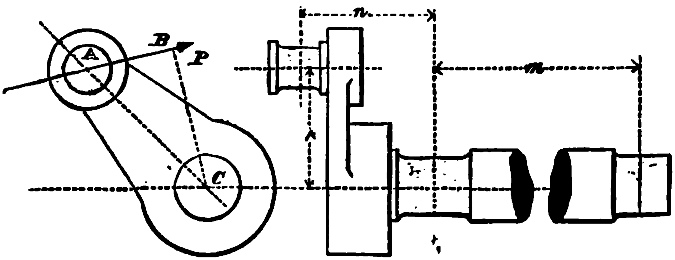


Fig. 127.

represent a side and end elevation of a crank and crank-shaft, and let it be required to determine the dimensions of the shaft-journal nearest the crank. Let P, acting in the direction A B, be the pressure transmitted to the crank-pin by the connecting rod, and let Q and R be the reactions due to P at the shaft journals. These forces may be taken to act at the centres of the journals.

$$\text{P}\, n = \text{Q}\, m \quad \text{and} \quad \text{P} + \text{Q} = \text{R}.$$

Hence,

$$\text{Q} = \text{P}\,\frac{n}{m}; \quad \text{R} = \text{P}\left(1 + \frac{n}{m}\right) \quad . \quad . \quad . \quad . \quad (19)$$

R is the reaction at the journal nearest the crank.

The force P produces a bending moment P *n*, at the centre

of the shaft journal, causing bending in a plane parallel to its direction. At the same time, the journal is subjected to a twisting moment $P \times CB = Pr \cos \theta$, where θ is the angle ACB, and r the crank radius. At the dead point, where the direction of P passes through C, $r \cos \theta = 0$; and the twisting moment vanishes. If P is constant, the twisting moment is greatest when the connecting rod is at right angles to the crank, and is then equal to Pr.

By the rules for combining bending and twisting action, § 44, the stress due to the combined moments is the same as that which would be produced by a simple bending moment.

$$M = \tfrac{1}{2} P n + \tfrac{1}{2} \sqrt{\{(Pn)^2 + (Pr)^2\}}$$

$$= \frac{P}{2} \left\{ n + \sqrt{(n^2 + r^2)} \right\} \quad . \quad . \quad . \quad . \quad . \quad (20)$$

$$= \frac{P}{2} (1{\cdot}84\, n + 0{\cdot}84\, r) \text{ nearly} \quad . \quad . \quad . \quad . \quad (20\,a)$$

Equating this to the moment of resistance of a circular section,

$$M = \frac{f d^3}{10{\cdot}2}$$

$$d = \sqrt[3]{\frac{5{\cdot}1}{f}} \sqrt[3]{\left\{ P\,(n + \sqrt{n^2 + r^2}) \right\}} \quad . \quad . \quad . \quad (21)$$

$$= \sqrt[3]{\frac{5{\cdot}1}{f}} \sqrt[3]{\left\{ P\,(1{\cdot}84\, n + {\cdot}84\, r) \right\}} \text{ nearly} \quad . \quad (21\,a)$$

where $\sqrt[3]{\frac{5{\cdot}1}{f}}$ has the values given above.

These equations exaggerate a little the straining action, because they neglect the distribution of the load on the surface of the journal. The length of the journal is to be calculated for the pressure R. Usually for crank-shafts $l = 1\tfrac{1}{2}$ to $1\tfrac{3}{4} d$, which allows a large margin for durability.

127. *Crank shaft journals when the load is uniformly distributed.*—The exact distribution of the load R on the crank

shaft journal is not really known. Two considerations show that it must be distributed in a tolerably uniform manner. If it were not, the intensity of pressure would squeeze out the lubricant and the journal would heat, and the journal would wear most rapidly at that part where the pressure was greatest. As the whole flexure of the journal due to the load is small, it is reasonable to suppose that the brasses soon wear to a form which is consistent with a uniform distribution of the pressure R over the whole length of the journal. If this is assumed, then the bending moment due to R distributed over the journal, if l is the length of the journal, is

$$\mathrm{R}\frac{l}{8}=\frac{\mathrm{P}\,l}{8}\left(1+\frac{n}{m}\right) \quad . \quad . \quad . \quad (22)$$

and the true bending moment at the centre of the journal is

$$\mathrm{P}\left\{n-\frac{l}{8}\left(1+\frac{n}{m}\right)\right\} \quad . \quad . \quad . \quad (23)$$

which should be substituted for $\mathrm{P}\,n$ in the equations above.

128. *Case IV. Two forces at right angles acting at the crank pin.*—The following case sometimes occurs. Two forces, P_1, P_2, act at the crank pin at right angles, but at different distances from the section at which the bending moment is to be estimated. If two forces not at right angles are given they can be resolved into two forces at right angles. Further, it is convenient in such cases to so resolve the forces that one is in the direction of the crank arm and the other at right angles to it.

Let P_1 be a force acting on the crank pin at n_1 from the centre of the shaft bearing, as in the figure above, and P_2 a force acting at a distance n_2. Further, let P_1 be at right angles to the crank arm and P_2 parallel to it. Let r be the radius of the crank and m the distance between the shaft journals.

The bending moments on the crank shaft due to P_1 and P_2 are, if the distribution of load on the journal is neglected, $\mathrm{M}_1=\mathrm{P}_1 n_1$, and $\mathrm{M}_2=\mathrm{P}_2\, n_2$, and the resultant bending moment is

$$\mathrm{M}=\sqrt{\mathrm{M}_1{}^2+\mathrm{M}_2{}^2}=\sqrt{\mathrm{P}_1{}^2\, n_1{}^2+\mathrm{P}_2{}^2\, n_2{}^2}.$$

If the distribution of the load along the length l of the shaft journal is allowed for,

$$M_1 = P_1 n_1 - \frac{P_1 l}{8}\left(1 + \frac{n_1}{m}\right)$$

$$M_2 = P_2 n_2 - \frac{P_2 l}{8}\left(1 + \frac{n_2}{m}\right),$$

and the resultant bending moment is

$$M = \sqrt{(M_1^2 + M_2^2)},$$

as before.

If P_1 is at right angles to the plane through the crank arm and crank shaft, the twisting moment is

$$T = P_1 r.$$

The bending moment equivalent to the combined bending and twisting moments is

$$M_e = \tfrac{1}{2} M + \tfrac{1}{2}\sqrt{(M^2 + T^2)}$$

$$= \tfrac{1}{2}\sqrt{(M_1^2 + M_2^2)} + \tfrac{1}{2}\sqrt{(M_1^2 + M_2^2 + T^2)} \quad . \quad (24)$$

From this the diameter of the shaft can be found by the equation

$$d = \sqrt[3]{\frac{10{\cdot}2}{f}}\sqrt[3]{M_e} \quad . \quad . \quad (25)$$

Pivot and Collar Bearings.

129. When a shaft is subjected to a force parallel to its axis, it requires a pivot, or collar, to prevent longitudinal displacement. A vertical shaft has usually a pivot at its foot, supporting the weight of the shaft and the gearing attached to it. The screw-shaft of a steam vessel is subjected to a longitudinal thrust, equal and opposite to the resistance of the ship. This thrust is usually transmitted to a collar-bearing, strongly connected to the framing of the ship.

130. *Friction of a pivot.*—Let d be the diameter of a flat pivot in ins., P the load on it in lbs., μ the coefficient of friction; N the revolutions per minute. The work expended in friction lies between the limits

$$\tfrac{1}{18}\,\pi\,\mu\,P\,N\,d \text{ and } \tfrac{1}{24}\,\pi\,\mu\,P\,N\,d \text{ ft. lbs. per min.}$$

Taking the former estimate, the heat evolved is

$$\frac{1}{772} \cdot \frac{1}{18} \cdot \pi \mu \mathrm{P} \mathrm{N} d \text{ units per min.}$$

If h units are conducted away per sq. in. of bearing surface.

$$\frac{\pi}{4} d^2 h = \frac{1}{772} \cdot \frac{1}{18} \pi \mu \mathrm{P} \mathrm{N} d$$

$$\therefore d = c \mathrm{P} \mathrm{N} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

where c is a constant determined by experience. According to Reuleaux, $c = 0{\cdot}0000272$.

131. When the speed is not great, it is desirable to limit the intensity of the pressure on the footstep, or there is danger of squeezing out the lubricant. Reuleaux gives the following values for the maximum intensity of the pressure on the pivot.

	$p=$
I. Wrought-iron pivot on gun-metal	700
II. Cast-iron pivot on gun-metal	470
III. Wrought-iron pivot on lignum vitæ . . .	1400

In order that the intensity of pressure may not exceed this limit, we must have

$$d = \sqrt{\frac{4}{\pi p}} \sqrt{\mathrm{P}} = k\sqrt{\mathrm{P}} \quad . \quad . \quad . \quad (27)$$

Allowing 10 per cent. of area for the oil-channels, $k = {\cdot}046$ in Case I.; $= {\cdot}057$ in Case II.; $= {\cdot}033$ in Case III.

Eq. 27 will give greater values of d than Eq. 26, and is to be preferred when

$$\mathrm{N} < \frac{k}{c\sqrt{\mathrm{P}}}$$

$\frac{k}{c\sqrt{\mathrm{P}}}$	P=	1000	2500	5000	7500	10,000	15,000
	I.=	53	34	24	20	17	14
	II.=	66	42	30	24	21	17
	III.=	38	24	17	14	12	10

If the number of revolutions is less than is given in this Table, eq. 27 is to be taken; if greater, eq. 26.

132. *Collar Bearings*, fig. 128, are used when a vertical shaft has to be suspended from a framing, an arrangement used for turbine and centrifugal pump-shafts, in order to bring the bearing to a position where it can be easily lubricated. They are also used for screw-propeller shafts.

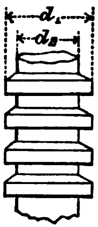


Fig. 128.

Let P be the load on the shaft (including its weight, if it is vertical); d_1 d_2 the outside and inside diameters of the collars; n, the number of collars; N, the number of revolutions per min. Then, by a similar process to that used for pivots, we get

$$n\,\frac{(d_1^2-d_2^2)^2}{d_1^3-d_2^3}=c\,\mathrm{P}\,\mathrm{N} \quad . \quad . \quad . \quad . \quad (28)$$

$$n\,(d_1^2-d_2^2)=k^2\,\mathrm{P} \quad . \quad . \quad . \quad . \quad (29)$$

where c and k have the same values as before. In using these equations, it is best to assume values for d_1 and d_2. Then n is easily found. The greater of the two values given by the equations is to be taken.

Axles and Shafts.

133. The terms *axle* and *shaft* are applied rather indiscriminately, to parts of machines which support rotating pieces, or which by their rotation convey and distribute motive power. They are usually cylindrical, but occasionally square or cross-shaped in section. They may be classified as follows :—

(1.) Axles loaded transversely, and subjected chiefly to bending action.

(2.) Transmissive shafting, subjected chiefly to torsion.

(3.) Crank-shafts and other shafts, subjected to combined torsion and bending.

134. *Axles loaded transversely.*—In designing axles of this kind, it is convenient to determine, first, the dimensions of the journals. If the axle is cylindrical, its diameter at any other point can be obtained from the journal diameter, if it is remembered that the diameters at any two points should be proportional to the cube roots of the bending moments at those points. If the section is not circular, it is still convenient to design a cylindrical axle, and then to replace the cylindrical sections by equivalent sections of any other form. If the axle rotates, the cylindrical form is the only one which is of equal strength in all positions. The mode of designing axles is best explained by examples.

135. *Example I.*—An axle is supported on two end

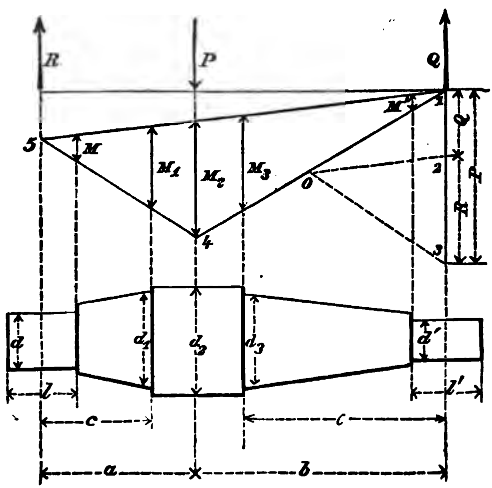


Fig. 129.

journals, and carries a load, P, at a point between the journals. Fig. 129 shows the axle. The load P is in equi-

librium with the reactions Q, R, acting at the centres of the journals.

$$Q=P\frac{a}{a+b}; \quad R=P\frac{b}{a+b}$$

These are the loads for which the journals are to be calculated. From the rules in §§ 113–124 will be determined d, d', l, l', and the projections which limit the end-play. The axle diameters which it is most necessary to determine are those marked d_1, d_2, and d_3. The bending moments at those points are $M_1=Rc$; $M_2=Ra$; $M_3=Qe$. The bending moments at the fixed ends of the journals are $M=R\frac{l}{2}$ and $M'=Q\frac{l'}{2}$.

Since at any section the diameter must be at least equal to $\sqrt[3]{\frac{5\cdot1}{f}}\sqrt[3]{\text{(Bending Moment)}}$,

$$\left.\begin{aligned}\frac{d_1}{d}&=\sqrt[3]{\frac{M_1}{M}}=\sqrt[3]{\frac{2c}{l}}\\ \frac{d_2}{d}&=\sqrt[3]{\frac{M_2}{M}}=\sqrt[3]{\frac{2a}{l}}\\ \frac{d_3}{d'}&=\sqrt[3]{\frac{M_3}{M'}}=\sqrt[3]{\frac{2e}{l'}}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (30)$$

The smallest values of the diameters consistent with the requirements of strength are, therefore, easily obtained from the journal diameters.

It is often convenient to measure the bending moments from the bending moment curve, which is easily drawn thus :—Take 13 on the direction of Q produced, and = P, on any scale; choose any pole O, and draw 1 O 4, meeting the direction of P produced in 4. Join 30 and draw 45 parallel to 30, meeting the direction of R in 5. Join 51, and draw O 2 parallel to it. Then 12, 23 are the values of Q and R on the scale assumed for P. The vertical ordinates of the

triangle 1 4 5, are proportional to the bending moments at the corresponding points of the axle. The values of M, M_1, M_2, etc., measured on the diagram, may be used in the preceding equations, in determining the diameters of the axle.

The boss at the loaded part of the shaft, is intended for keying on the wheel, or other part supported by the axle. Its projection must therefore be sufficient for cutting a key-way, § 87, even if it is then larger than is necessary for strength. If at any part the axle is not circular, it is only necessary to equate the modulus of a section of the required form, to the modulus of the circular section previously determined. Thus, if the section is to be square, the equation

$$0{\cdot}118\, s^3 = {\cdot}0982\, d^3$$

will give the side of the square, the values of the moduli having been taken from Table IV. § 29. If the axle rotates, the value of the modulus must be that which corresponds to the position in which it is weakest.

136. *Example II.*—The axle supports two parallel loads between the journals. The bending moment curve is drawn thus: Let $a\,b$ be the centres of the journals, $c\,d$ the points at which the loads P Q are applied. At the points a and b the reactions R, S are produced by the action of P and Q. On the direction of S set off 12, 23, equal to Q and P on any scale. Choose a pole, O; join O 1, intersecting the direction of Q in 4. Join O 2 and draw 45 parallel to it, intersecting the direction of P in 5. Join O 3, and draw 5 6 parallel to it, intersecting the direction of R in 6. If, now, O 7 is drawn parallel to the closing

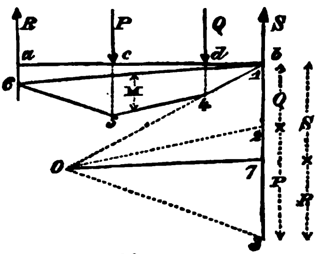


Fig. 130.

line 6 1, 3 7 and 7 1 will be equal to the reactions R and S. Also, 6 1 4 5 is the bending moment polygon, the breadth of which at any point, measured parallel to the forces, is proportional to the bending moment at the corresponding point of the axle. Having, therefore, the bending moments, the diameters of the axle may be obtained from the journal diameters, as before.

When $P=Q$ and $a\,c=d\,b$, the case is one which occurs very commonly in practice, in which it will be found that the bending moment is uniform from c to d. A railway carriage axle is in this position when the carriage is at rest, or moving along a straight portion of line. In passing round curves, however, it is subjected to torsion as well as bending; and in consequence of the pressure of the wheel flange against the rail, the forces are no longer parallel. Then the bending action between c and d is no longer uniform. It is for this reason that railway axles are tapered a little towards the centre.

The bending moment curve for forces not parallel, is drawn in the same way as before, the only difference being that the lines 1 2, 2 3, 3 7, 7 1, parallel to the forces, no longer fall on a single line, but form a closed polygon.

137. *Shafts transmitting power, and subjected to torsion only.*—Rotating shafts are very extensively used, in transmitting the energy of prime-movers to the various parts of the factory or workshop in which it is applied to useful purposes. Such shafting was at one time of timber, then cast iron was adopted, and later still, wrought iron has almost entirely superseded cast iron, except in a few cases, where the shafting is not subjected to much impulsive action. Shafts up to 4 inches in diameter are usually turned from rolled bars; larger shafts from forged bars. In transmitting power shafts are subjected to torsion, but they are also subjected to bending action, due to their own weight, the weight of the wheels and pulleys they support, to the thrust of the gearing and the tension of the belting

connected with them, and to other causes. This bending action is, to a great extent, indeterminate; it will, therefore, be convenient to consider, first, the torsion due to the power transmitted, and then to examine how an allowance can be made for the other straining actions.

Let H. P. be the indicated horses' power transmitted.

N the number of revolutions of the shaft per minute.

P the twisting force in lbs., acting on a shaft at a radius R in ins.

f the greatest safe stress for the material of the shaft, in lbs. per sq. in.

d the diameter of the shaft in ins.

The mean twisting moment is in statical inch lbs. (§ 22)—

$$T = P\,R = 63{,}024\,\frac{H.\,P.}{N} \qquad . \quad . \quad . \quad . \quad . \qquad (31)$$

The moment of resistance of a circular section with respect to torsion is (§ 36) $0{\cdot}196\, d^3 f$. Hence,

$$d = \sqrt[3]{\frac{5{\cdot}1}{f}}\sqrt[3]{P\,R} = \alpha \sqrt[3]{P\,R} \qquad . \quad . \quad . \quad . \qquad (32)$$

$$= \sqrt[3]{\frac{63{,}024}{0{\cdot}196 f}}\sqrt[3]{\frac{H.\,P.}{N}} = \beta\,\sqrt[3]{\frac{H.\,P.}{N}} \qquad . \quad . \quad . \qquad (33)$$

Taking f=9,000 for wrought iron; =4,500 for cast iron; and = 13,500 for steel; we get,

	α=	β=
For wrought iron	·08275	3·294
For cast iron	·1042	4·150
For steel	·0723	2·877

138. *Ratio of greatest to mean twisting moment in shafts driven by steam engines.*—In the case of a steam engine the twisting moment exerted on the crank shaft varies with the variation of the steam pressure and with the variation of the leverage of the crank as it rotates. For a single engine working with little expansion $T_{max} = 1{\cdot}3\ T_{mean}$, a rule which

may be used for locomotive cranks for instance. But if the engine works with much expansion the difference between the maximum and mean twisting moment is greater. Mr. Milton has found the ratio ρ of the greatest twisting moment to the mean twisting moment (that given by equation 31), from actual indicator diagrams, to be as follows in certain selected actual engines:—

	$\rho=$	$\sqrt[3]{\rho}=$
A. Single engine	2·1	1·28
B. Engine with equal cylinders, cranks at right angles	1·37	1·11
C. Compound engine cranks at right angles	1·48	1·14
D. Compound engine, cranks at 130° .	1·77	1·21
E. Compound cranks at 135°. Three bearings to crank shaft . . .	1·94	1·25

The diameters of a shaft directly connected with a steam engine, such as the propeller shaft of a ship, calculated from the indicated horses-power by the rules above, must be multiplied by $\sqrt[3]{\rho}$ to allow for the difference between the mean and maximum twisting moment.

139. *Shafts subjected to torsion and bending.*—Let T be the twisting moment (calculated by equation 31), and M the bending moment, at any section of a shaft. Then, the combined straining action is equivalent to that which would be produced by a twisting moment T_e, given by the following equation, which is eq. 29*a* of § 44.

$$T_e = M + \sqrt{(M^2 + T^2)} \quad . \quad . \quad . \quad . \quad . \quad (34)$$

Let $M = kT$, in any given case, so that k is a known fraction. Then

$$T_e = (k + \sqrt{k^2 + 1})T \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

$$= (1{\cdot}83\,k + {\cdot}83)\,T \text{ approximately} \quad . \quad . \quad . \quad (35\,a)$$

Then the preceding formulæ may be used in designing the

shaft, if the equivalent twisting moment T_e is substituted for the actual twisting moment PR in eq. 32. Hence,

$$d=\sqrt[3]{(k+\sqrt{k^2+1})}\sqrt[3]{\frac{5\cdot1}{f}}\sqrt[3]{T_e} \quad . \quad . \quad . \quad (36)$$

Or, if d is the proper diameter of the shaft, calculated for the combined bending and twisting action, and d' is the diameter calculated for the twisting action alone, by eq. 32 or 33; then

$$d=n\,d' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

where n is equal to $\sqrt[3]{(k+\sqrt{k^2+1})}$, or to $\sqrt[3]{(1\cdot83\,k+0\cdot83)}$ nearly. The following Table gives some values of n for given values of k.

k=0·25	0·50	0·75	1·0	1·25	1·50	1·75	2·0	3·0
n=1·09	1·17	1·26	1·34	1·42	1·49	1·56	1·62	1·83

It appears, from some calculations of Prof. Rankine, that for such cases as the propeller shafts of steam-vessels, where the straining action, additional to the torsion transmitted, is chiefly due to the weight of the shaft itself, k=0·25 to 0·5, and the diameter of the shaft should be 1·09 to 1·17 times the diameter, calculated from the torsion alone. For line shafting in mills, the bending action is often much greater, and the twisting moment is not constant, but rises above the mean value, calculated from the power transmitted. Practical experience appears to show, that for ordinary light shafting, k is 0·75 to 1, and the diameter of the shafting is 1·26 to 1·34 times the diameter, calculated from the mean torsion alone. For crank-shafts and heavy shafting subjected to shocks, k=1 to 1·5, and the diameter is 1·34 to 1·49 times that calculated from the torsion alone. Cases occur in which still greater allowance must be made.

Cranked shafts of marine engines.—Mr. Milton has calculated for the same engines mentioned above, the ratio

of the greatest equivalent twisting moment, due to twisting and bending, to the greatest simple twisting moment. From these we can easily obtain values of k; allowing for the variation of twisting moment mentioned above, the diameter of the cranked shaft should be greater than that calculated from the mean twisting moment (as given by equation 31), in the ratio

$$\sqrt[3]{p}\ \sqrt[3]{\{k+\sqrt{(k^2+1)}\}} \text{ to } 1.$$

Values of $\sqrt[3]{p}\ \sqrt[3]{\{k+\sqrt{(k^2+1)}\}}$ for these engines are given in the following table.

	Ratio of greatest to mean twisting moment p	Ratio of greatest equivalent twisting moment to greatest simple twisting moment	$k=$	$\sqrt[3]{p\{k+\sqrt{(k^2+1)}\}}$
Engine A	2·10	1·38	·33	1·43
,, B	1·37	1·15	·14	1·16
,, C	1·48	1·11	·10	1·23
,, D	1·77	1·05	·05	1·33
,, E	1·94	1·21	·19	

Diameters of Wrought-iron Shafts for given Twisting Moments.

The following table is calculated for a stress of 9,000 lbs. per sq. in. which is suitable for wrought iron and in many cases for steel. For cast iron multiply the diameters in the table by 1·26. For steel multiply by 1 to 0·874 according to the strength and soundness assumed.

The shaft diameter is found from the known mean twisting moment in the column corresponding to the assumed ratio k of bending to twisting.

Twisting moment P R in inch lbs.	Diameter for twisting moment only d	Diameter for twisting and bending moment for $k=$ 0·5	0·75	1·0	1·5
125	·41	·48	·52	·55	·61
250	·52	·61	·66	·70	·73
500	·66	·77	·83	·88	·99
750	·75	·88	·94	1·00	1·12
1,000	·83	·97	1·05	1·11	1·24
1,500	·95	1·11	1·20	1·27	1·42
2,000	1·04	1·22	1·31	1·37	1·55
2,500	1·12	1·31	1·41	1·50	1·67

Twisting moment P R in inch lbs.	Diameter for twisting moment only d	Diameter for twisting and bending moment for k= 0·5	0·75	1·0	1·5
3,000	1·19	1·39	1·50	1·60	1·73
4,000	1·31	1·53	1·65	1·75	1·95
5,000	1·42	1·66	1·79	1·90	2·11
6,000	1·50	1·75	1·89	2·00	2·24
7,500	1·62	1·90	2·04	2·08	2·42
10,000	1·78	2·08	2·24	2·39	2·66
12,500	1·92	2·22	2·42	2·57	2·86
15,000	2·04	2·39	2·57	2·73	3·04
17,500	2·15	2·52	2·70	2·88	3·20
20,000	2·25	2·63	2·83	3·01	3·36
25,000	2·42	2·83	3·05	3·24	3·60
30,000	2·57	3·00	3·23	3·44	3·83
35,000	2·70	3·16	3·40	3·62	4·02
40,000	2·83	3·32	3·57	3·80	4·22
45,000	2·94	3·44	3·70	3·94	4·40
50,000	3·05	3·57	3·85	4·10	4·55
60,000	3·24	3·80	4·10	4·33	4·82
70,000	3·41	4·00	4·30	4·57	5·10
80,000	3·57	4·17	4·50	4·77	5·31
90,000	3·71	4·35	4·70	4·97	5·52
100,000	3·84	4·50	4·85	5·15	5·72
110,000	3·97	4·65	5·00	5·30	5·90
120,000	4·08	4·80	5·15	5·47	6·10
130,000	4·19	4·90	5·30	5·61	6·25
140,000	4·30	5·05	5·40	5·76	6·40
150,000	4·40	5·15	5·55	5·90	6·55
175,000	4·63	5·45	5·85	6·20	6·90
200,000	4·84	5·65	6·10	6·50	7·20
250,000	5·21	6·10	6·55	7·00	7·76
300,000	5·54	6·50	7·00	7·42	8·27
400,000	6·10	7·15	7·70	8·20	9·10
500,000	6·57	7·70	8·30	8·80	9·80
600,000	6·98	8·20	8·80	9·40	10·40
750,000	7·53	8·80	9·50	10·05	11·20
1,000,000	8·28	9·70	10·40	11·10	12·30
1,250,000	8·92	10·42	11·20	12·00	13·30
1,500,000	9·47	11·10	11·95	12·70	14·10
1,750,000	9·97	11·70	12·60	13·35	14·90
2,000,000	10·42	12·20	13·10	14·00	15·50
2,250,000	10·84	12·70	13·65	14·50	16·20
2,500,000	11·23	13·15	14·10	15·10	16·70
3,000,000	11·93	14·00	15·00	16·00	17·80
3,500,000	12·57	14·70	15·90	16·80	18·70
4,000,000	13·14	15·40	16·55	17·60	19·60
4,500,000	13·67	16·00	17·25	18·30	20·20
5,000,000	14·16	16·60	17·85	19·00	21·10

Diameters of Shafts, when the Horses Power and Revolutions per minute are given, calculated by Eq. 33.

Divide the H.P. by the number of revolutions per minute. The diameter will be found opposite the nearest number to the quotient in the following table :—

Horses power / Revolutions $\frac{H.P.}{N.}$	Diameter for twisting moment only d	Diameter for twisting and bending moment for $k=$ 0·5	0·75	1·0	1·5
·012	0·753	0·881	·948	1·009	1·121
·025	0·963	1·126	1·213	1·290	1·434
·050	1·213	1·428	1·528	1·625	1·806
·075	1·389	1·624	1·750	1·862	2·070
·1	1·529	1·788	1·926	2·050	2·277
·15	1·750	2·047	2·202	2·345	2·607
·2	1·926	2·255	2·425	2·58	2·87
·25	2·075	2·427	2·615	2·78	3·09
·3	2·205	2·58	2·78	2·95	3·28
·35	2·321	2·71	2·92	3·11	3·46
·4	2·428	2·84	3·06	3·25	3·62
·45	2·524	2·95	3·18	3·38	3·76
·5	2·614	3·06	3·29	3·50	3·90
·6	2·777	3·25	3·50	3·73	4·14
·7	2·925	3·42	3·68	3·92	4·36
·8	3·06	3·58	3·85	4·10	4·56
·9	3·18	3·72	4·01	4·26	4·74
1·0	3·29	3·85	4·14	4·41	4·90
1·25	3·55	4·15	4·47	4·76	5·29
1·5	3·77	4·41	4·75	5·05	5·62
1·75	3·97	4·64	5·00	5·32	5·92
2·0	4·15	4·86	5·23	5·56	6·18
2·25	4·32	5·05	5·44	5·79	6·44
2·5	4·47	5·23	5·63	6·00	6·66
2·75	4·61	5·39	5·81	6·18	6·87
3·0	4·75	5·56	5·98	6·37	7·08
3·25	4·88	5·71	6·15	6·54	7·27
3·5	5·00	5·85	6·30	6·70	7·45
3·75	5·12	5·99	6·45	6·86	7·63
4·00	5·23	6·12	6·59	7·01	7·80
4·25	5·34	6·25	6·73	7·16	7·96
4·5	5·44	6·36	6·85	7·30	8·11
4·75	5·54	6·48	6·98	7·43	8·26
5·0	5·63	6·59	7·10	7·55	8·39
5·5	5·82	6·81	7·33	7·80	8·67
6·0	5·99	7·01	7·55	8·03	8·93
6·5	6·15	7·20	7·75	8·25	9·17

Horse power / Revolutions $\frac{\text{H.P.}}{\text{N.}}$	Diameter for twisting moment only d	Diameter for twisting and bending moment for $k=$ 0·5	0·75	1·0	1·5
7·0	6·30	7·37	7·94	8·45	9·39
7·5	6·45	7·55	8·13	8·65	9·61
8·0	6·59	7·71	8·31	8·84	9·82
9·0	6·85	8·02	8·63	9·19	10·20
10	7·10	8·31	8·95	9·52	10·58
11	7·33	8·58	9·24	9·83	10·91
12	7·53	8·81	9·50	10·10	11·21
13	7·75	9·07	9·77	10·40	11·54
14	7·94	9·29	10·00	10·65	11·82
15	8·12	9·50	10·23	10·90	12·10
16	8·30	9·71	10·46	11·13	12·37
17	8·47	9·91	10·67	11·36	12·62
18	8·63	10·09	10·87	11·57	12·86
19	8·79	10·28	11·08	11·80	13·10
20	8·93	10·45	11·25	11·98	13·30
21	9·08	10·62	11·44	12·18	13·52
22	9·23	10·80	11·63	12·38	13·74
23	9·38	10·97	11·81	12·58	13·97
24	9·51	11·12	11·98	12·75	14·17
25	9·66	11·30	12·16	12·95	14·39
26	9·76	11·41	12·29	13·08	14·54
27	9·88	11·55	12·44	13·25	14·71
28	10·00	11·70	12·60	13·40	14·90
29	10·12	11·84	12·75	13·56	15·08
30	10·23	11·95	12·89	13·70	15·24

If the shaft is of cast iron or steel, or if bending action is to be allowed for, proceed as indicated for the preceding table.

140. *Mill shafting.*—Ordinary mill shafting for textile manufactures is calculated for $k=1$; that is, the diameter is 1·34 times that which would be necessary, if there were no bending. When as usual it is of wrought iron, we have from eq. 33—

$$d = 1{\cdot}34\,\beta\sqrt[3]{\frac{\text{H.P.}}{\text{N}}}$$

$$= 4{\cdot}414\sqrt[3]{\frac{\text{H.P.}}{\text{N}}} \quad . \quad . \quad (38)$$

$$\frac{\text{H.P.}}{\text{N}} = 0{\cdot}01163\,d^3. \quad . \quad . \quad (38\,a)$$

From this the following table has been computed, giving the values of $\frac{\text{H.P.}}{\text{N}}$ for the most ordinary sizes of shaft.

Horses Power transmitted by Wrought-iron Mill Shafting.

Multiply the tabular number in the column $\frac{\text{H.P.}}{\text{N}}$ by the number N of revolutions per minute, the result is the horses power which the shaft will transmit.

Diameter of shaft in ins.	$\frac{\text{H.P.}}{\text{N.}}$	Diameter of shaft in ins.	$\frac{\text{H.P.}}{\text{N.}}$
$1\frac{3}{4}$	0·0623	5	1·4536
2	0·0930	$5\frac{1}{2}$	1·9344
$2\frac{1}{4}$	0·1325	6	2·5112
$2\frac{1}{2}$	0·1817	$6\frac{1}{2}$	3·1944
$2\frac{3}{4}$	0·2418	7	3·9888
3	0·3139	$7\frac{1}{2}$	4·9056
$3\frac{1}{4}$	0·3993	8	5·9536
$3\frac{1}{2}$	0·4986	$8\frac{1}{2}$	7·1440
$3\frac{3}{4}$	0·6132	9	8·4800
4	0·7442	10	11·6288
$4\frac{1}{4}$	0·8930	11	15·4752
$4\frac{1}{2}$	1·0600	12	20·0896
$4\frac{3}{4}$	1·2470		

141. *Hollow shafts.*—At the present time the use of compressed steel permits shafts to be made hollow; in this way, since the least effective portion of the section of the shaft is removed, the weight is diminished in a much greater ratio than the strength. Let d be the diameter of a solid shaft, and d_1 d_2 the external and internal diameters of a hollow shaft of the same material. Then the shafts will be of equal strength when the moduli of the two sections with respect to torsion are equal, that is when (§ 36)—

$$d^3 = \frac{d_1^4 - d_2^4}{d_1}$$

Let $d_2 = x d_1$

$$d_1 = \sqrt[3]{\left\{\frac{d^3}{1 - x^4}\right\}} \quad . \quad . \quad (39)$$

which gives the external diameter of a hollow shaft in terms of the diameter of a solid shaft of the same strength, when the fraction of the diameter removed at the centre is fixed. Suppose a 10-inch shaft has a hollow 4 ins. diameter. Its weight will be 16 per cent. less than that of a solid 10-inch shaft, but its strength is only 2·56 per cent. less. A hollow shaft with a hole $\frac{4}{10}$ths of the diameter, and equal in strength to a 10-inch solid shaft, would have a diameter of

$$d_1=\sqrt[3]{\frac{10^3}{1-0{\cdot}4^4}}=10{\cdot}09 \text{ inches.}$$

142. *Forged cranked shafts.*—Forged cranked shafts are very extensively used, for inside cylinder locomotives and for marine engines of all kinds. Fig. 131 shows the ordinary

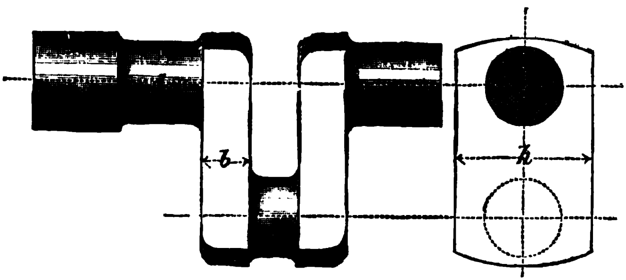


Fig. 131.

form of such a cranked shaft. In marine engine practice the diameter may be obtained thus:—

H.P. = indicated horse power.
N = number of revolutions per minute.
d = diameter of shaft.

$$d=4{\cdot}55\sqrt[3]{\left(\frac{\text{H.P.}}{\text{N}}\right)} \quad . \quad . \quad (40)$$

which allows for bending as well as twisting action. The arms of the crank may be so proportioned that

$$b\,h^2 = c\,d^3$$

where $c = 0{\cdot}9$ to $1{\cdot}0$.

The objection to the use of cranked shafts is their liability to fracture, and it is one of the principal reasons for the preference of outside to inside cylinder locomotives, that in the former cranked shafts are not required. On this point Mr. Milton makes the following remark:[1] 'Most of the flaws for which cranked shafts are condemned occur at the angle between the web of the crank and the journal or crank-pin, at the place where the forging is most likely to be defective, and they are evidently produced more by the bending than by the twisting strains. At these places the change of form of the shaft throws great local stresses on the material, and there can be no doubt that, if these parts are made with a large radius, the strength of the shaft is materially increased. Among the causes which tend to throw great strains on the shafting may be mentioned, the presence of water in the cylinders, slackness of the brasses, and the bearings being out of line.'

Steel screw propeller shaft.—The shafting is made hollow. Each length is made from a hollow cylindrical ingot which, while in the molten state, is subjected to a heavy hydraulic pressure, thus insuring the exclusion of all gas cavities. The ingot is afterwards reheated and placed on a mandril, and it is then forged and drawn by hydraulic pressure until it assumes the form of a double collared shaft (fig. 136). Hollow steel cranked shafts are also used, but these are built up, and will be treated of in the chapter on Cranks.

143. Usually in line shafting the power is taken off at various points in the length of the shaft. Hence, the shaft ... gradually reduced in diameter, and thus material is

[1] ... on Crank-shafts,' Proc. Inst. of Naval Architects, 1880.

economised and the friction diminished. A long shaft consists of lengths of shafting, each of uniform diameter, the reduction of diameter being made in passing from one length to the next. When a shaft is calculated very closely in size, its ends must have bosses, to receive the keyways for fixing the couplings. The plan of graduating the size of the shaft to the work transmitted, has some serious disadvantages. The bearings which support the shaft and the couplings, wheels and pulleys fixed to it, are not interchangeable in position, if the diameter of the shaft is variable. This gives rise to much trouble and expense, if the machinery requires to be rearranged. Shafts of uniform diameter are now often adopted, and for such shafts the forging of bosses on the ends is unnecessary, because at most points the shaft has surplus strength.

Small shafts often give trouble from insufficient stiffness, although they have ample strength. For such shafts, $\frac{3}{8}$ in. to $\frac{5}{8}$ in. may be added to the diameter, which is sufficient for strength, in order to secure stiffness and freedom from vibration. For long shafts, and when $\frac{\text{H. P.}}{\text{N}}$ is less than 1, the diameter may be calculated by Redtenbacher's rule, which makes the angle of torsion a fixed proportion of the length of the shaft. Then,

$$d = \beta \sqrt[4]{\frac{\text{H. P.}}{\text{N}}} \quad . \quad . \quad . \quad . \quad (41)$$

where β has the same value as before, and the diameter is to be multiplied by the same values of n, to allow for bending action.

The span between the bearings of shafting should be so arranged as to limit the deflection of the shaft to a fixed proportion of its length. Let L be the span between the bearings, in inches. Then

$$\text{L} = \gamma \sqrt[3]{d^2} \quad . \quad . \quad . \quad . \quad (42)$$

where $\gamma = 60$ to 75, for shafting supporting its own weight only, and $\gamma = 54$ to 60, for shafting carrying the ordinary proportion of pulleys or gearing. Very commonly the bearings of mill shafting are 10 to 11 feet apart.

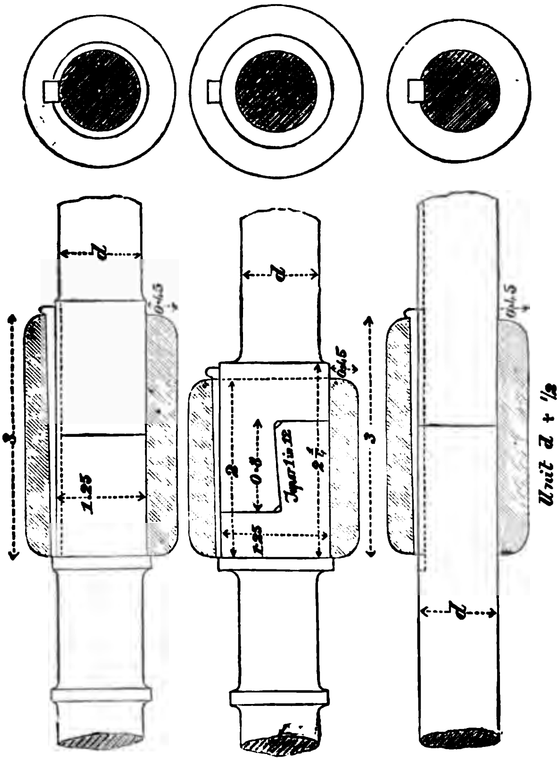


Fig. 132. Fig. 133. Fig. 134.

COUPLINGS.

144. It is inconvenient to make shafting in long lengths, and hence, lines of shafting are composed of separate lengths of 20 to 30 feet each coupled together. The couplings should

be placed as near bearings as possible, and on the side farthest from the driving point. The coupling may be a permanent, or *fast coupling*, which can only be removed by unscrewing bolts or slacking keys, or a disengaging, or *loose coupling*, which is provided with arrangements for throwing part of the shafting out of gear as often as necessary. In some loose couplings, the connection is a frictional one only, and if any sudden strain comes on the machinery, the coupling slips. Special forms of coupling are used when the connected shafts are not in one line.

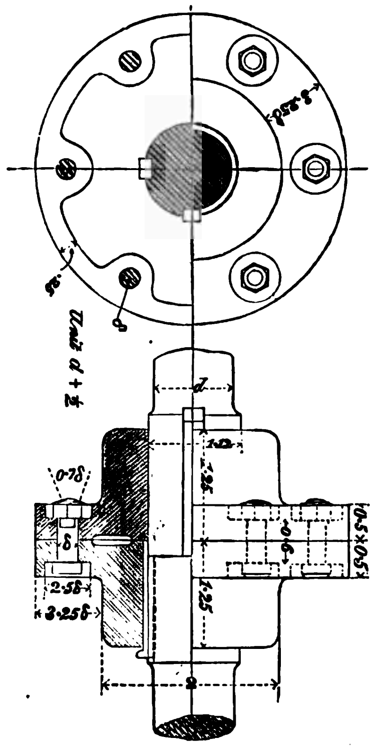


Fig. 135.

145. *Fast couplings.*—Figs. 132, 133, and 134, show three forms of coupling, known as 'box' or 'muff' couplings. In these a cast-iron box, or hollow cylinder, is fitted over the ends of the shafts. In figs. 132 and 134, the coupling is termed a butt coupling, and relative movement of the shafts is prevented by a wrought-iron key, which lies in a key-way, cut half into the box and half into the shaft ends. Fig. 133 is a half lap coupling, the shaft-ends overlapping, so as to prevent relative motion independently of the key, whose chief function is to fix the box rigidly in place. The keys are proportioned according to the rules in § 87. In fig. 133,

the key is a saddle-key. The other dimensions may be obtained from the proportional numbers. The half-lap coupling is an excellent coupling for shafts not exceeding 5 ins. diameter, but is somewhat expensive. The butt coupling is cheaper, but less secure. Both forms are free from projections likely to catch the clothes of a workman.

146. Fig. 135 shows a flange, or face-plate coupling. It consists of two parts of cast iron, firmly fixed by keys to the two shaft ends. The face of each coupling is turned after it has been keyed on the shaft, so that it is accurately perpendicular to the axis of the shaft. The couplings being brought together are fixed by bolts, which prevent relative movement by their resistance to shearing. In the coupling shown, the bolt-heads are sunk in the substance of the flange, for safety. The bolt-holes must be drilled, and the bolts carefully fitted. The number of bolts may be

$$n=3+\frac{d}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

the nearest even number being usually taken.

Let R be the radius of the bolt circle. Then, when the shaft is strained to its limit of elasticity, eq. 32 gives, for the shearing force on the bolts,

$$P=\frac{f d^3}{5{\cdot}1\,R}$$

The resistance of the bolts to shearing is $\frac{\pi}{4} n \delta^2 f_s$. Equating this to the shearing force, we get, for the diameter of the bolts,

$$\delta=0{\cdot}577\sqrt{\left(\frac{d^3}{n\,R}\right)} \quad . \quad . \quad . \quad . \quad (44)$$

In practice the bolts are often a little larger, and may be

$$\delta=\frac{d}{n}+\tfrac{1}{4} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

To keep the shafts in line, the end of one shaft may enter into the coupling on the other $\frac{1}{4}$ to $\frac{1}{2}$ inch.

147. *Propeller shaft coupling.*—Fig. 136 shows a coupling formed by flanging the shaft itself. In this case the shaft is a hollow compressed steel shaft. The unit for

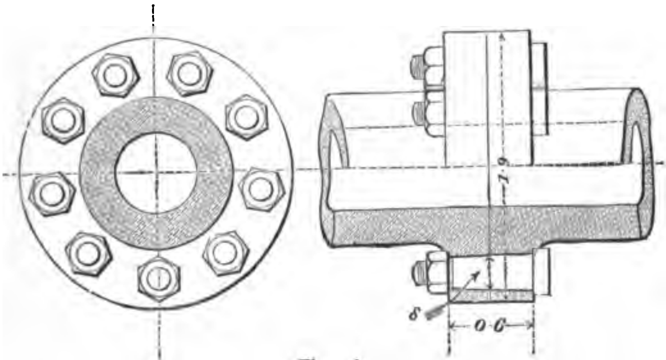


Fig. 136.

the proportions may be the diameter of an equivalent solid shaft. Let d_1, d_2 be the internal and external diameters of the shaft; then,

$$\text{Unit}=d=\sqrt[3]{\left(\frac{d_2^4-d_1^4}{d_2}\right)}$$

The bolt diameter is given by the equation

$$\delta=0{\cdot}75\sqrt{\frac{d^3}{n\,\mathrm{R}}} \qquad . \qquad . \quad (46)$$

where n=number of bolts; R=radius of bolt circle.

148. *Sellers's Double Cone Vice Coupling.*—With box couplings it is generally necessary to forge bosses on the shaft-ends, to receive the couplings. This prevents pulleys and wheels being put on the shafts from the ends. The face-plate coupling depends for its solidity on a taper key, and cannot be often loosened without danger of impairing its accurate adjustment relatively to the axis of the shaft. Mr. Sellers has introduced a coupling which obviates these

difficulties, and which does not require such perfect fitting. Fig. 137 shows this coupling in longitudinal section, and end elevation and cross section. It consists of an outer cylindrical muff, or barrel, enclosing the ends of the shafts. The inside of this is turned to a double conical form. Between the barrel and the shaft are two sleeves, the outsides of which are conical, and fit the box, and the insides are cylindrical, and fit the shaft. These sleeves are pressed together by three screw-bolts, parallel to the shaft. The bolts are square in section, and rest in slots cut into the sleeves and the barrel. To give elasticity to the sleeves, they are completely cut through on one side, at the bottom of one of the bolt slots. Each sleeve is drawn inwards with equal force, and grasps the shaft with equal tightness. A key is driven into each shaft-end, as an additional precaution, but these keys should fit sideways only, and not at top and bottom. They do not then exercise any bursting force on the

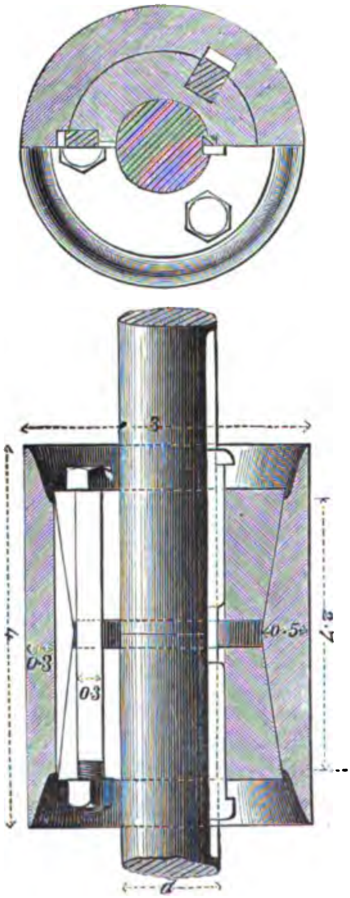


Fig. 137.

coupling. Absolute equality of size of the shafts is unnecessary. When two shafts of unequal size are connected, the larger is turned down, at the end, to the size of the smaller. These couplings are sometimes difficult to disconnect. To obviate this, it has been proposed to tap a set screw through the barrel, having a conical end pressing against the two inner sleeves. When this set screw is turned, it separates the sleeves. If the parts are well oiled before they are put together, there is no great difficulty in disconnecting. The bolts are taken out, and the coupling

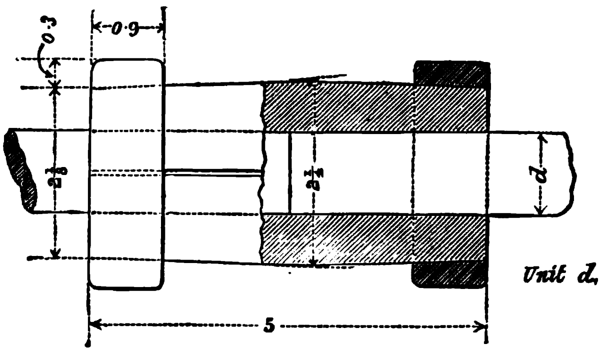


Fig. 138.

struck with a wooden mallet, or a wedge is driven into the split through the sleeves.

149. A simple and effective friction coupling, used in France and Germany, is shown in fig. 138. The coupling consists of a pair of semicircular cast-iron clips. These are placed together with a thin plate or sheet of paper interposed, and bored out to the diameter of the shaft. When placed on the shaft the clips are held by two wrought-iron rings shrunk on. The ends of the shafts are turned, but not polished. The usual proportions are given on the figure, the outside of the clips being turned slightly taper. Sometimes bolts are used to connect the two clips instead

of rings, and then the clips have a somewhat different form.

Supposing the shaft transmitting a twisting moment only (eq. 32)

$$d=a\sqrt[3]{\text{P R}};$$

or for wrought-iron shafts, P estimated at the circumference of the shaft is

$$\text{P}=3530\,d^2.$$

Let p be the pressure of the clip on each shaft end per unit of circumference; μ = the coefficient of friction. Then

$$\mu\,\pi\,d p \overline{\gt}\, 3530\,d^2.$$

Taking $\mu=0{\cdot}2$,

$$p \overline{\gt}\, 5616\,d.$$

Such a bursting pressure produces in the ring a tension of $\frac{1}{2}\,p\,d$ lbs. (eq. 2, § 26). Taking the safe stress in the wrought-iron rings at 9,000 lbs. per sq. inch, each ring must have a section

$$\frac{p\,d}{2\times 9000}=\frac{5616\,d^2}{18000}=0{\cdot}312\,d^2\,;$$

or if the width of the ring is $0{\cdot}9\,d$, its thickness must be at least $0{\cdot}35\,d$. The thickness given above is a little less than this; but on account of allowance for bending, the torsion transmitted is rarely as great as that assumed above. With a stress of 9,000 lbs. the ring will extend

$$\frac{l}{\text{L}}=\frac{f}{\text{E}}=\frac{1}{3200}$$

of its circumferential length. Consequently its internal diameter before shrinking on must be $\frac{1}{3200}$ less than the external diameter of the coupling.

150. *Claw coupling.*—For very large, slowly-rotating shafts, it is desirable that the coupling should have a slight

amount of play, so that when the shafts are a little out of line, the coupling accommodates itself to the obliquity, with-

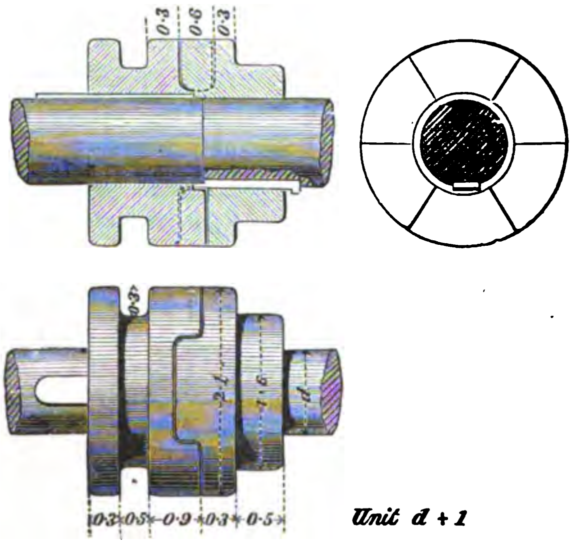


Fig. 139.

out straining the shafts. The claw coupling may then be used, either as a fast coupling, or so arranged that one half can be slid back, and the shafts thrown out of gear. Fig. 139 shows this coupling, arranged for disengaging. It consists of two parts, like the face-plate coupling; but each part has projections, which fit in recesses in the opposite coupling. In the coupling shown, the left-hand part is prolonged, and has a groove cut round it. In this fit the jaws of a lever, for sliding it back. The right-hand part is firmly keyed on its shaft. The left-hand part slides on a fixed key, or feather, which is not tapered. The claws of one coupling fit a little loosely in the recesses of the other, so as to permit a small amount of play.

151. *Universal coupling.*—When the axes of two shafts which are not in line intersect, they may be connected by a Hooke's joint, or universal coupling, shown in fig. 140. The velocity ratio of the shafts is then variable, but if their directions make a small angle, the variation is not great, and is generally unimportant. The proportional unit for the dimensions is $d+\frac{1}{2}$ or $d+1$.

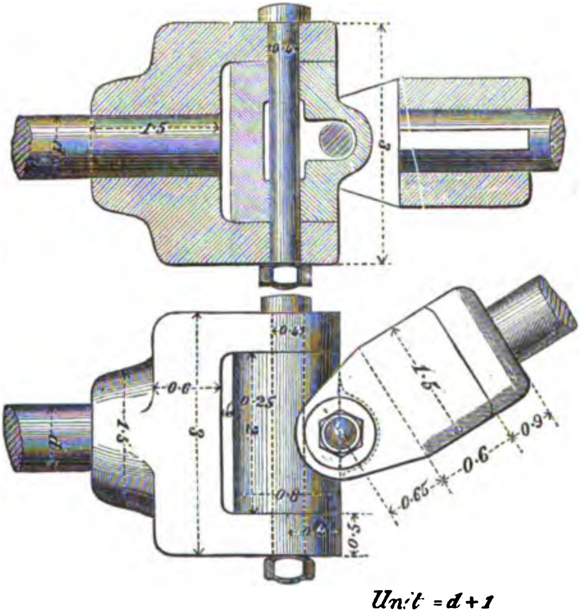


Fig. 140.

152. *Planished wrought-iron shafting.*—The round bars which come from the rolling-mill are rough and slightly crooked. Shafting made from such bars must be turned from end to end in the lathe, to obtain uniformity of diameter and smoothness of surface. A process has, however, lately

been introduced which promises to supersede turning in many cases. By passing the bar while still hot between rapidly revolving bevilled rollers, the scale is cleaned off and the bar rendered so straight and regular that it may be used for shafting, after having been merely polished with a file and emery stick, either in the lathe or in place.

153. *Expansion of shafts.*—If the shaft is of great length, especially if it carries bevil wheels, at distances of more than 40 feet from the journal with collars, which prevent end motion, its alteration of length from changes of temperature becomes troublesome. In such cases other collar bearings are provided near the wheels, and an expansion coupling is introduced at some intermediate point. Ordinary claw couplings are sometimes used as expansion couplings, but more commonly a box coupling is used. One shaft end is fixed in the coupling box by a taper key. The other is secured from rotating relatively to the box by two parallel keys on opposite sides fitted accurately but easily, and allowing free expansion.[1]

The expansion of iron is about 0·0012 of its length for a rise of temperature of 180° Fahr. Hence, if t_1, t_2, are the highest and lowest temperatures to which a shaft is exposed, a point l inches from the bearing with collars will move a distance

$$0{\cdot}0012 \frac{t_1 - t_2}{180} l \text{ inches,}$$

in consequence of variation of temperature.

Centrifugal whirling of shafts.—Prof. Rankine showed that the centrifugal force of a slightly bent shaft and the elastic stress tending to straighten it would become equal at any given speed for a certain distance between the bearings. With a greater distance such a shaft is liable to continue revolving in a bent form, if any small deflection is given to

[1] Sutcliffe, 'Machinery for the Production and Transmission of Motion,' Proc. Inst. C. E. vol. lviii.

it. For a shorter distance it restores itself to straightness. The limit of length between the bearings which should not be exceeded, for a wrought-iron or steel shaft d inches in diameter, making N revolutions per minute, is,—

$$175 \sqrt[2]{\frac{d}{N}} \text{ feet.}$$

With a less distance between the bearings centrifugal whirling does not occur. This does not, however, take into account the effect of rotating masses fixed on the shaft, and not near the bearings.[1]

Fencing of Shafting.—To protect workmen from accident, shafting and gearing not out of reach should be fenced by covers of tin plate. Shafting between detached buildings should be at a considerable height above the ground, or in a covered trench below it.

[1] Rankine, 'Millwork,' p. 549.

CHAPTER VIII.

BEARINGS FOR ROTATING PIECES.

PEDESTALS.

154. THE simplest form of journal bearing is a cylindrical hole in the frame supporting the rotating piece. Such a hole wears oval in the course of time, and does not admit of readjustment. The hole may be lined with a brass sleeve or bush, or with soft metal; it can then be restored to its original condition by a new brass bush, or a new lining of soft metal. A better, though still a very simple, form of bearing is shown in fig. 141. In this the bearing is still in part formed in the frame of the machine, but it is in two parts, which are so arranged that the upper part can be tightened down on the journal by bolts, as it wears. The projections on the cap prevent any horizontal movement. The wear, being most often due to the weight of the pieces supported, takes place vertically. When this is not the case, the division of the bearing should be at right angles to the direction of the resultant pressure on the journal.

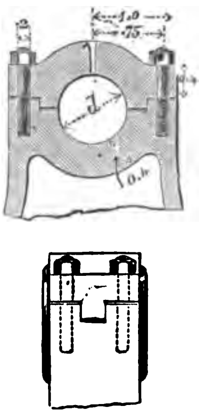
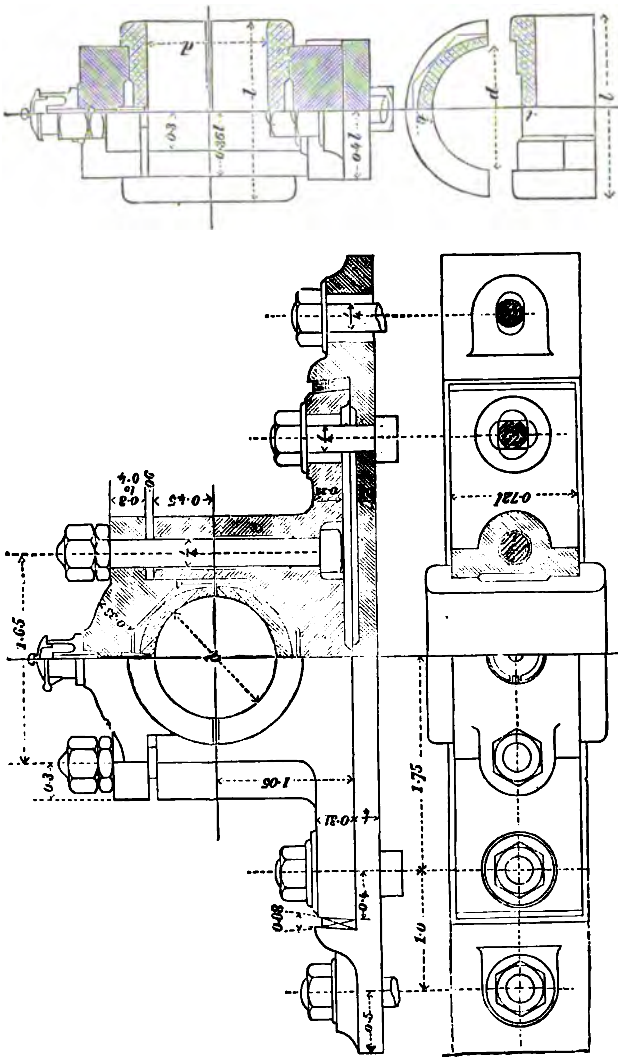


Fig. 141.



Unit $d + \frac{1}{3}$

Fig. 142.

By making the bearing separate from the framing of the machine, a means of initially adjusting its position is secured. Further, by lining the bearing with brasses or steps it is made practically independent of wear. The steps are divided so as to permit adjustment from time to time, and they can be removed and replaced by new ones when so much worn that their adjustment is no longer sufficient to keep the journal steady. When such a bearing instead of being fixed on the framing of a machine has to be fixed on masonry or brickwork, a wall plate or foundation plate is commonly used. This is adjusted in position tolerably accurately, so that the final placing of the bearing can be effected with little trouble. The foundation plate serves also to spread the pressure of the pedestal over a sufficiently large area of the masonry.

Fig. 142 shows a typical pedestal, with the foundation, or wall plate, on which it is fixed. This wall plate spreads the pressure of the pedestal over a larger area, and affords a levelled surface, on which the pedestal can be adjusted with less trouble than on the rough masonry of a wall. The steps are shown externally of octagonal form, the shape most convenient for hand fitting. They are often cylindrical, and are then turned in the lathe, and the pedestal is bored out to receive them. The steps have flanges, to prevent lateral movement. The under surface of the pedestal, and the upper surface of the wall plate, have narrow chipping strips, to facilitate the adjustment of level. The bolt holes in the wall plate and pedestal base are oblong, so that the pedestal can be shifted laterally in either direction. When adjusted to its true position, it is fixed by hard wood or iron wedges, driven between the ends of the pedestal and jaws cast on the wall plate.

The diameter and length of the steps are the same as those of the journal. The other dimensions may be obtained from the proportional figures, the unit for which is $d + \frac{1}{2}$.

155. *Brasses or steps.*—Fig. 143 shows some ordinary forms of steps for journal bearings. In many cases the pressure of a journal on its support acts always in one sense. In such cases a single step is sufficient, as at *a*, which represents the arrangement adopted for railway axles. The part of the journal not in contact with the step is then protected from dust by a shell cap. More commonly the pressure of a journal acts alternately in opposite senses, and then two steps are required as shown at *b*, *c*, *d*, *e*. The dividing plane of the steps is placed in the direction in

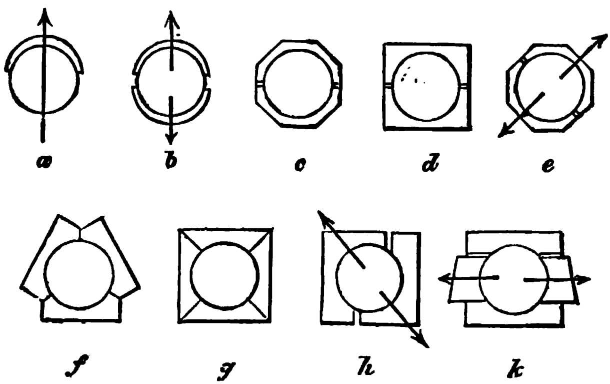


Fig. 143.

which the wear is least, or at right angles to the direction of the resultant pressures. When the direction of the pressure varies, more complex adjustment is necessary, and three or four separately adjustable steps may be used, as shown at *f* and *g*; *h* is an arrangement adopted where the pressure of the journal is inclined, but where the adjustment can be most conveniently made horizontally; *k* is an arrangement permitting horizontal adjustment.

156. Fig. 144 shows sections of three ordinary forms of brass or gun-metal steps, and a half-plan, half-longitudinal, section of a step. The step is fitted to the pedestal for a

portion of its width only, at each end, the intermediate part being recessed, and left rough. When the step is turned, instead of being fitted by hand, it sometimes bears on the pedestal over its whole width. The hexagonal step cannot turn in its seat. The cylindrical step requires a snug, to

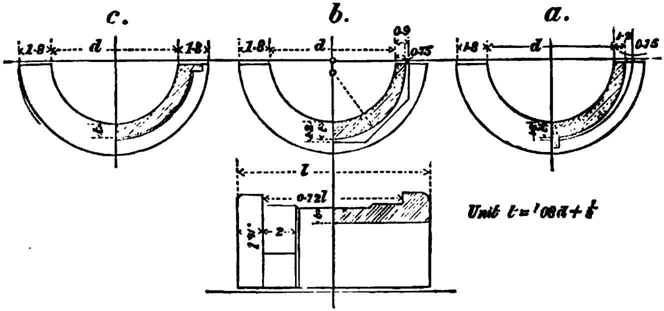


Fig. 144.

prevent turning. The composition of the gun-metal, white brass and phosphor bronze, used for steps, is given in Chapter I. When antifriction metal is used, it is usually applied as a lining to a gun-metal step, and is cast in shallow recesses formed to receive it.

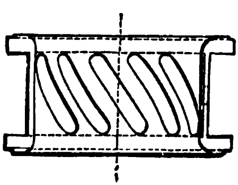


Fig. 145.

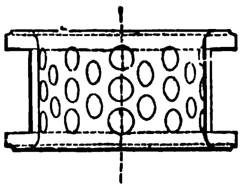


Fig. 146.

Figs. 145, 146, show two good methods of applying antifriction metal or white brass in pedestal steps. The shallow recesses in which the white metal is placed are either helical shallow slots or circular shallow depressions. It appears that from some difference of expansion of the

white metal and brass, channels for distributing the lubricant are formed when the white metal is applied in this way, and the bearing wears better than when a complete white metal lining is applied.

The thickness of the steps at the bottom, where the wear is greatest, may be

$$t = 0{\cdot}07d + \tfrac{1}{8} \text{ to } 0{\cdot}1d + \tfrac{1}{8}.$$

At the sides the thickness may be $\frac{3}{4}t$. The proportional unit for the dimensions of the steps is t.

Table of Pedestal Proportions.

Diameter of journal, in ins.	Length of bearing, in ins.	Height to centre	Diameter of bolts	Size of bolt holes	Length of base	Centres of cap bolts	Centres of base bolts	Thickness of step at bottom.
$1\frac{1}{2}$	$2\frac{1}{2}$	$2\frac{1}{8}$	$\frac{1}{2}$	$\frac{5}{8} \times 1$	$8\frac{7}{8}$	$3\frac{1}{2}$	$7\frac{1}{4}$	$\frac{1}{4}$ to $\frac{5}{16}$
2	3	$2\frac{3}{4}$	$\frac{5}{8}$	$\frac{3}{4} \times 1\frac{1}{4}$	11	$4\frac{3}{8}$	9	$\frac{5}{16}$ to $\frac{3}{8}$
$2\frac{1}{2}$	$3\frac{1}{2}$	$3\frac{1}{4}$	$\frac{3}{4}$	$\frac{7}{8} \times 1\frac{1}{2}$	$13\frac{1}{4}$	$5\frac{1}{4}$	$10\frac{7}{8}$	$\frac{5}{16}$ to $\frac{7}{16}$
3	4	$3\frac{3}{4}$	$\frac{7}{8}$	$1 \times 1\frac{5}{8}$	$15\frac{1}{2}$	$6\frac{1}{8}$	$12\frac{5}{8}$	$\frac{3}{8}$ to $\frac{1}{2}$
$3\frac{1}{2}$	$4\frac{1}{2}$	$4\frac{5}{16}$	1	$1\frac{1}{8} \times 1\frac{3}{4}$	$17\frac{1}{2}$	7	$14\frac{3}{8}$	$\frac{3}{8}$ to $\frac{1}{2}$
4	5	$4\frac{7}{8}$	$1\frac{1}{8}$	$1\frac{1}{4} \times 2$	20	$7\frac{7}{8}$	$16\frac{1}{4}$	$\frac{7}{16}$ to $\frac{9}{16}$
5	6	6	$1\frac{3}{8}$	$1\frac{1}{2} \times 2\frac{1}{4}$	24	$9\frac{5}{8}$	$19\frac{7}{8}$	$\frac{1}{2}$ to $\frac{3}{4}$
6	7	7	$1\frac{5}{8}$	$1\frac{7}{8} \times 2\frac{3}{8}$	$28\frac{1}{2}$	$11\frac{3}{8}$	$23\frac{3}{8}$	$\frac{9}{16}$ to $\frac{13}{16}$
7	8	$8\frac{1}{2}$	Two $1\frac{1}{8}$	$1\frac{5}{8} \times 2\frac{1}{4}$	...	$12\frac{1}{4}$	...	$\frac{5}{8}$ to $\frac{7}{8}$
8	9	$9\frac{3}{8}$	,, $1\frac{1}{4}$	$1\frac{3}{4} \times 2\frac{5}{8}$	...	14	...	$\frac{11}{16}$ to 1
9	10	$10\frac{1}{4}$	,, $1\frac{3}{8}$	$1\frac{7}{8} \times 2\frac{1}{2}$	...	$15\frac{3}{4}$	...	$\frac{3}{4}$ to 1
10	11	$11\frac{1}{2}$	,, $1\frac{3}{4}$	$2 \times 2\frac{3}{4}$	...	$17\frac{1}{2}$	...	$\frac{7}{8}$ to $1\frac{1}{8}$
12	13	$13\frac{1}{2}$	,, $2\frac{1}{8}$	$2\frac{3}{8} \times 3\frac{1}{8}$	...	21	...	1 to $1\frac{1}{8}$

From 7″ upwards, the pedestals have two bolts on each side, both in cap and base plate.

157. Large steps when heated by the friction of the journal tend to grip the journal at the sides. They should therefore be eased so as to fit the journal a little loosely at the sides. At one or more bearings of a shaft, collars are used to prevent longitudinal motion, and the steps at these bearings sometimes give trouble from heating. Mr. Sutcliffe points out that the steps heat more and expand more rapidly than the shaft. Hence some clearance should be

provided between the ends of the steps and the collars. Generally a very small amount of clearance is sufficient. But in shafts driven by belting, where a small amount of longitudinal motion is unobjectionable, the clearance may be one-tenth of the length of the journal. The longitudinal play tends to make the journal wear uniformly.

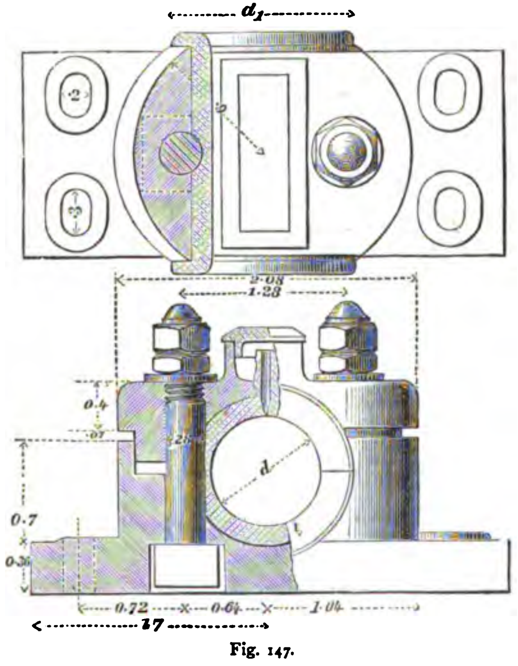


Fig. 147.

158. *Weight of pedestals.*—The approximate weight of the cast iron in pedestals is given approximately by the following equation :—

$$w = 1 \cdot 1 d^3 + 18 \text{ lbs.}$$

and the weight of a pair of steps is

$$w = 0 \cdot 23 d^3 + 6 \text{ lbs.}$$

159. At times a pedestal requires to be contracted in dimensions. Fig. 147 shows a very neat and compact pedestal, designed by Mr. Arthur Rigg, C.E. The cap fits in a cylindrical recess in the body, which can be turned out in the lathe. The bolt holes are bored out, and the recess for the steps also. The pedestal may be still further contracted, by making the cap bolts double-ended, and using them both for attaching the cap to the body, and the body to its support. The base is then absent.

160. *Long bearings for high-speed shafts.*—When a shaft runs at a high speed, the bearings must be long, to secure durability. The steps are then often of cast iron, which answers well as a support for wrought iron, if sufficient bearing surface is given. But the longer the bearings are, the more important it becomes that they should be exactly concentric, and in line. For long shafts, it is then desirable to give the steps a spherical seat, so that they may, to some extent, adjust themselves to the position of the shaft. In America, fast-running shafts, supported on cast-iron bearings four diameters long, have been extensively used; and for carrying these shafts, Mr. Sellers has introduced the pedestal shown in fig. 148. The steps are supported on the spherical parts, and can rotate slightly, either horizontally or vertically. The spherical parts ought, in strictness, to be struck from the same

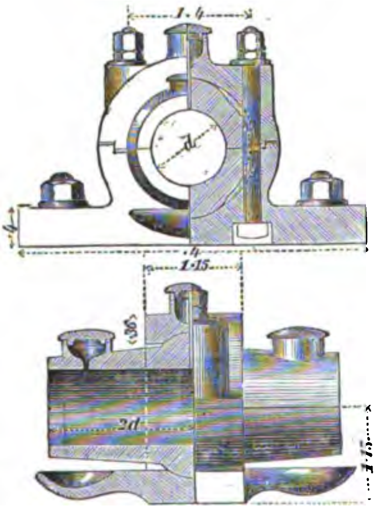


Fig. 148.

centre on the axis of the shaft. Drawings of these pedestals are given in a paper by Mr. Sellers in the 'Journal of the Franklin Institute,' 1872. The lubrication of these pedestals is peculiar. The ordinary lubrication is at the centre of the pedestal; in addition to this, two cup-shaped hollows are formed near the ends of the top step. These are filled with a mixture of tallow and oil, which is solid at ordinary temperatures, and melts at about 100° F. If the step heats from failure of the ordinary lubrication, the tallow melts, and prevents injury to the shaft. A drip cup is provided under each end of the pedestal.

161. *Self-lubricating pedestals.*—Many pedestals have been designed with oil reservoirs, which enable the pedestal

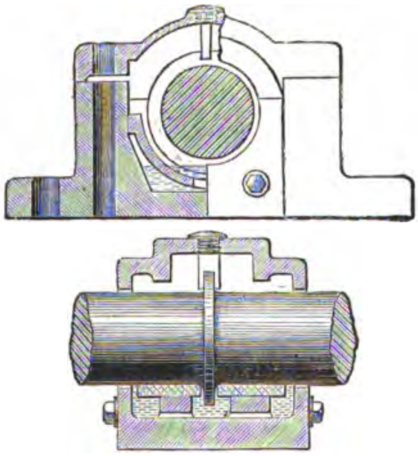


Fig. 149.

to run six months, without additional lubrication. Fig. 149 shows Möhler's pedestal. This has a lower brass only, divided into two portions by a collar on the shaft. The lower part of the pedestal is hollow, and forms a reservoir, into which the collar dips. As the shaft revolves, the collar lifts the oil, and distributes it to the shaft on either side.

The surplus oil flows back into the reservoir. The objection to these pedestals is that they require a large supply of oil at first, which gradually becomes viscid by absorption of oxygen, and is then useless.

162. *Lignum vitæ bearings.*—In a few cases where ordinary oil lubrication is difficult, lignum vitæ bearings are used. Thus the stern tube bearing of propeller shafts consists of strips of lignum vitæ, 1 to 4 inches wide, placed parallel to the shaft. The strips are fixed in a cast-iron tube, into which they are dovetailed. They project $\frac{1}{4}$ to $\frac{3}{8}$ inch, and have spaces of about $\frac{1}{2}$ to 1 inch wide between them. Through these spaces the sea water finds free access to the rubbing surfaces, acting as a lubricant and keeping the surfaces cool. The shaft is cased with gun metal in order that there may be no corrosion of the rubbing surface. The bearing varies in length from 2 feet for engines of 100 I.H.P. to 6 feet for engines of 1,000 I.H.P.

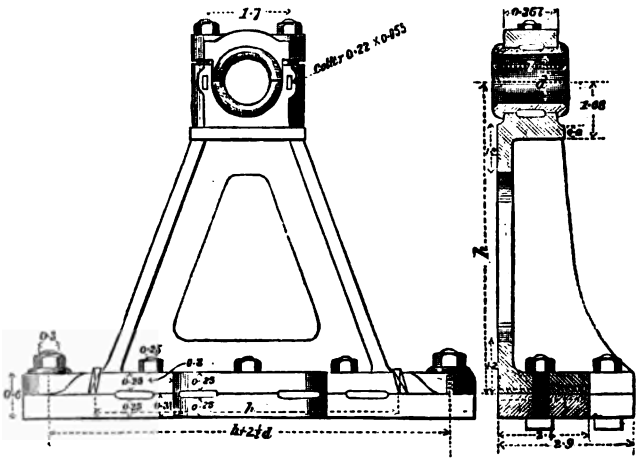


Fig. 150.

163. When a pedestal requires to be elevated above its support, the form shown in fig. 150 is used. The propor-

tions of the steps, cap and cap bolts, are the same as for an ordinary pedestal. The other dimensions are given on the figure, the proportional unit being, as before, $d+\frac{1}{2}$.

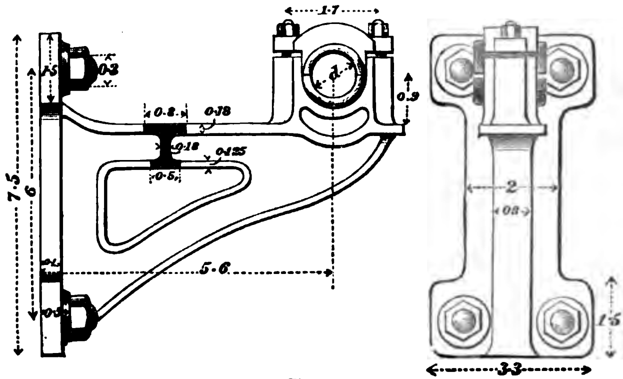


Fig. 151

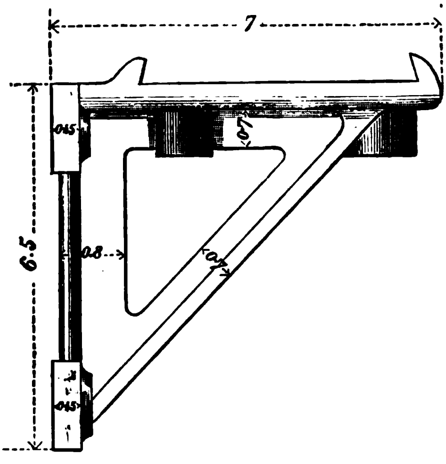


Fig. 152.

Sometimes a pedestal has to be fixed to a wall. Then the bracket pedestal shown in fig. 151 is used. The unit

for the proportions is $d+\frac{1}{2}$. An ordinary pedestal may be

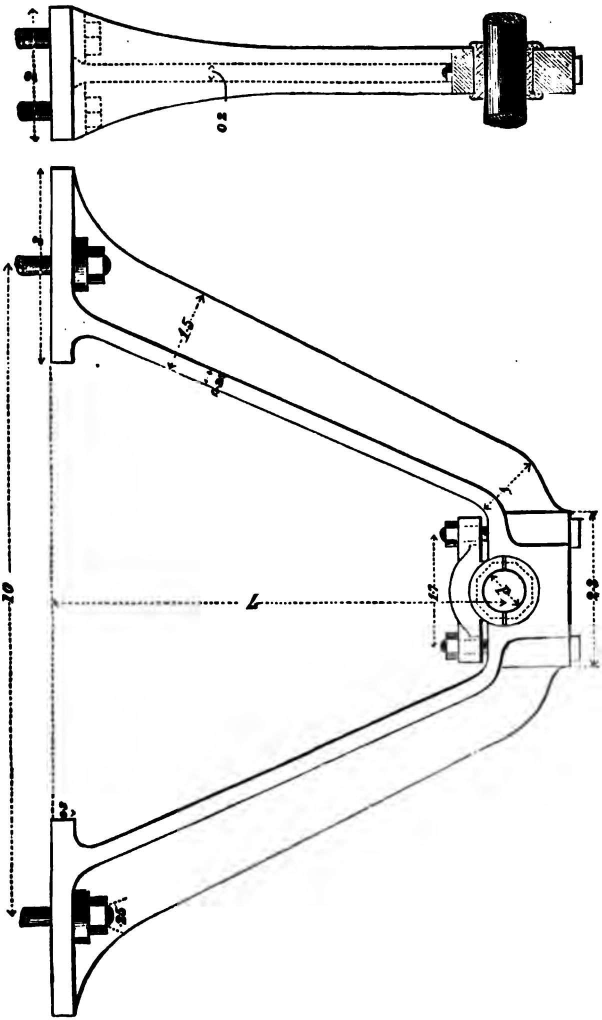


Fig. 153.

used, fixed on a bracket such as that shown in fig. 152. The

recess under the bearing in fig. 151 serves to receive a tin dish, which catches the oil drippings.

164. *Hangers.*—When a shaft is supported from the ceiling girders, the pedestal is modified in form, and is

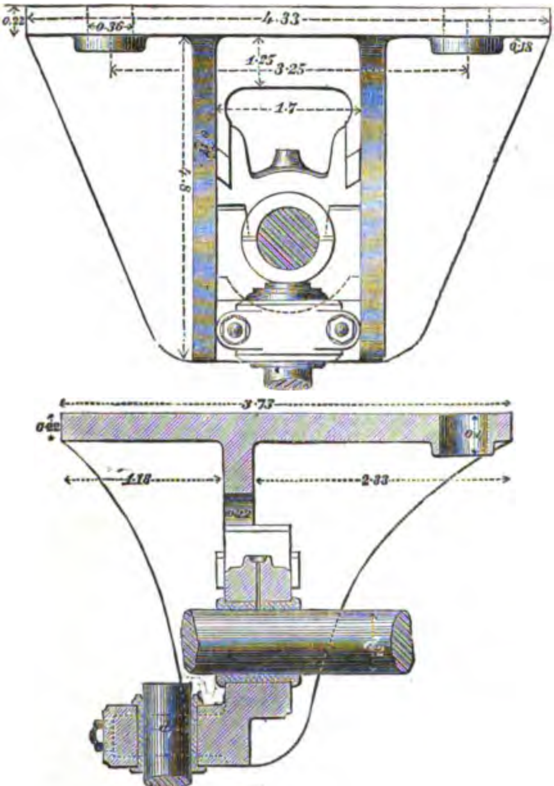


Fig. 154.

termed a hanger. Two forms are used; in fig. 153, the pedestal base is bent round and upwards, and attached to the ceiling on both sides; in the other, the pedestal is supported on one side only. The latter arrangement facilitates

the dismounting of the shaft, but requires more metal in the hanger.

165. Fig. 154 shows a hanger for two shafts, whose directions intersect. This happens when one shaft drives another by bevil gearing. The cap of the upper pedestal is kept in place by keys. The steps, bolts, and caps of this hanger may be designed as for ordinary pedestals. The proportional unit for the remainder of the pedestal is

$$d_2 + 0{\cdot}4d_1 + \tfrac{1}{2},$$

where d_2 is the diameter of the greater, and d_1 that of the smaller, of the two shafts.

166. *Wall Fixings.*—When a pedestal is fixed in a wall, a wall box is used. These wall boxes (fig. 155) not only

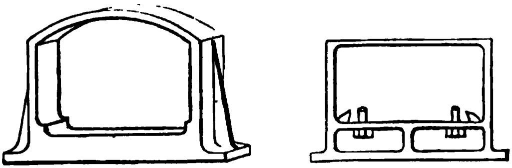


Fig. 155.

give a firm and level support to the pedestal, but they carry the wall over the opening, and give a regular form to the opening.

When wall boxes are provided and built into the wall during its erection, they may have broad outside flanges on each face of the wall to prevent movement. They are also often conveniently built of four separate plates, instead of being in a single casting (fig. 156). All wall boxes giving a clear opening through a wall should be provided with internal flanges, to which wrought-iron plates can be bolted to form a fireproof barrier. (Sutcliffe, *Proc. Inst. C.E.*, vol. lviii.)

Figs. 156, 157, taken from Mr. Sutcliffe's paper, are examples of the most modern type of wall fixings, for carrying shafts of considerable size in well-arranged factories. In fig. 157 a small wall box is shown, which forms

a support for a bracket fixing, on which a pedestal is to be placed. The wall box is built into the wall between two ashlar stones, which carry the pressure of the ironwork

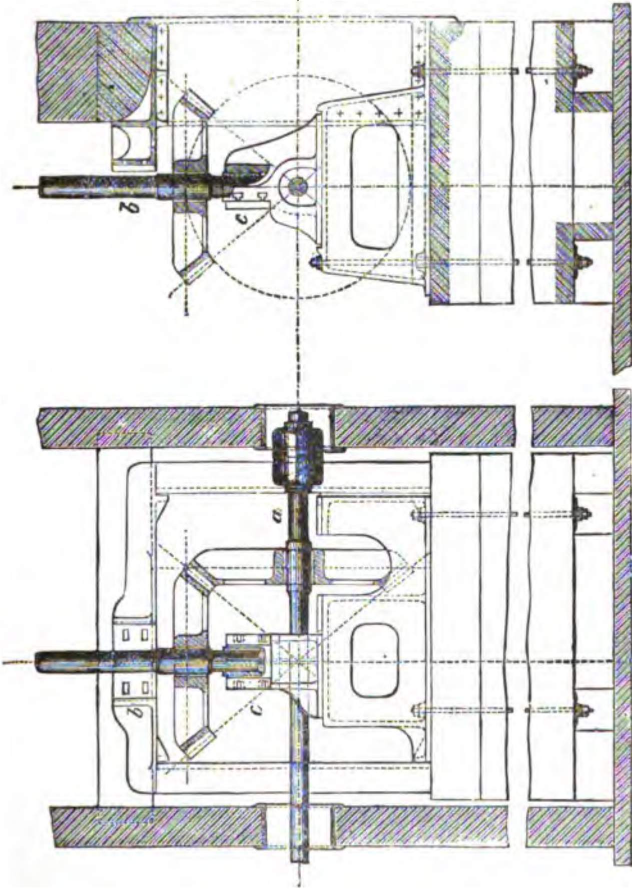


Fig. 156.

better than the brickwork of the wall. The face of the iron wall box is planed to receive the base plate of the bracket, and is provided with two vertical grooves which receive the

T-shaped heads of the bolts which fix the bracket in place. Thus a very accurate vertical adjustment of the bracket can be effected. When this is done, the bracket is additionally secured by wedges or floats driven between the ends of the bracket and snugs on the wall box. The horizontal adjust-

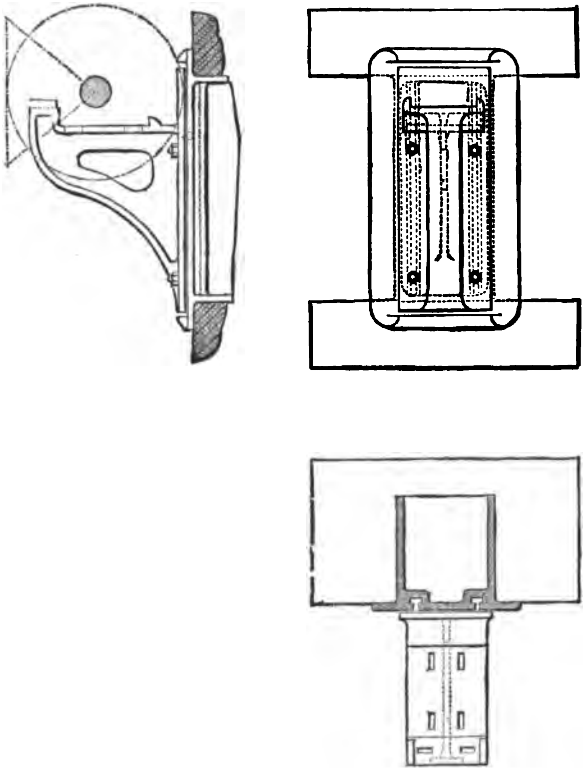


Fig. 157.

ment of the pedestal can be similarly effected, and thus the adjustment of the position of the shaft is completely provided for.

In most mills the engines drive a horizontal shaft, which gives motion to a vertical shaft passing up through all the

floors of the mill, and in turn driving the shafting on each floor. The vertical shaft, therefore, carries the whole power of the engines, and any disarrangement of its action affects the whole factory. Great care, therefore, is taken in the fixing of this shaft. The shaft is now often placed in a separate walled tower with stages or floors at each pair of wheels. The footstep is carried on a massive plate, which rests on a masonry and brick pier carried up from a solid foundation. Openings to the tower are closed by iron doors to prevent the communication of fire from floor to floor. Fig. 156 shows the arrangement of the fixing at the foot of such an upright shaft on a scale of $\frac{1}{6}$th of an inch to the foot. The drawing shows the horizontal shaft, with its coupling, and the vertical shaft with its pivot. At *a* is the position of the pedestal for the horizontal shaft; at *b*, that of the pedestal for the upright shaft. The bevil wheels work in the aperture formed by the wall box. The casting on which the footstep is fixed is partly built into the wall, partly fixed by long foundation bolts. The front plate of the footstep *c* is fixed by T-headed bolts. When this is removed the footstep can be taken out.

Axle Boxes.

167. Axle-boxes are peculiarly formed journal-bearings, by which the weight of locomotive engines and railway carriages is transmitted to the axles. The axle-boxes of carriages are of very various forms, and will not be treated here. The axle-boxes of locomotive engines are more simple, and may be briefly considered, as illustrations of modified pedestals. In a good axle-box the lubrication should be constant, and not wasteful; the journal should be protected from grit; and should fit easily in the step, with a moderate amount of end play. Axle-boxes consist of an outer casing, a step of gun-metal or other alloy, and a hollow shell, closing the under side of the box, and receiving the surplus oil. The outer casing is accurately faced on

both sides, to fit the space prepared for it in the horn plates, between which it slides vertically, and it is provided with flanges, which permit a small amount of lateral play. The upper part is formed into an oil box, from which copper tubes conduct the oil to the journal. It has also usually a socket, to receive the end of the spindle, through which the

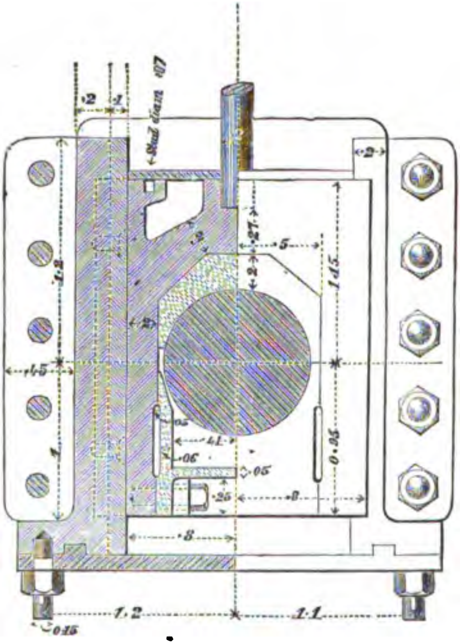


Fig. 158.

weight of the engine is transmitted to the box. This rod bears on a plate spring at its upper end. External axle-boxes are closed in front, and being of rather complex form, are of cast iron. Axle-boxes for internal framings are simpler, and are of forged wrought iron, or of cast steel. In the older axle-boxes, wedges were provided on each side, to com-

pensate for wear, but these are now generally omitted. The step is like that of an ordinary pedestal, and is sometimes lined with soft metal.

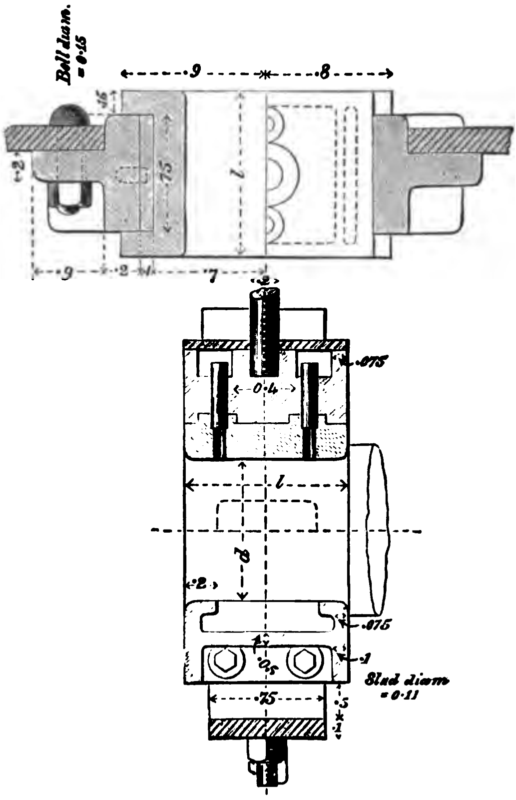


Fig. 159.

Let d be the diameter, l the length of a journal. Then the product $d\,l$ is called the bearing surface of the journal. Let P be the load on the journal, then $P \div d\,l$ is the intensity of the pressure on the bearing surface.

The following table gives the dimensions of some actual axle journals, the ordinary speed and the pressure allowed per unit of bearing surface.

Material of axle		Size of journal	Revolutions per minute	Load per unit of bearing surface in lbs.
Steel	Passenger carriage .	6″ × 3″	240 to 360	340
,,	Goods waggon . .	6″ × 3½″	180	380
,,	Tender axle . . .	7′ × 3½″	240 to 360	330
,,	Locomotive axle .	6″ × 4½′	240	210
Wrought iron	Goods engine axle .	7″ × 5¼″	180	250
,,	Goods waggon . .	6″ × 3½′	180	380

The axle-box shown in figs. 158, 159, is a trailing axle-box of cast iron. Let D be the diameter of the cylinder. The bearing surface on each side may be $0{\cdot}4\ D^2$. The thickness of the step is $\frac{d}{5}$ to $\frac{d}{6}$, where d is the journal diameter. Lengthways, the step may be $\frac{1}{16}$th inch shorter than the journal, to allow a little end play. The unit for the proportional figures is $d + \frac{1}{2}$.

FOOTSTEP BEARINGS.

168. When a shaft is vertical, its lower end rests in a kind of pedestal, termed a footstep. The ordinary arrangement is shown in fig. 160. The end of the shaft is steeled or has a steel end welded to it. Lateral motion is prevented

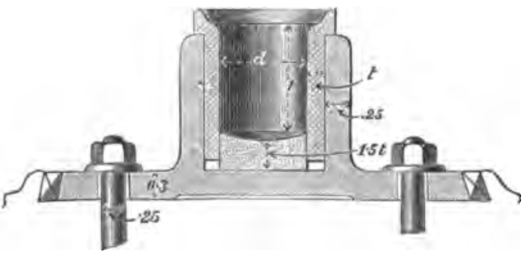


Fig. 160.

by a brass bush, fitting in a cast-iron fixing, and end movement, by a tough brass or steel, slightly cup-shaped, disc, on

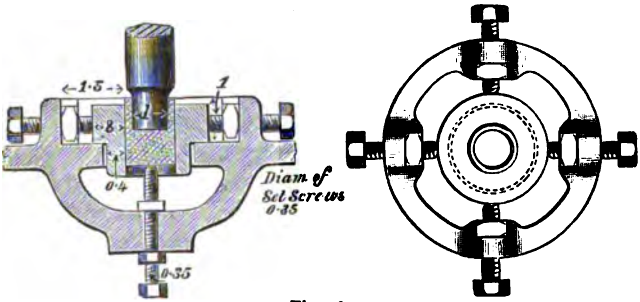


Fig. 161.

which the shaft pivot revolves. The thickness of the brass may be $t = 0{\cdot}07d + \frac{1}{8}$. The unit for the other dimensions is $d + \frac{1}{2}$.

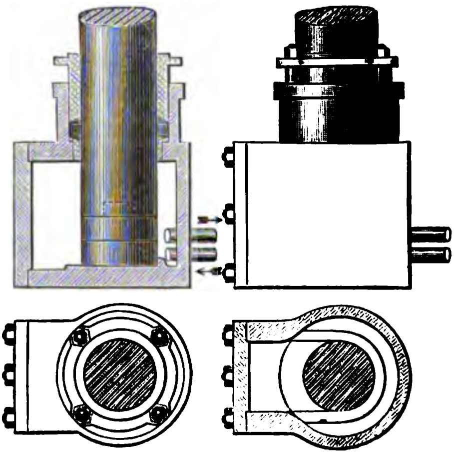


Fig. 162.

In the case of important upright shafts of factories, the weight of the shaft is so great that the brass disc is liable to split. To prevent this, it may be hooped with a wrought-

iron ring shrunk on. Fig. 156 shows a pivot footstep in position, and has already been described.

When exact vertical and lateral adjustment of the footstep is necessary, the arrangement in fig. 161 is adopted. The lateral adjustment is effected by four set screws, and the vertical adjustment by a single set screw. The horizontal screws are tapped into the casting, and fixed by lock nuts. The vertical screw has two nuts. Unit $d+\frac{1}{2}$.

When a footstep works under water, there is difficulty in ensuring proper lubrication of the pivot. Fig. 162 shows a turbine pivot, enclosed in an oil casing, through which a flow of oil can be ensured. Two small copper pipes are connected with the casing, and these are conducted to points above the water level. The shaft passes into the casing through a stuffing box. The end of the shaft is provided with a steel disc, working on a similar disc fitted in the casing.

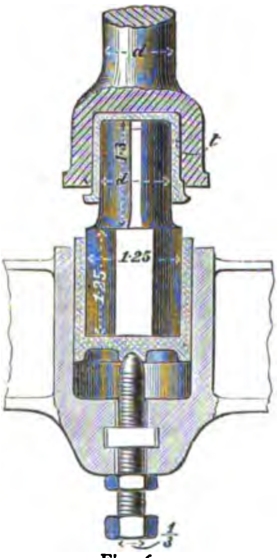


Fig. 163.

Another arrangement which is very effective, is to dispense with the ordinary metal pivot, and replace it with a pivot of lignum vitæ. For metal on wood, water is an excellent lubricant, and such bearings work with very little wear under great pressures. The pivot, fig. 163, is inverted, so that grit is less likely to enter. The pivot is adjusted vertically by a set screw. The end of the shaft is enlarged, bored out, and fitted with a brass step. A groove is cut round the pivot, which being always filled with water, ensures proper lubrication.

169. *Pivot bearing for suspended shaft.*—In the construc-

tion of turbines which have a vertical shaft, the difficulty of lubricating the pivot under water has led most continental constructors to adopt the arrangement shown in fig. 164. A is a fixed wrought-iron pillar carrying at its top the footstep C, which is in any position above the water in which the turbine works convenient for access. D is the wrought-iron shaft carrying the power of the turbine which rests on the pivot, and which is coupled to a hollow cast-iron shaft B B surrounding the fixed pillar A, and carrying at its lower end the turbine wheel. The lower hollow shaft is, therefore, suspended on the pivot. The diameter of the upper wrought-iron shaft d is calculated by the rules for wrought-iron shafting. The pillar has a diameter d_3 which is often the same as d; but it must be considered if this diameter is sufficient to carry the weight of the turbine and gearing when the pillar is treated as a long column virtually rounded at the ends (Rule II. Table VII. p. 64). The inside diameter, d_1, of the

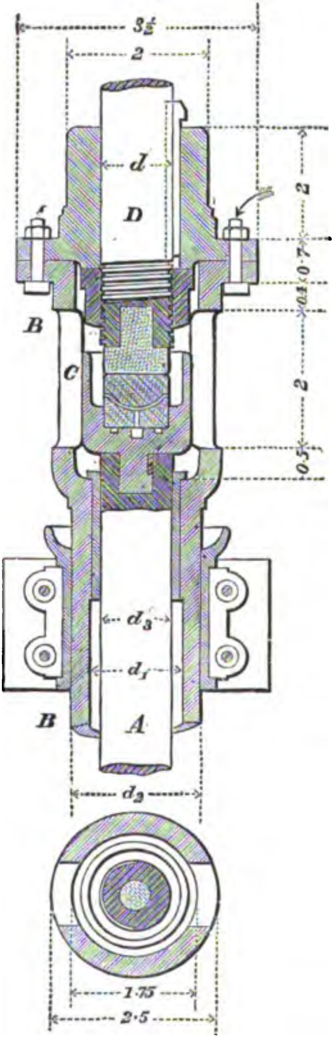


Fig. 164.

hollow shaft must be sufficient to allow room for the brass sleeve which steadies the top of the pillar A. Usually $d_1 = 1{\cdot}35\, d_3$. Lastly, d_2 is calculated so that the section of the hollow shaft is equivalent in torsional resistance to the solid shaft of diameter d. If the two shafts were of the same material, we should have

$$\frac{d_2^4 - d_1^4}{d_2} = d^3.$$

Taking the shearing resistance of wrought iron to be three times as great as that of cast iron, we must have

$$3\frac{d_2^4 - d_1^4}{d_2} = d^3.$$

Generally d_1 is known. Then

$$d_2^4 - \tfrac{1}{3} d_2 d^3 - d_1^4 = 0,$$

an equation difficult to solve directly. Let δ_2 be any approximation to the value of d_2. Then

$$\delta_2 - \frac{\delta_2^4 - \frac{1}{3}\delta_2 d^3 - d_1^4}{4\delta_2^3 - \frac{1}{3} d^3}$$

is a nearer approximation. By repeating the calculation with the new value a still nearer approximation is obtained.

CHAPTER IX.

FRICTION AND TOOTHED GEARING.

170. GEARING is a general term for the means of transmitting motion, but it is especially employed to denote the wheels by which motion is transmitted from one shaft to another. The wheels employed for transmitting motion are almost always toothed wheels, but it is convenient to study first the action of toothless rollers, because each kind of toothed wheel is equivalent cinematically to a toothless roller.

In the following chapter the notation and units employed will be as follows:—

R=radius, d=diameter of wheel in inches.
N=number of rotations per minute.
p=pitch in inches. h=height of tooth in inches.
b=width of face of wheel in inches.
t=thickness of tooth in inches.
T=number of teeth.
P=pressure of one wheel on another, measured in direction of motion or along tangent to pitch line, in lbs.
nP=load on one tooth in lbs.
f=safe stress in lbs. per sq. in.
H=horses power transmitted.
V=velocity of pitch line in ft. per sec.

CONSTANT VELOCITY RATIO.

Parallel shafts.—Let two accurately turned cylindrical rollers be keyed on the shafts, of such a size that they are in contact. Then, if one shaft revolves, the other must revolve also, provided the resistance to motion is not greater than the frictional resistance of the rollers to slipping. If there is no slipping, the velocities of the rollers at the point of contact must be equal.

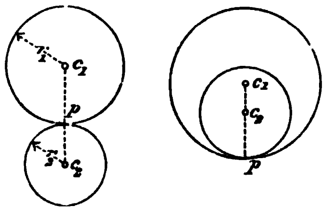


Fig. 165.

Let N_1 N_2 be the number of revolutions of the shafts per minute, and R_1, R_2 the corresponding radii of the wheels. The velocities at the point of contact are $2\pi R_1 N_1$, and $2\pi R_2 N_2$. Since these are equal,

$$\frac{N_1}{N_2} = \frac{R_2}{R_1} \quad . \quad . \quad . \quad . \quad (1)$$

and the velocity ratio is constant. Toothed wheels corresponding to rollers of this kind, are called spur wheels. The surfaces of the rollers are termed *pitch surfaces*. Planes

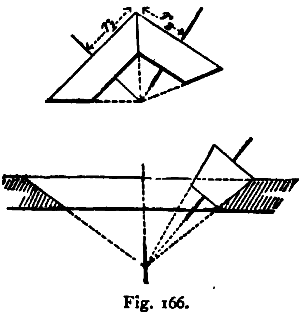


Fig. 166.

normal to the shafts cut the pitch surfaces in circles termed *pitch lines*. The point of contact, *p*, is the *pitch point*.

Shafts the directions of which intersect.

Let two conical rollers (fig. 166) be placed on the shafts, then one will drive the other with uniform velocity ratio, as in the last case. Toothed wheels corresponding to these rollers are termed bevil wheels. In practice, the shafts are in most cases at right angles. The radius, or diameter, of bevil wheels is conventionally measured at the larger end.

Shafts the directions of which are not parallel and do not intersect.

Place on the shafts frusta of hyperboloids, generated by the revolution of a straight line about an axis to which it is not parallel. Then one shaft will drive the other with con-

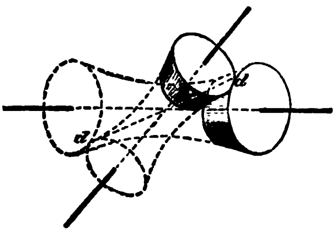


Fig. 167.

stant velocity ratio, though there will be sliding in the direction of the line of contact. Toothed wheels corresponding to these rollers are termed skew bevil wheels.

Velocity Ratio Variable.

In all the preceding cases the sections of the rollers by planes perpendicular to the shafts are circular, and the ratio $\frac{R_2}{R_1}$, which is also the velocity ratio, is constant. If the rollers are of elliptical or of some other form of section, the ratio $\frac{R_2}{R_1}$ and the velocity ratio are variable. In any case

the condition $R_1 + R_2 =$ constant must be fulfilled, or the contact of the rollers during rotation will cease.

171. With toothless rollers it is difficult to transmit much force, without causing a slipping of the rollers. The rollers may be covered with india-rubber or leather, which compresses a little, and thus neutralises the effect of inaccuracy of form, but such arrangements are rarely used. A better plan is to cut circumferential wedge-shaped grooves in the rollers, and to place them so that the projections on one roller fit the recesses of the other. Gearing of this kind is termed frictional gearing. The resistance to slipping is greater than with smooth rollers, but slipping is not altogether prevented. In some circumstances this is an advantage. A much more generally adopted modification of smooth rollers is to form teeth upon them; the teeth of one wheel fit the spaces in the other, and slipping is prevented if the teeth do not break.

Friction Gearing.

172. If two wheels of any of the forms just described are pressed together by a force acting normally to the surfaces at the line of contact, there is a frictional resistance to the slipping of one wheel on the other. Hence, if one wheel is rotated, the other will rotate also, provided the resistance to motion, measured at the pitch surfaces, is less than the frictional resistance to slipping.

Fig. 168 shows simple friction gearing of this kind. The wheels at A are for parallel shafts, those at B for shafts at right angles. In both these cases, if the arrangements are perfect there is simple rolling contact of the wheels. Such wheels may be used (*a*) when the power to be transmitted is not very great; (*b*) when the speed is so high that toothed wheels would be noisy; (*c*) when the shafts require to be frequently put into or out of gear. At C, fig. 168, is shown another form of friction gearing, which has been used with a different object. Here the wheels take the form of discs, and the shafts are at right angles. By moving the

smaller wheel towards or away from the axis of the larger wheel the velocity ratio is varied, and that while the gear is in motion and without any abrupt change. As, however, the smaller wheel must have a sensible thickness, its edge is in contact with parts of the larger disc having different velocities. Consequently there must be sliding and wear of the surfaces in contact. To avoid this, the arrangement shown at D has been invented by Prof. James Thomson. *a* is a disc, *b* a heavy metal sphere, and *c* a cylinder. The

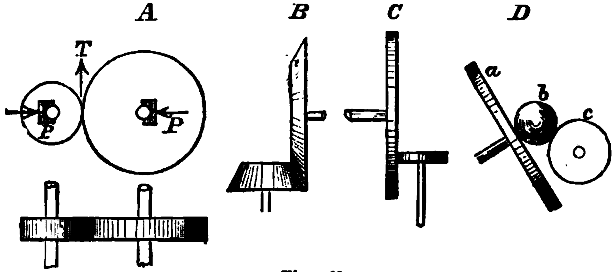


Fig. 168.

disc rotates the sphere by friction at the point of contact, and this in turn communicates motion to the cylinder. The contact is simple rolling contact. By moving the sphere across the face of the disc, the velocity ratio is altered.

Let P be the pressure acting between two friction wheels normally to the surfaces at the line of contact; μ the coefficient of friction; T the tangential resistance to motion of the driven wheel, at the pitch surface. Then

$$\mu P \geq T$$

$$P \geq \frac{T}{\mu}$$

Let V be the velocity of the pitch surfaces in feet per second, H the horses power transmitted. Then

$$T = \frac{550\,H}{V}$$

$$P \geq \frac{550\,H}{\mu V}.$$

R

This gives the magnitude of the external force which must be applied to prevent slipping. We may take for the coefficient of friction, the surfaces being dry,

For metal on metal . .	0·15 to 0·20
„ wood on metal . .	0·25 to 0·30
„ millboard on metal . .	0·20.

In the case of bevil wheels, B, fig. 168, the pressures acting on each wheel may be resolved into a force Q parallel and a force N normal to the shaft. Putting N_1 Q_1 for the forces acting on one wheel, and N_2 Q_2 for those acting on the other; P for the normal pressure at the pitch surfaces of the wheels; r_1, r_2, for the radii of the wheels; and δ for the angle the tangent of which is $\frac{r_1}{r_2}$; we get

$$Q_1 = P \sin \delta = N_2$$
$$Q_2 = P \cos \delta = N_1$$

in which equations P has the value given above, and the friction of the supports of the shafts is neglected. Q_1, Q_2, are the external forces which must be applied along the shafts to prevent the slipping of the wheels.

Friction wheels may be of metal, but generally one of the pair has a surface of wood, of millboard, or of leather, to secure greater resistance to slipping.

Fig. 169 shows the construction of bevil and spur wheels with wood faces; and millboard may be also used in the same way. It is best to make the wheel with wood or millboard face the driver, as it is then less liable to wear irregularly if slipping occurs. It appears that friction gearing of this kind is frequently used in America, especially for driving sawing machinery. From details given by Mr. Wicklin[1] it appears that it is the practice to make the width of face of the wheels equal to the width of a single leather belt to transmit the same power at the same speed, or that

[1] Cooper, 'Use of Belting,' p. 288.

the tangential force t per inch width of face is about 30 lbs. for maple wood, and about 15 to 20 lbs. for pine or other soft wood. Hence if β is the width of face,

$$\mathrm{T}=t\beta=\frac{550\,\mathrm{H}}{\mathrm{V}}$$

where t has the values given above. The pressure between the wheels must be $3\mathrm{T}$ to $4\mathrm{T}$. In the case of a wheel with millboard face, the force transmitted was observed by the author to be much greater, amounting to about 80 lbs. per inch width of face.

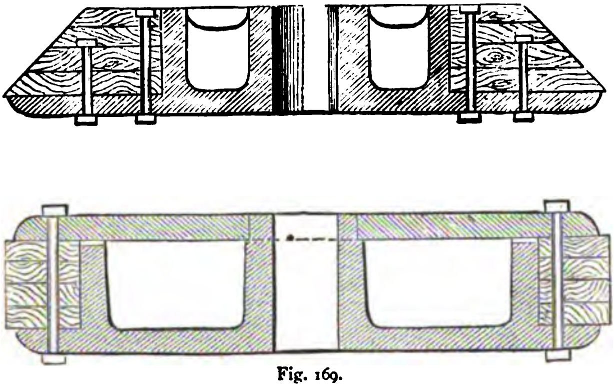


Fig. 169.

Since a very slight movement puts friction wheels out of gear, they are convenient when rapid disconnecting is necessary. Fig. 170 shows an arrangement for disconnecting. The end of the shaft of the driven wheel is carried in an eccentric disc, a slight rotation of which puts the wheels into or out of gear.

A lever and weight is shown for producing the pressure necessary to prevent slipping. This is so arranged as to move both ends of the shaft simultaneously.[1]

[1] Fig. 170 is a diagrammatic sketch of the gearing of a crab invented by Mr. H. Lewis of the firm of Lewis and Lewis, and erected at the Great Eastern Railway Goods Station.

173. *Wedge gearing* (fig. 171) is a modification of friction gearing intended to secure a given resistance to slipping with less pressure at the pitch surfaces, and therefore with less wear of the supports of the wheels than ordinary friction gear. The circumferences of the wheels are cut into wedge-shaped projections by circumferential grooves. Let N be the normal pressure on all the wedge surfaces in contact,

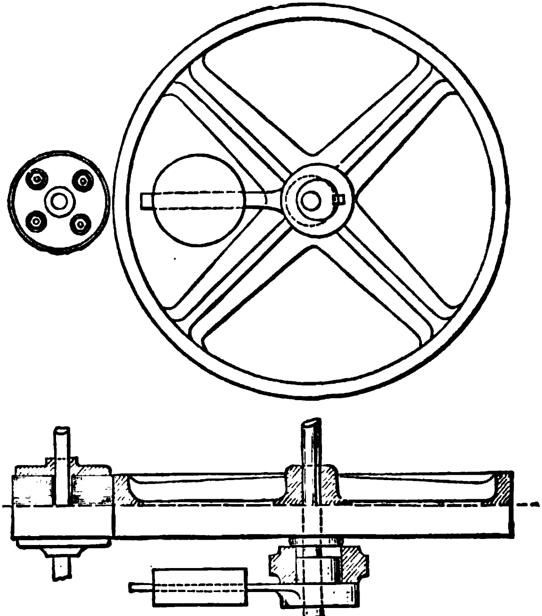


Fig. 170.

P the force pressing the wheels together, T the tangential force transmitted—

$$P = N\,(\sin a + \mu \cos a)$$

where $2\,a$ is the inclination of the sides of the wedges

$$\mu N \geq T$$

$$P \geq \frac{T}{\mu}\,(\sin a + \mu \cos a)$$

The objection to these wheels is that the contact is sliding contact, and therefore the wheels wear and in some cases are noisy. To diminish this evil, the depth of the surfaces in contact should be made small. The inclination of the sides of the wedge projections is usually 30°. The

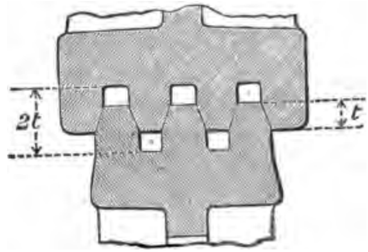


Fig. 171.

number of projections on each wheel is usually 1 to 6, but sometimes a greater number are used. The number of projections has no influence on the power transmitted. The depth t of the acting surface may be taken about

$$t = 0{\cdot}025\sqrt{\text{T}}.$$

The rim and arms may be of the same strength as those of a spur wheel transmitting the same power.

Toothed Gearing.

If the teeth are properly formed, the motion of the wheels is identical with that of two imaginary smooth rollers, whose surfaces form what are termed the pitch surfaces of the wheels. Hence, if R_1 R_2 are the radii of the wheels, measured to the pitch surfaces,

$$\frac{N_1}{N_2} = \frac{R_2}{R_1} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

(§ 170). The importance of forming the teeth so that the velocity ratio is constant, is very great. If the velocity of the driven wheel varies a little during the contact of each pair of teeth, an irregular and injurious motion is imparted to the machinery. In addition, since the inertia of the driven machinery resists alteration of velocity, the tooth of

the driven wheel will be carried forward relatively to the driving tooth, and will strike against the next tooth in front. The wheels then work with noise and vibration. This action is termed back lash.

174. *Material of gearing.*—Ordinary gearing is made of cast iron, and two methods are adopted in moulding wheels. The older plan is to construct an accurate pattern in wood of the entire wheel; from this pattern any number of wheels can be moulded. The other plan is to mould the rim of the wheel, from a pattern of two or three teeth only, in a wheel-moulding machine. The pattern of the teeth is fixed to a radial arm, which can be revolved very accurately, through any required fraction of the circumference. The arms of the wheel are formed by dry sand-cores, moulded in core boxes. Very small wheels are cast with a blank rim, and the teeth cut out in a wheel-cutting machine. The process of moulding wheels by machinery was introduced by Mr. P. R. Jackson, of Manchester, about the year 1855.

Pattern-moulded wheels are less accurate than machine-moulded wheels, partly because the wood pattern is liable to warp, and partly because in moulding the wheel, it is necessary to give a slight taper, or draught, to the teeth. The wheels should be placed in gear with the draught of one wheel in the opposite direction to the draught of its fellow, but this is not always attended to. The result is that the teeth bear on each other chiefly on one side. Machine-cut wheels are expensive, and the teeth are generally inferior in form to those of moulded wheels.

175. *Mortice wheels.*—When wheels are run at high velocities, the teeth of one of each pair are of wood, and are termed cogs. These cogs are morticed into an iron rim, and shaped by hand. The iron wheel, which works with a mortice wheel, is usually 'pitched and trimmed,' that is, the rough surface of the teeth is chipped away, and the teeth are filed perfectly smooth. This ensures greater accuracy in the form of the teeth, and prevents the destruction of the wood cogs by the rough surface of the casting.

Table giving Diameter or Number of Teeth of Wheels in Terms of the Pitch.

Pitch in inches	$\frac{\pi}{p}$	$\frac{p}{\pi}$
$\frac{1}{2}$	6·2832	0·1592
$\frac{5}{8}$	5·0266	0·1989
$\frac{3}{4}$	4·1888	0·2387
$\frac{7}{8}$	3·5904	0·2786
1	3·1416	0·3183
$1\frac{1}{8}$	2·7926	0·3581
$1\frac{1}{4}$	2·5132	0·3979
$1\frac{3}{8}$	2·2848	0·4377
$1\frac{1}{2}$	2·0944	0·4775
$1\frac{3}{4}$	1·7952	0·5570
2	1·5708	0·6366
$2\frac{1}{4}$	1·3963	0·7162
$2\frac{1}{2}$	1·2566	0·7958
$2\frac{3}{4}$	1·1424	0·8754
3	1·0472	0·9549
$3\frac{1}{4}$	0·9668	1·0345
$3\frac{1}{2}$	0·8976	1·1141
$3\frac{3}{4}$	0·8377	1·1936
4	0·7854	1·2732
$4\frac{1}{4}$	0·7392	1·3528
$4\frac{1}{2}$	0·6981	1·4324
$4\frac{3}{4}$	0·6615	1·5120
5	0·6283	1·5916
$5\frac{1}{2}$	0·5711	1·7507
6	0·5236	1·9099
$6\frac{1}{2}$	0·4833	2·0691
7	0·4488	2·2283
$7\frac{1}{2}$	0·4188	2·3875
8	0·3927	2·5465
9	0·3491	2·8647
10	0·3142	3·1829
11	0·2856	3·5014
12	0·2618	3·8200

With machine-moulded wheels, it is only necessary to clear off the sand from the surfaces of the teeth, and to file them smooth. No chipping is necessary.

When wheels are subjected to vibration and shock, gun-metal, phosphor-bronze or malleable cast iron is preferable to cast iron. In a few cases, wheels have been made of wrought iron. Excellent cast steel wheels can also now be obtained.

176. *Relation between the number of teeth and the radius of the wheel.*—The distance measured along the pitch line, from the centre of one tooth to the centre of the next, is termed the *pitch* of the wheel. Let p = the pitch, d = the diameter, T = the number of teeth in a wheel. For reasons which will appear presently, the pitch for any one wheel is made uniform all round the circumference. Then, $p\,\mathrm{T}$ is equal to the circumference πd of the wheel, hence

$$\left.\begin{aligned} d &= \frac{p}{\pi}\,\mathrm{T} \\ \mathrm{T} &= \frac{\pi}{p}\,d \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The preceding table gives values which facilitate the use of these equations.

To lay off the pitch on the pitch line.—The following construction is convenient, when the wheel is so large that it is impossible to find the exact pitch, by stepping round the pitch-line. Let the circle, fig. 172, be the pitch line. At any point, a, draw the tangent $a\,b$. Make $a\,b$ = the pitch. Take $a\,c = \frac{1}{4}\,a\,b$. With centre c, and radius $c\,b$, draw the arc $b\,d$. Then the arc $a\,d$ is $= a\,b$, and is the pitch laid off on the pitch line. When the wheel has many teeth the arc $a\,d$ sensibly coincides with its chord, but, if it has few teeth, there is an appreciable error in taking the chord $a\,d$ equal to the pitch.

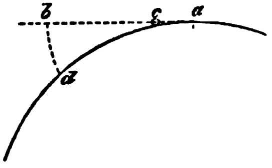


Fig. 172.

177. *Parts and proportions of teeth.*—Fig. 173 shows the general form of wheel teeth, drawn, for convenience, on a straight pitch line. *b d* is the thickness of a tooth ; *d e*, the side clearance ; *e b*, is the width of a space ; *b a* is the face, and *b c*, the flank of a tooth ; *g h* is the bottom clearance.

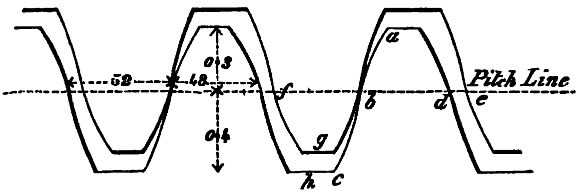


Fig. 173.

In working, the faces of the teeth of one wheel come into contact with the flanks of the teeth of the other wheel. Also, while the point of contact is approaching the line of centres, the flank of the driving acts on the face of the driven tooth ; while the point of contact is receding from the line of centres, the face of the driving acts on the flank of the driven tooth. The arc of the pitch-circle, through which the wheel turns during contact, is called the arc of contact, and the portions into which it is divided by the pitch point, are the arcs of approach and recess. The length of the arc of approach depends on the length of face of the driven tooth, and the arc of recess on the length of the face of the driving tooth.

In ordinary gearing, the teeth have the following proportions :—

Pitch $= p$
Thickness of tooth $= b\,d =$ ·48 p to ·485 p
Width of space $= b\,e =$ ·52 p to ·515 p
Height of tooth $=$ ·6 p to ·75 p
Height above pitch line $=$ ·25 p to ·33 p
Depth inside pitch line $=$ ·35 p to ·42 p

The following proportions are good :—

Thickness of tooth	$= 0.48\,p - .03$,	for pattern-moulded wheels.
"	$= 0.485\,p - .03$,	for machine-moulded wheels.
Width of space	$= 0.52\,p + .03$,	for pattern-moulded wheels.
"	$= 0.515\,p + .03$,	for machine-moulded wheels.
Height above pitch line	$= 0.3\,p$	
Depth below "	$= 0.35\,p + 0.08$	

Mortice wheels usually have the wooden cogs of one wheel thicker than the iron teeth of the other wheel, and the teeth are sometimes a little shorter than those of ordinary gearing. The clearance also may be very much reduced. The following proportions are good :

Thickness of iron teeth . . .	$0.395\,p$
" wood cogs . . .	$0.595\,p$
Height outside pitch line . .	$0.25\,p$
Depth inside pitch line . . .	$0.29\,p + .08$

It is safer to obtain increased strength by increasing the pitch instead of the width, but adding to the width gives greater durability. The average practice is to make the width of face from 2 to $2\frac{1}{2}$ times the pitch.

Conditions determining the Form of Teeth.

178. *Condition of continuous contact of a pair of teeth.*—Let A and B be two spur wheels rotating at any moment with angular velocities $+\omega_1$ and $-\omega_2$. Nothing will be changed in the relative motion of the two wheels, if a rotation $-\omega_1$ is impressed on each. Then A's angular velocity of rotation will be $\omega_1 - \omega_1 = 0$, that is, it will be at rest. The centre of B will rotate about A with the velocity $-\omega_1$, and B

will rotate about its own centre with the velocity $-(\omega_1+\omega_2)$. Hence, the motion of B will be the same as if it were at the moment rotating about an axis, placed at a point dividing the line of centres in the ratio $\omega_1 : \omega_2$, or it will roll on the pitch line of A. Now let a tooth be fixed to B. In order that that tooth may remain continuously in contact with a tooth on A, the form of the latter must be the envelope of the successive positions of the tooth on B, as it moves round A. The form of the tooth on B is not arbitrary. Only certain forms give to the envelope shapes which are practically realisable as wheel teeth.

If two solids, such as two wheel teeth, move in contact, they must have equal velocities in the direction of their common normal. For if the velocities were not equal, one tooth would be penetrating into the space occupied by the other. But in the case above one tooth is at rest. Therefore the other can have no velocity in the direction of the normal to its surface at the point of contact. Hence the point of contact must always fall in such a position that the normal to the tooth at that point passes through the axis of rotation at the moment, that is, the point which divides the line of centres in the ratio $\omega_1 : \omega_2$, which coincides with what is commonly termed the pitch point.

Condition that the teeth may act simultaneously.—In order that all the pairs of teeth may have the same work to do, they must come into action at the same point, and remain in action while the wheel turns through the same angle. All the teeth should therefore be similar in form. In actual wheels, two pairs of teeth (sometimes more) are simultaneously in contact. Let α and β be two positions of the simultaneous points of contact of two teeth a and b. Then, under the conditions assumed above, when A is reduced to rest and B moves round it, the two envelopes passing through α and β are simultaneously described. Therefore ω_1, ω_2 must be the same for all the teeth, both in the position α and the position β. Since a corresponding relation must hold for

all pairs of contact points, ω_1 and ω_2 must be constant for the whole period of contact, that is, the velocity ratios of the wheels must be constant.[1]

Condition of constancy of velocity ratio.—To a certain extent this is secured by having the teeth numerous and small. But in addition, the single and sufficient condition which ensures the constancy of the velocity ratio, during the action of each pair of teeth, is this:—The common normal to two teeth at the point of contact, must always pass through the pitch point,[2] a condition which is fulfilled if one tooth is the envelope of the relative positions of the other.

Influence of the form of the tooth on its strength.—It will be seen presently, that the teeth tend to break across at the root. The teeth are stronger the shorter they are, and the thicker they are at the root. They cannot be shortened without reducing the arc of contact, and their length should be such as to ensure a sufficient, but not excessive, arc of contact. The thickness at the root depends on the form selected for the teeth. Involute teeth are generally stronger than cycloidal teeth. With cycloidal teeth, the teeth are stronger the smaller the diameter of the describing circle used for the flanks. In no case should the flanks be described with a rolling circle the diameter of which is greater than half the diameter of the pitch line, inside which it is rolled.

Conditions of durability.—The rolling of the teeth over each other, so as to spread the contact over a considerable length of tooth, is advantageous, but the sliding of the teeth against each other is injurious. The amount of sliding action is the difference of the length of the face of one tooth and the acting part of the flank of its fellow. To ensure durability these two should be nearly equal. The wider the face of the wheel, the more area there is to resist

[1] This was pointed out to the Author by Professor Huët of Delft.

[2] If the pitch line is not circular, and the common normal to the teeth always passes through the pitch point, the wheels have the same varying velocity ratio as smooth rollers, coinciding with the pitch lines.

the pressure. According to Willis, the pushing friction during the approach of the teeth to the pitch point, is more injurious than the drawing friction when receding from it. For this reason it would be advantageous to shorten the faces of the teeth of the driven wheel, even at the cost of having to lengthen those of the driving wheel.

In wheels for clockwork, where the friction is specially prejudicial, it is usual in certain cases to design the wheels so that the driving teeth have no flanks and the driven teeth no faces. The contact is then entirely confined to the period of recess from the pitch point. Prof. Willis gives the following table of the lowest numbers of teeth which will work together when the action is during recess only :

Driver	54	30	24	20	17	15	14	13	12	11	10	9
Driven	11	12	13	14	15	16	17	18	19	21	23	27

179. *Arc of contact.*—If from the points where the contact of a pair of teeth begins and ends, radii are drawn to the centre of the pitch circle, they will cut off an arc of the pitch circle, termed the arc of contact. This is divided by the pitch point into two parts, termed the arcs of approach and recess. To ensure continuous working, one pair of teeth must always be in gear, and hence the arc of contact must be at least equal to the pitch. Wheels are sometimes made so that contact occurs only during the receding of the teeth from the line of centres. Then the arc of recess must be at least equal to the pitch. Ordinary gearing is so constructed that the wheels are reciprocal, that is, either wheel of a pair may be the driver. In that case, the arcs of approach and recess are equal. In ordinary gearing the arc of contact varies from 1·6 to 2 times the pitch.

180. *Given the form of tooth of one wheel, to find the proper form of the tooth of another wheel, to gear with it.*—Let *a b c* be the given tooth ; *b* B, *b′* B the pitch lines of the wheels ; B, the pitch point, or point of contact of the pitch

lines. From any points, $a\ c$, draw normals, $a\ \alpha$, $c\ \gamma$, to the curve of the given tooth, cutting the pitch line in α, γ. Then the points $a\ b\ c$ should be points of contact, when α, b, γ, are at the pitch point. From B set off B A$=a\ \alpha$, B C$=c\ \gamma$. Then A is the point where a is in contact; B the point where b is in contact, and C the point where c is in contact, and some line, passing through A B C, is the path of contact. Through A, C, draw circles A a', C c'. Set off arc B α'=arc B α; also $\alpha'\ a'=\alpha\ a$. Then a' is a point in the tooth of the second wheel, which will come in contact with a at A, and will have a common normal, passing through the pitch point. Set-off arc B b'=arc B b; then b' will come in contact with b at B. Also, set-off arc B γ'= arc B γ, and $\gamma'\ c'=\gamma\ c$; then c' will come in contact with c at C. A curve $a'\ b'\ c'$ through the points so found, will be the required tooth. In certain cases the construction becomes impossible, and the given tooth is of unsuitable form. Forms should be avoided which make a tooth entirely concave.

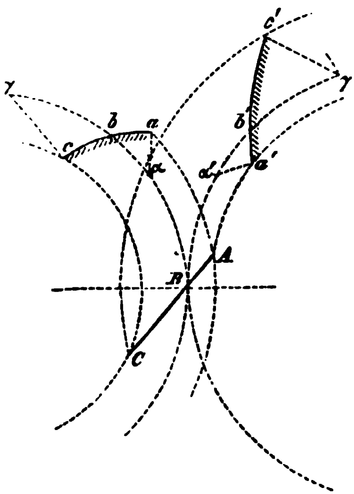


Fig. 174.

Cycloidal Teeth.

181. If a circle rolls on the circumference of another circle, a point in its circumference describes a curve, termed an epicycloid. If the rolling circle rolls inside the base circle, a point in its circumference describes a curve, termed a hypocycloid. It is shown, in treatises on applied me-

chanics, that, if the faces of the teeth of each wheel, and the flanks of the teeth of the other, are respectively epicycloids and hypocycloids, generated by the same rolling circle, rolling outside and inside the given pitch lines, then the teeth will move so that the common normal at the point of contact will pass through the pitch point, and the condition of uniform velocity ratio will be fulfilled. With teeth of this kind, the point of contact moves over an arc of a circle of radius equal to that of the rolling circle, and having its centre on the line of centres.

External Cycloidal Teeth.—Let A_1 A_2, fig. 175, be the pitch circles, p the pitch point, a_1 c_1 the tooth of A_1, and a_2 c_2 that of A_2. The face p a_1 and the flank p c_2 work together, and are described by the rolling circle R, rolled

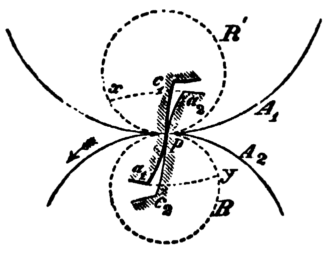


Fig. 175.

outside A_1 and inside A_2. The face p a_2 and the flank p c_1 work together, and are described by the rolling circle R′, rolled outside A_2 and inside A_1. The rolling circles in the figure are so drawn that their centres are on the line of centres. Draw arcs a_2 x, a_1 y concentric with the pitch circles. Then, $x p y$ is the path of contact. Suppose the wheels turn in the direction of the arrow, and that the lower one is the driver. During approach, the flank of the driver is in contact with the face of the driven tooth, and contact begins at y. During recess, the face of the driver acts on the flank of the driven tooth, and contact ends at x. Radii drawn through x and y will mark off on the

pitch lines the arcs of approach and recess. Conversely, if the arc of approach or recess is marked off on the pitch line, and radii drawn cutting the rolling circles, the points x and y will be found, which define the height of the teeth for given arcs of contact.

Internal cycloidal teeth.—Let A_1 A_2, fig. 176, be the pitch lines, and p the pitch point; a_1 c_1 the tooth belonging to A_1, and a_2 c_2 the tooth belonging to A_2. The flank p c_1 works on the face p a_2; both are epicycloids described by the rolling circle R, rolling outside A_1 and A_2. The face p a_1 and the flank p c_2 work together, and are hypocycloids described by R′ rolling inside A_1 and A_2. Through a_1 draw an

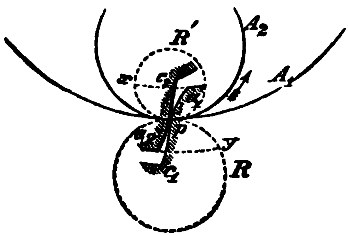


Fig. 176.

arc a_1 x concentric with A_1, and through a_2 an arc a_2 y concentric with A_2, cutting the rolling circles in x and y, then x p y is the path of contact. Radii drawn from the centres of A_1 A_2 to x and y, will mark off the arcs of approach and recess.

182. *Choice of the diameter of the Rolling Circle.*—The diameter of the rolling circle is not usually greater than the radius of the pitch circle, inside which it is rolled. When it is equal to the radius, the flanks of the teeth become radial straight lines. Teeth with radial flanks were at one time much used. Two rolling circles were then taken for each pair of wheels, the diameters of which were the radii of the wheels. When a pair of wheels only are required, the rolling circle, for both faces and flanks of both

wheels, may be made, with convenience, equal in diameter to the radius of the smaller wheel.

When a set of wheels are to be constructed, any two of which will gear together, the same rolling circle must be taken for the faces and flanks of all the wheels of the set. It is then best to take, for the diameter of the rolling circle, the radius of the smallest wheel of the set. Let p be the pitch, d the diameter of the rolling circle, thus determined, T the number of teeth of the smallest wheel of the set.

$$d = \frac{\mathrm{T}\,p}{2\pi} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

T=	d=
11	1·751 p
12	1·910 p
13	2·068 p
14	2·228 p
15	2·387 p
16	2·546 p
20	3·183 p
25	3·981 p

Since the introduction of machine-moulding, it is less an object than it used to be to make wheels in sets.

Fig. 177 shows the influence of the size of the describing circle on the form of the teeth. Let R be the radius of the wheel; r the radius of the describing circle. The tooth curve 1 p 1 is described with rolling circles of radius $r = \frac{1}{4}$ R, and $x'\,y'$ is the corresponding path of contact. The tooth curve 2 p 2 is described with $r = \frac{1}{2}$ R, the flank of the tooth is radial, and $x''\,p\,y''$ is the path of contact. The tooth 3 p 3 is described with $r = \frac{1}{8}$ R, and $x'''\,p\,y'''$ is the path of contact. It will be seen that the smaller the rolling circle, the stronger the tooth is at the root. On the other hand, the smaller the rolling circle, the shorter is the path of contact and the greater the obliquity of action for a given length of tooth.

The arcs of contact are marked off at $a'\,a'$, $a''\,a''$, $a'''\,a'''$.

Taking the pitch at 2 ins., which corresponds to the height of tooth shown, the arcs of contact are 1·07, 1·4, and 0·7 times the pitch. The corresponding maximum angles of obliquity are $x' p \text{T} = 24°$; $x'' p \text{T} = 15°$ and $x''' p \text{T} = 38°$.

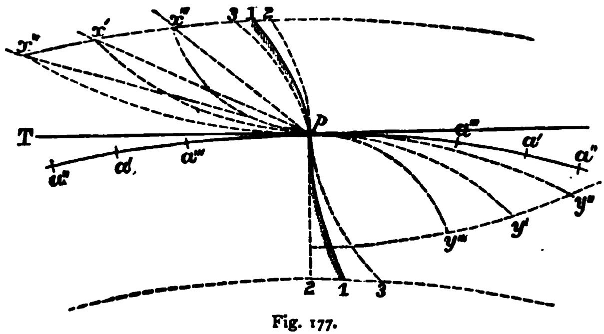


Fig. 177.

183. *Gee's Patent Gearing.*—Messrs. Jackson of Manchester have introduced a peculiar form of tooth, which is 35 per cent. stronger than the usual form. In this gearing the driving faces of the teeth, fig. 178, are of the usual form. The other faces have much more obliquity than ordinary

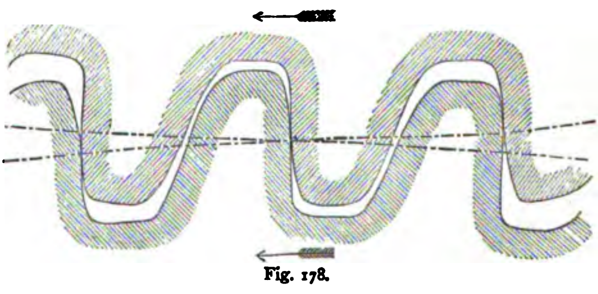


Fig. 178.

teeth. Gearing of this kind should therefore be used to drive in one direction only. The back faces may be cycloidal curves described with very small describing circles, or a form may be selected for the back of the tooth of one

wheel, and the suitable curve for the tooth of the other may be obtained by the process given in § 180.

184. *Methods of drawing cycloidal teeth.*—The curves of the teeth may be found by rolling a templet of the size of the rolling circle, inside and outside templets of the size of the pitch circle. A pencil held in contact with the rolling templet, describes the required curve. The curves may also be obtained by the ordinary rules for describing cycloidal curves. When they have been drawn, it is usually convenient to replace the cycloidal curves by circular arcs, sensibly coinciding with them, and which can be used by the pattern-maker more conveniently than the true curves. In proceeding thus, two sources of error are introduced. It is not easy to draw small cycloidal arcs very exactly, and in fitting circular arcs to them, a new source of error is introduced. To obviate these objections, it was proposed by Professor Willis, to find directly the centres of circular arcs which would approximate to the cycloidal arcs. The method of Professor Willis, however, does not give a very good approximation, the teeth being too thin at the points, and too thick at the roots.

The following method, founded on Rankine's rules for rectifying circular arcs, gives a much nearer approximation to the true curves; in fact, with teeth of ordinary size, there is no appreciable difference between the cycloidal and circular arcs. The method is also much easier in practice than that of drawing first the true curves. The method is based on this principle. For each cycloidal arc a circular curve is found, which coincides with it at the pitch line, and at $\frac{2}{3}$ its length from the pitch line, and which has at the latter point a common normal with it.

In fig. 179, the strongly marked circle $b\,p\,b'$ is the pitch circle, and p the pitch point. The complete dotted circles are the rolling circles. The height of the tooth outside the pitch line is $p\,r$, and its depth within it is $p\,s$, so that the circles through r and s are the addendum and root circles.

The arcs *v p w* mark the path of contact. As the rolling circle R rolls to the position R′, the tracing point moves from *p* to *m*, marking out the epicycloid *p m*, which forms the face of the tooth.

Method I.—Take $p\,c=\frac{2}{3}\,p\,r$, and draw the arc *c e* concentric with the pitch line. Step off arc $p\,b=$arc $p\,e$. Take the chord *p e* in the compasses, and with centre *b* mark off $b\,m=p\,e$; then *m* is a point of the true epicycloid, and *m b*

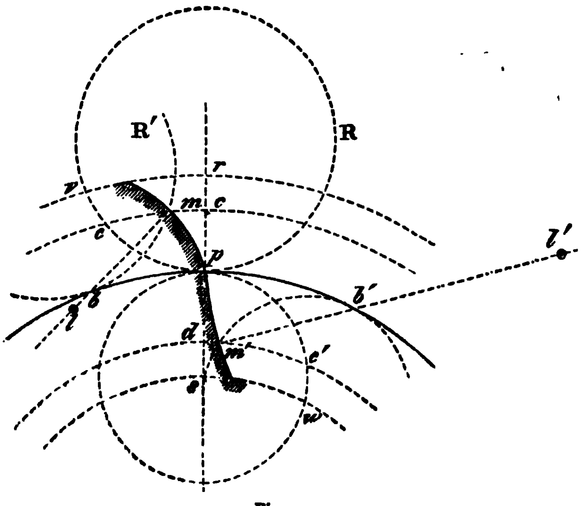


Fig. 179.

is the normal to the curve at *m*. It is then easy to find in *m b*, by trial, a centre *l* for a circular arc, which will pass through *m* and *p*. That circular arc will be the required approximation to the epicycloid.

For the flank of the tooth, make $p\,d=\frac{2}{3}\,p\,s$, and draw the arc *d e′*. Step off with the compasses, the arc $p\,b'=$ arc $p\,e'$. With centre *b′* and radius=chord *p e′*, cut *d e′* in *m′*. Then *m′* is a point in the hypocycloid, and *m′ b′* is the normal to the

curve at m'. Find, by trial, a centre, l' on $m' b'$, for a circular arc, passing through m' and p. That arc is the required approximation to the hypocycloid.

Method 2.—The following method (fig. 180) is the same as the last, except that all the points are found by construction, instead of by trial. Take, as in the last method, $p c = \frac{2}{3}$ of the height of the tooth, outside the pitch line, and $p d = \frac{2}{3}$ the depth within the pitch line. Draw the arcs

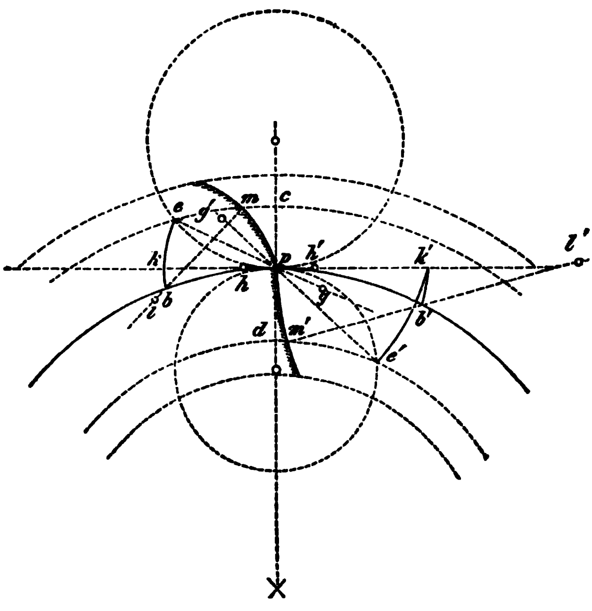


Fig. 180.

$c e$, $d e'$ concentric with the pitch line. Through the pitch point p, draw a tangent to the pitch circle. Join $e p$, produce it, and make $p g = \frac{1}{2} p e$. With centre g, and radius $g e$, describe an arc $e k$, cutting the tangent at the pitch point

in k. Then, $p\,k=$ arc $p\,e$. In $p\,k$ take $p\,h=\frac{1}{4}p\,k$. From centre h, with radius $h\,k$, describe an arc $k\,b$, cutting the pitch line in b. Then, arc $p\,b=$arc $p\,e$. With centre b and radius=chord $p\,e$ cut $c\,e$ in m. Join $m\,b$, and in $m\,b$ find a centre l of a circular arc, passing through m and p. That point l may be found by joining $m\,p$, and drawing a line, bisecting $m\,p$ at right angles. The line so drawn will intersect $m\,b$, produced in l. Then the arc $p\,m$, drawn with centre l and radius $l\,m$, is the required approximation to the epicycloid. The same construction gives the flank of the tooth, and the same description is applicable, if accented letters are substituted for unaccented letters.

185. *Method* 3. *Mr. Heys' method.*—The following method, adopted by Mr. Heys of Manchester, gives very good approximations, so far as the author has tested it. Let O A be the line of centres, P the pitch line, and x the pitch point. At x, draw the tangent $x\,g$, and make $x\,g=0{\cdot}571$ of the diameter of the rolling circle. Through g, parallel to O A, draw B C. Make g B $=g\,x$, and gC=diameter of rolling circle. Join O B, O C, and produce O C. Draw $a\,y$, $b\,z$, parallel to $x\,g$, and at a distance from it $=\frac{1}{8}\,x\,g$. Take $b\,z'=b\,z$. Then, y is the centre for a circular arc, approximating to the hypocloidal flank of the teeth, and $y\,x$ is its radius. Also, z' is the

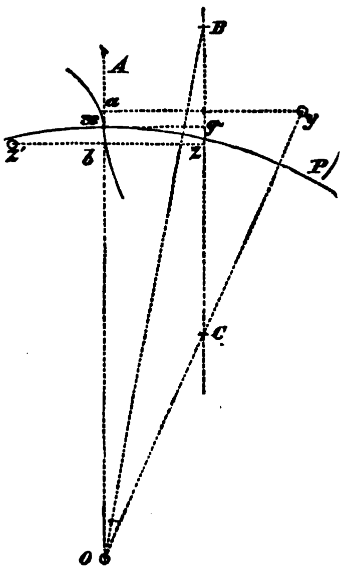


Fig. 181.

centre for a circular arc, approximating to the face of the

tooth, and $s' x$ is its radius. In practice, it is accurate enough to take, if p = pitch,

$$x\,g = g\,\mathrm{B} = 1{\cdot}125\,p$$

$$\mathrm{BC} = 3\,p$$

$$x\,a = x\,b = \tfrac{1}{7}\,p$$

Involute Teeth.

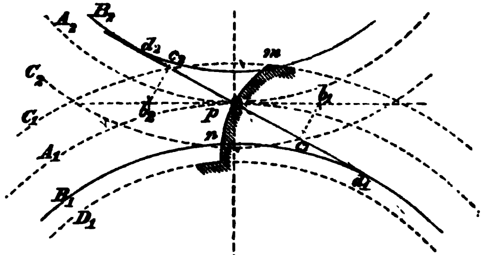
186. When the path of contact is a straight line, inclined to the line of centres, the form of the teeth is an involute of a base circle, concentric with the pitch circle, and having the path of contact for a tangent. The pressure between the teeth is in the direction of their common normal very nearly, and this normal coincides in involute teeth with the path of contact. Hence, if the path of contact deviates much from the tangent to the pitch circle at the pitch point, the pressure is correspondingly oblique to the direction of motion, and considerable stress is thrown on the supports of the wheel. Usually, the path of contact makes an angle of $74\frac{1}{2}°$ with the line of centres, and $15\frac{1}{2}°$ with the tangent to the pitch circle. Then the diameter of the base circle from which the involute is described, is $\frac{63}{65}$ths of the diameter of the pitch circle.

In fig. 182, let $\mathrm{A}_1\ \mathrm{A}_2$ be the pitch circles of the wheels, and p the pitch point. Draw the path of contact $d_1\,p\,d_2$, making the desired angle with the line of centres. The base circles $\mathrm{B}_1\ \mathrm{B}_2$ are concentric with $\mathrm{A}_1\ \mathrm{A}_2$, and touch the path of contact at $d_1\ d_2$. The greatest possible length of the path of contact is $d_1\ d_2$. If the length of the path of contact is given, draw the tangent $b_1\,p\,b_2$. Take $b_1\,p$, $p\,b_2$ = the given arcs of approach and recess. Drop perpendiculars $b_1\,c_1$, $b_2\,c_2$, on the path of contact. Then $c_1\ c_2$ is the length of the actual path of contact. A circle, C_1, concentric with A_1 and passing through c_2, will mark off the length of the teeth of A_1, and a circle C_2 concentric with A_2, passing through

e_1, will mark off the length of the teeth of A_2. The root circle D_1 must be drawn, so as to fall below C_2 at the line of centres, by a distance equal to the bottom clearance, which is 0·08 to 0·1 of the pitch.

The involute is not difficult to describe, but the following method gives a very accurate circular approximation.

Fig. 182.



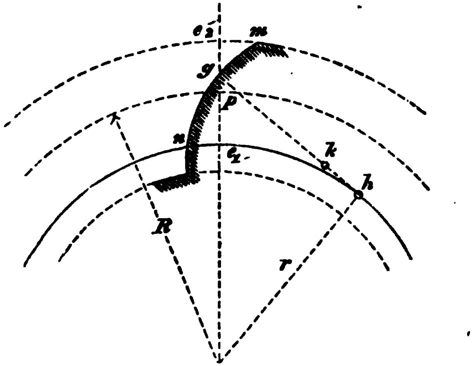


Fig. 183.

Let $e_1 e_2$, fig. 183, be the working height of the teeth, or the distance between circles C_1 C_2, in fig. 182 measured along the line of centres. Take $e_1 g = \frac{2}{3} e_1 e_2$. Draw a tangent $g h$ to the base circle. Take $h k = \frac{1}{4} h g$. Then, a circle $m n$, struck from k with radius $k g$, will be the required approximation to the involute. It will coincide with the involute at n and

g, and will have the same normal at *g*. The part of the tooth below the base circle may be a tangent to the arc at *n*. This part does not come in contact with the teeth of the other wheel.

Involute teeth have two remarkable properties. All involute wheels, whose teeth have the same pitch and the same obliquity of the line of contact, work well together. A pair of involute wheels may be drawn a little further apart, without the accuracy of action of the teeth being impaired, though the arc of contact is diminished. Involute wheels cannot be made with very long teeth, because then the obliquity of the line of contact must be great. Hence, the centres cannot be moved much further apart than their normal distance, without too much reducing the arc of contact. But this property of involute wheels is a valuable one, as it neutralises the injurious effect of wear of the supports of the wheels. With the angle of obliquity given above, the smallest number of teeth in an involute wheel should be twenty-five. With fewer teeth the arc of contact is too small. The obliquity of action is ordinarily alleged as a serious objection to involute wheels. Its importance has perhaps been overrated.

Teeth of Bevil Wheels.

187. The teeth of bevil wheels may be cycloidal or involute, and are described in the same way as the teeth of spur wheels, upon a development of the conical surfaces which limit their length. Let fig. 184 represent the section of a bevil wheel rim. o *a* is the intersection of the conical pitch surface with the plane of the paper, o *d* the axis of the wheel Let *a c* be the width of face of the wheel. Draw *a d*, *c e*, perpendicular to o *a*, cutting the axis of the wheel in *e* and *d*. Then the teeth are limited in length by the conical surfaces, whose intersections with the paper are *e c*, *d a*, and which have o *d* as axis. With centre *d* and radius *d a*, draw a circle. That circle is the virtual pitch-line of the

ends of the teeth, and the teeth are described on that circle as if it were the actual pitch line.

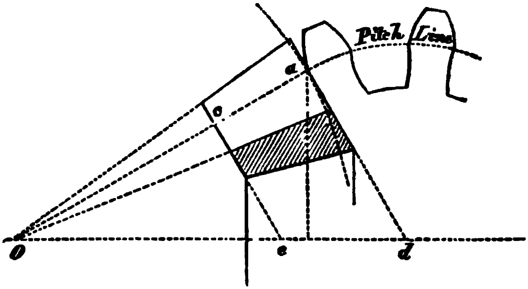


Fig. 184.

SUMMARY OF CURVES FOR TOOTHED WHEELS.

188. R_1, R_2=the radii of the wheels; r=radius of rolling circle; p the pitch of the wheels; T the number of teeth in the smallest wheel of the set; ρ_1 ρ_2 radii of base circles of involutes.

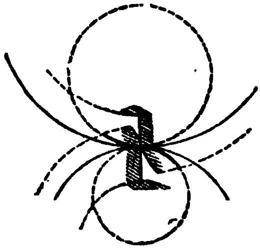


Fig. 185.

EXTERNAL CONTACT.

Cycloidal Curves.

Case I.—*Pair of wheels* (fig. 185).
Face of R_1, epicycloid, $r=\frac{1}{2} R_2$.
Flank of R_1, radial line, $r=\frac{1}{2} R_1$.
Face of R_2, epicycloid, $r=\frac{1}{2} R_1$.
Flank of R_2, radial line, $r=\frac{1}{2} R_2$.

Case II.—*Set of wheels of which any two are to work together* (fig. 186).

Faces of R_1 and R_2, epicycloids.

Flanks of R_1 and R_2, hypocycloids.

Radius of rolling circle for all the cycloidal curves $r = \frac{1}{2}\frac{pT}{2\pi}$.

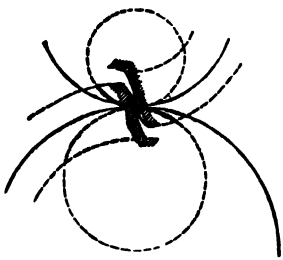


Fig. 186.

Case III.—*Pair of wheels, contact during recess only* (fig. 187).

Flank of R_1, radial line.

Face of R_2, epicycloid, $r = \frac{1}{2} R_1$.

R_2 is the driver.

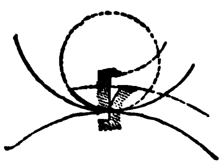


Fig. 187.

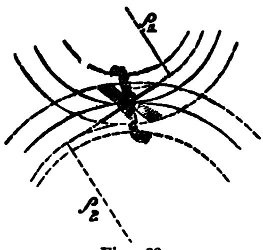


Fig. 188.

Involute Curves.

Case IV.—*Set of wheels, any two of which work together.*

The curve of each tooth (fig. 188) is an involute, the base circles being chosen so that

$$\frac{\rho_1}{\rho_2} = \frac{R_1}{R_2}.$$

The parts of the teeth beyond the region of contact may be tangents to the curves.

Internal Contact.

Cycloidal Curves.

Case V.—*Set of wheels, any two of which work together* (fig. 189).

Face of R_1 and flank of R_2, epicycloids.
Flank of R_1 and face of R_2, hypocycloids.

Radius of rolling circle for all the curves $r = \frac{1}{2} \cdot \frac{p\,T}{2\,\pi}$.

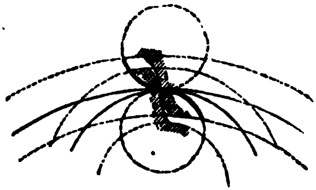


Fig. 189.

Case VI.—*Two wheels having contact only during recess.*
Face of R_1, epicycloid, $r = \frac{1}{2} R_2$.
Flank of R_2, radial line.
R_1 is the driver (fig. 190).

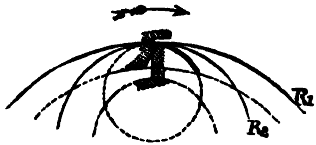


Fig. 190.

Involute Curves.

Case VII.—*Set of wheels, any two of which gear together.*

The curve of each tooth is an involute (fig. 191), the base circles being chosen so that

$$\frac{\rho_1}{\rho_2} = \frac{R_1}{R_2}.$$

Beyond the region of contact the tooth of R_2 may be tangential to the curve.

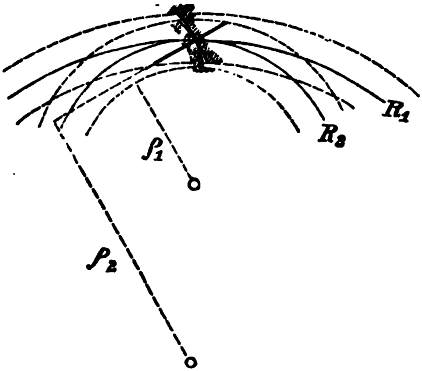


Fig. 191.

PINION AND RACK.

Case VIII.—*Set of wheels to work with one rack* (fig. 192).

Face of pinion tooth, epicycloid.
Flank of pinion tooth, hypocycloid.
Face and flank of rack tooth, cycloids.

Radius of rolling circle for all the curves $r=\frac{1}{2}\frac{p\,T}{2\,\pi}$.

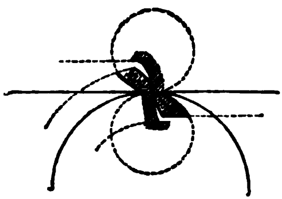


Fig. 192.

Case IX.—*Single wheel to work with rack* (fig. 193).

Face of pinion tooth an epicycloid described with $r=\infty$, consequently an involute.

Flank of pinion tooth, hypocycloid, $r=\frac{1}{2}\,R_1$.

Face of rack tooth, cycloid, $r = \frac{1}{2} R_1$

Flank of rack tooth, hypocycloid described with $r = \infty$, and therefore a straight line perpendicular to the pitch line.

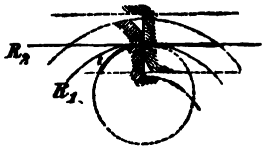


Fig. 193.

Involute Curves.

Pinion tooth an involute, with tangential prolongation beyond the region of contact.

Rack tooth, a straight line perpendicular to the path of contact.

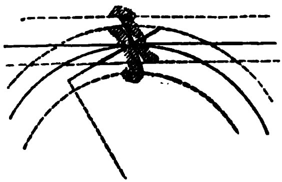


Fig. 194.

In all the figures, the pitch lines are thin full lines, the rolling or base circles dotted lines, the path of contact a thick full line.

Strength of Wheel Teeth.

189. In determining the strength of wheel teeth, it is not usually necessary to take into account their curved form. It is sufficiently accurate to treat the tooth as a rectangular cantilever (fig. 195), of thickness 36, uniform and equal to the thickness of the actual tooth at the pitch-line. Usually at least two pairs of teeth are simultaneously in contact. The pressure transmitted is therefore shared

by two or more pairs of teeth. The wheels cannot be made accurately enough to ensure an equal distribution of the pressure. Hence, if P is the whole pressure transmitted, the greatest pressure on one pair of teeth is n P, where n is a fraction lying between $\frac{1}{2}$ and 1. The teeth are in contact at a line which, in spur wheels, is parallel to the axis of rotation. The line of contact varies in position during the action of the teeth, and either at the beginning or end of contact coincides with the extreme edge of the tooth. Ordinarily, in teeth which have worn a little by mutual friction, the pressure will be distributed with approximate uniformity along the edge of the tooth, and will tend to break the tooth across at its root along its whole breadth. Another contingency less favourable to the strength of the tooth is possible. From inaccurate form in the teeth or inaccurate fixing of the wheels, the pressure may be restricted to a small portion of the edge of the tooth. In that case, to ensure safety, the tooth must be strong enough to sustain the pressure n P applied at one corner, as shown in fig. 195.

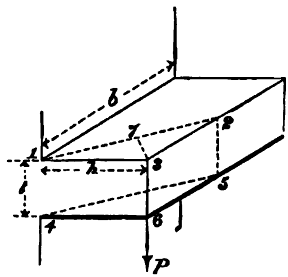


Fig. 195.

Let P be the whole pressure of one wheel on the other, estimated in the direction of motion; H the number of horses' power transmitted by the wheel; N the number of revolutions of the wheel per minute; R its radius in inches; V the velocity of the pitch line in ft. per sec.

$$V=\frac{2\pi R N}{12\times 60}=\cdot 00873 \text{ R N} \quad . \quad . \quad . \quad (4)$$

$$P=\frac{550 \text{ H}}{V}=63{,}020\frac{\text{H}}{\text{R N}} \quad . \quad . \quad . \quad (5)$$

Let p be the pitch of the wheels in inches, T the number

of teeth. Then the velocity of the pitch line in feet per second is

$$V=\frac{p\,T\,N}{12\times 60};$$

consequently the pressure on the teeth in lbs. may be expressed in the form

$$P=\frac{550\times 12\times 60\times H}{p\,T\,N};$$

$$=396{,}000\,\frac{H}{p\,T\,N}\quad . \quad . \quad . \quad (5a).$$

Let the height of the tooth 13, fig. 143, $=h$; its thickness $14=t$; the width of face$=b$. Then, if the pressure $n\,P$ is applied at a corner, it tends to break off a triangular prism, bounded by a plane 1 2 5 4, which passes through the root of the tooth. Draw 37 perpendicular to that plane, and let the angle $2\ 1\ 3=\theta$; then,

$$\overline{37}=\overline{13}\sin\theta=h\sin\theta.$$

$$\overline{12}=\overline{13}\sec.\theta=h\sec.\theta.$$

The bending moment of $n\,P$, with respect to the section 1 2 5 4, is $n\,P\,h\sin\theta$. The moment of resistance of that section to bending is $\frac{1}{6}f\,h\,t^2\sec.\theta$. Equating the bending moment and moment of resistance, we get for the greatest stress due to bending,

$$f=\frac{3\,n\,P}{t^2}\sin 2\,\theta,$$

which will be a maximum when $\theta=45°$ and $\sin 2\,\theta=1$. Then,

$$f=\frac{3\,n\,P}{t^2}.$$

If f is the greatest safe stress,

$$\left.\begin{aligned} P&=\frac{1}{3}\frac{f\,t^2}{n} \\ t&=\sqrt{\left(\frac{3\,n\,P}{f}\right)} \end{aligned}\right\}\quad . \quad . \quad . \quad . \quad (6)$$

190. It is convenient to express t in terms of the pitch p. For unworn teeth $t=0{\cdot}48\ p$ for cast-iron teeth. Since, however, the teeth must be strong enough when worn, we may take $t=0{\cdot}36\ p$. For mortice teeth of hard wood $t=0{\cdot}595\ p$ when the teeth are new, and we may take $t=0{\cdot}45\ p$ for worn teeth. Then, introducing these values in eq. 6,

$$\left.\begin{aligned} p &= 4{\cdot}8\sqrt{\frac{n\,\text{P}}{f}}\ \text{for iron teeth} \\ &= 3{\cdot}85\sqrt{\frac{n\,\text{P}}{f}}\ \text{for wood teeth} \end{aligned}\right\} \quad . \quad . \quad . \quad (7)$$

These formulæ cease to be applicable if $b<h$, but this does not occur in wheels of ordinary proportions.

In obtaining the above formulæ, some assumptions are made, and the value of n is undetermined. For different wheels p is simply proportional to $\sqrt{\text{P}}$, and we may therefore write,

$$p=\text{K}\sqrt{\text{P}} \quad . \quad . \quad . \quad . \quad (8)$$

and determine K from existing wheels. K will be found to vary considerably in different cases. In slowly moving gearing, especially in gearing worked by hand and not subjected to much vibration or shock, $\text{K}={\cdot}04$ for iron wheels. In ordinary mill-gearing, running at a greater speed and subjected to considerable vibration, $\text{K}={\cdot}05$, and in wheels subjected to excessive vibration and shock, as in the gearing which drives machine tools, $\text{K}=0{\cdot}06$. For mortice gearing $\text{K}={\cdot}06$.

If, now, it is assumed that $n=\frac{2}{3}$, which cannot be very far from the truth, we get for the values of f for cast iron, corresponding to the three cases above, 9,600, 6,100, and 4,300 lbs. per sq. in.; values which are quite consistent with ordinary practice in the use of cast iron, to resist transverse straining actions. For hard wood, when $\text{K}={\cdot}06$, $f=2{,}740$.

Safe Pressure P *on Wheel Teeth from Equation* 8.

Pitch in ins.	Safe pressure on teeth in lbs.			
	Iron teeth, Little shock	Iron teeth, Moderate shock	Iron teeth, Excessive shock	
1	625	400	277	The pressures in the last column are applicable to mortice teeth calculated by eq. 8.
$1\frac{1}{4}$	975	624	432	
$1\frac{1}{2}$	1,406	900	623	
$1\frac{3}{4}$	1,912	1,224	848	
2	2,500	1,600	1,108	
$2\frac{1}{4}$	3,162	2,024	1,402	
$2\frac{1}{2}$	3,906	2,500	1,732	
$2\frac{3}{4}$	4,726	3,024	2,094	
3	5,625	3,600	2,493	
$3\frac{1}{4}$	6,600	4,224	2,926	
$3\frac{1}{2}$	7,658	4,900	3,393	
$3\frac{3}{4}$	8,787	5,624	3,895	
4	10,000	6,400	4,432	
$4\frac{1}{2}$	12,656	8,100	5,608	
5	15,625	10,000	6,924	
$5\frac{1}{2}$	18,906	12,100	8,379	
6	22,500	14,400	9,972	

191. *Strength of teeth when the influence of the width of face is taken into account.*—In the foregoing investigation, the pressure is assumed to be concentrated at the corner of the tooth, and consequently the strength is independent of the width of the tooth. For well-constructed and carefully erected mill-gearing this is a very improbable condition. If the pressure is distributed along the edge of the tooth, the bending moment at its root is $n \text{P} h$. The moment of resistance of the section of the tooth is $\frac{1}{6} f b t^2$. Equating these,

$$\text{P} = \frac{1}{6} \frac{b t^2}{n h} f.$$

Let $t = 0{\cdot}36 p$ for iron, and $0{\cdot}45 p$ for wood teeth; $h = 0{\cdot}7 p$ for iron, and $0{\cdot}6 p$ for wood teeth; $n = \frac{2}{3}$.

$$\left.\begin{aligned} \text{P} &= {\cdot}046\, b p f \text{ for iron teeth} \\ &= {\cdot}084\, b p f \text{ for wood teeth} \end{aligned}\right\} \quad . \quad . \quad . \quad (9)$$

The following is the most convenient form of these equations:

$$p = \text{K}_1 \sqrt{\frac{p}{b}} \sqrt{\text{P}} \quad . \quad . \quad . \quad . \quad (10)$$

Where K_1 in practice is about 0·0707 for iron wheels, and 0·0848 for mortice wheels, when the breadth of face is not less than twice the pitch. These values give for the stress on the teeth 4,400 lbs. per sq. in. for iron teeth, and 1,650 lbs. per sq. in. for wood. These are low enough values to allow for some inequality in the distribution of the pressure along the edge of the tooth. The following table facilitates the use of this equation :

$\frac{b}{p} =$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	
$K_1\sqrt{\frac{p}{b}} =$	·0500	·0475	·0447	·0408	·0378	·0354	Iron teeth.
$=$	·0600	·0565	·0536	·0490	·0453	·0424	Wood teeth.

Safe Pressure P, *on Teeth of Ordinary Cast-Iron Mill Gearing, by Equation* 10.

Pitch in ins.	Pressure on teeth in lbs. when $\frac{b}{p}$ =					
	2	$2\frac{1}{4}$	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4
1	400	450	500	600	700	800
$1\frac{1}{4}$	624	702	780	936	1,092	1,248
$1\frac{1}{2}$	900	1,012	1,150	1,400	1,550	1,800
$1\frac{3}{4}$	1,224	1,377	1,530	1,836	2,142	2,448
2	1,600	1,800	2,000	2,400	2,800	3,200
$2\frac{1}{4}$	2,024	2,277	2,530	3,036	3,542	4,048
$2\frac{1}{2}$	2,500	2,812	3,125	3,750	4,375	5,000
$2\frac{3}{4}$	3,024	3,402	3,780	4,536	5,292	6,048
3	3,600	4,050	4,500	5,400	6,300	7,200
$3\frac{1}{4}$	4,224	4,752	5,280	6,336	7,392	8,448
$3\frac{1}{2}$	4,900	5,512	6,125	7,350	8,575	9,800
$3\frac{3}{4}$	5,624	6,327	7,030	8,436	9,842	11,248
4	6,400	7,200	8,000	9,600	11,200	12,800
$4\frac{1}{2}$	8,100	9,112	10,125	12,150	14,175	16,200
5	10,000	11,250	12,500	15,000	17,500	20,000
$5\frac{1}{2}$	12,100	13,612	15,125	18,150	21,175	24,200
6	14,400	16,200	18,000	21,600	25,200	28,800

The pressures for mortice wheels may be taken at $\frac{7}{10}$ths of those for iron wheels.

No specific rule can be given to decide between the cases in which eq. 8 and eq. 10 should be used. It is really a question of the degree of security against accident which is desired.

192. The equation above may be put in another convenient form. For the most usual proportion, $b=2\frac{1}{2}p$, and iron teeth

$$p=0{\cdot}0447\sqrt{P}$$

but

$$P=\frac{550\,H}{V}=\frac{550\,H\times12\times60}{p\,T\,N}$$

Inserting this value,

$$p=28{\cdot}13\sqrt{\left(\frac{H}{p\,T\,N}\right)} \quad . \quad . \quad . \quad (10a)$$

or inverting, we get the number of teeth of a given pitch necessary for strength.

$$T=791\,\frac{H}{p^3\,N} \quad . \quad . \quad . \quad . \quad (10b)$$

From this equation the following table is calculated. It gives the least number of teeth suitable for strength, when $\frac{H}{N}$ is known and p assumed.

For mortice wheels, when $\frac{b}{p}=3$, we have

$$p=0{\cdot}049\sqrt{P};$$

and this gives

$$T=951\,\frac{H}{p^3\,N}. \quad . \quad . \quad (10c)$$

Consequently, if the numbers in the table are multiplied by 1·2 (or, what is the same thing, if the numbers are increased one-fifth), they will be the proper numbers for mortice wheels.

193. *Limiting velocity of toothed wheels.*—If the wheels are run at a sufficiently high velocity, the wheel rim bursts in consequence of the centrifugal tension. Toothed wheels

Table giving least Numbers of Teeth for a Wheel of given Pitch and Speed, from equation 10b.

Pitch in inches p	p^3	Least number of teeth for $\frac{HP}{N}=$ 0·012	·025	·05	·075	·1	·15	·2	·25	·3	·35	·4	·45	·5	·6	·7	·8	·9	1·0	1·25	1·5	1·75	2·0	2·25	2·5	2·75	3·0	3·5	4·0	4·5	5·0	6·0	7·0	8·0
1	1·00	10	20	39	60	79	119	158	200	236	[illegible]76	316	356	394																				
$1\frac{1}{4}$	3·37			1[illegible]	17	24	3[illegible]	47	59	70	82	94	106	114	141	164	187	211	234	293	351													
$1\frac{1}{2}$	5·36				11	15	22	30	37	44	52	59	67	74	89	103	118	133	148	184	241	258	295											
2	8·00	..	..	.	..	10	15	20	25	30	35	40	45	50	59	69	79	8[illegible]	99	124	148	173	198	223										
$2\frac{1}{4}$	11·4						11	14	18	21	25	28	31	35	42	49	56	63	70	87	104	123	138	156	173	191	208	243						
$2\frac{1}{2}$	15·6							10	13	16	18	21	23	25	31	36	41	46	51	63	76	89	103	114	127	139	153	177	202					
$2\frac{3}{4}$	20·8								10	12	14	16	17	19	23	26	31	34	38	43	57	67	76	86	9[illegible]	105	114	133	152	171	183	228	266	
3	27·0	..	..	.	..	.	..	..	..	..	11	12	13	15	18	21	24	27	30	37	44	52	59	66	73	81	88	103	117	132	146	176	205	234
$3\frac{1}{4}$	34·3													12	14	16	19	21	23	29	3[illegible]	41	47	52	5[illegible]	64	69	81	93	104	115	138	161	185
$3\frac{1}{2}$	42·9														11	13	15	16	19	23	28	33	37	42	46	51	55	65	74	83	92	110	129	147
$3\frac{3}{4}$	52·7															11	12	14	15	19	23	26	30	34	37	41	45	53	6[illegible]	68	75	89	105	120
4	64·0	..	..	.	..	.	..	..	..	..	..	..	..	..	..	..	..	11	13	16	19	22	25	28	31	34	37	43	50	56	62	74	87	99
$4\frac{1}{4}$	76·8																			13	16	18	21	24	26	29	31	36	41	46	51	62	72	82
$4\frac{1}{2}$	91·1																				13	16	18	20	22	24	26	31	35	39	44	52	61	70
$4\frac{3}{4}$	107·2																					13	15	17	19	21	22	26	30	34	37	45	52	59
5	125·0	..	..	.	..	.	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	13	15	16	18	19	22	26	29	32	38	44	51
$5\frac{1}{2}$	166·4																							11	12	13	14	17	19	22	24	29	34	38
6	216·0																									10	11	13	15	17	19	22	26	30

RULE.—To find the least number of teeth in either wheel of a pair, which will insure sufficient strength, divide the horses power transmitted by the revolutions of the wheel per minute; under the nearest number to the quotient so obtained, and opposite the pitch selected, is the required least number of teeth.

are in this respect materially in a worse position than pulleys, because the teeth add considerably to the weight of the rim without adding to the section which resists bursting. No increase of the pitch or section of the rim renders the wheels safe, because the increase of weight increases the centrifugal tension in the same ratio as the increase of section. For very high velocity wheels must be made of a stronger material than cast iron.

From the equation previously given (§ 191), we have for the load on the teeth

$$P = 0{\cdot}046\, b p f$$

where f is the safe stress allowed for the breaking across of the teeth. The actual section of the rim is about $0{\cdot}5\, b p$. But if the teeth were thrown into the rim the section would be about $0{\cdot}85\, b p$. Hence the weight of the rim per foot of length (12 cubic inches of iron weighing 3·36 lbs.) is $0{\cdot}85 \times 3{\cdot}36\, bp = 2{\cdot}86\, bp$ lbs.

Each foot of rim has a radial centrifugal force of $\dfrac{2{\cdot}86\, bp}{g} \cdot \dfrac{v^2}{R}$ lbs. where v is the velocity of the rim in feet per second, and R its radius in feet. The resultant centrifugal force of half the rim is

$$\frac{2{\cdot}86\, bp}{g} \cdot \frac{v^2}{R} \times 2\,R = \frac{5{\cdot}72\, bp v^2}{g} \text{ lbs.}$$

This is balanced by the stress on two radial sections of the rim. Hence the stress due to rotation is

$$\frac{5{\cdot}72\, bp v^2}{bp} = \frac{5{\cdot}72\, v^2}{g} \text{ lbs. per sq. in.}$$

The stress in the rim due to the pressure on the teeth will be on the average $\frac{1}{2}\,P$; the load being transmitted half to the arm in advance and half to the arm behind the teeth in contact. But as the proportion transmitted each way will depend on the relative nearness of the arms, it seems probable that the maximum stress due to the load may

amount to twice the mean value, or $\frac{P}{0{\cdot}5\,b\,p}$ lbs. per sq. in. Putting in the value of P above, this becomes

$$0{\cdot}092\,f.$$

Consequently the whole stress per sq. in. in the rim is

$$f_2 = 0{\cdot}092\,f + \frac{5{\cdot}72\,v^2}{g}.$$

For wheels run at high speed, we may take $f = 4{,}000$ lbs. per sq. in. The safe limit of tensional resistance for cast iron is about 3,000 lbs. per sq. in.; but looking to the fact that there are initial stresses in wheels due to contraction in cooling, and bending stresses due to the oblique action of the teeth, which have been neglected, it does not appear safe to take f_2 at more than 2,000 lbs. per sq. in. Then the limiting safe velocity is

$$v = \sqrt{\left\{\frac{9}{5{\cdot}72}\,(2000 - 0{\cdot}092 \times 4000)\right\}}$$
$$= 96 \text{ feet per sec. nearly.}$$

In a case which came under the author's notice, a pair of very large and well-constructed wheels run at about this speed, actually broke up, apparently from the action of the centrifugal force. Hence it is doubtful if even this calculation allows quite margin enough in the case of heavy wheels.

194. *Wheels for high speeds.*—At high speeds the influence of shocks and vibrations becomes more serious. Reuleaux has proposed to allow for this by making the value of the stress f decrease inversely as the cube root of the velocity of the pitch line. Then for cast iron

$$f = \frac{10{,}000}{\sqrt[3]{V}} \quad . \quad . \quad . \quad . \quad (11)$$

195. *Strength of bevil wheels.*—In stating the size of bevil wheels, the pitch at the outer circumference of the wheel is always given, but in estimating their strength the pitch at the inner circumference of the rim should be taken.

Let p_1, p_0 be the pitches at the inner and outer circumferences, r_1 and r_0 the corresponding radii of the smaller, and R_1, R_0 those of the larger wheel. Width of face $= b$. Let $R_0 + 0{\cdot}4\, r_0 = m$. Then,

$$p_1 = p_0 \frac{r_1}{r_0} = p_0 \frac{m-b}{m} \text{ nearly.}$$

It is p_1, not p_0, which should be taken in estimating the strength of the wheel. For all other purposes p_0 is used.

196. *Shrouded wheels.*—The teeth of wheels are sometimes united at the ends by annular rings cast with the wheel, and the wheel is then said to be shrouded. The shrouding may extend the whole depth of the teeth of the pinion of a pair of wheels. In that case the shrouding has the effect of neutralising the weakness of the teeth, which in very small wheels are of a weak form. With the pinion shrouded, it is stronger than the wheel, but it wears more rapidly than the wheel, so that the shrouding may be regarded as a provision against the failure of the pinion in consequence of wear. If both wheel and pinion are shrouded to half the depth of the teeth, the strength of the pair of wheels is considerably increased. But this arrangement is seldom adopted, and the casting of the wheels is difficult.

197. *Width of face of wheel.*—The durability of wheels is increased by making the wheels wider. In practice, b is rarely less than $1\frac{1}{2}\, p$ in wheels used to transmit power, and that width answers well for wheels moving slowly or intermittently. For ordinary mill-gearing $b = 2\, p$ to $4\, p$.

Wear of wheels.—No exact data of the wear of wheels in given circumstances have yet been recorded. The following theory may be useful as a guide when there is a doubt as to the width to be given to a pair of wheels.

Let R_1, R_2 be the radii of a pair of wheels, making N_1, N_2 revolutions per minute, and transmitting H horses' power,

Let p be the pitch, and b the width of face of the wheels. Then the work lost in friction is proportional to

$$p\left(\frac{1}{R_1}+\frac{1}{R_2}\right) H \text{ ft. lbs. per min.} \qquad . \quad . \ (12)$$

The wearing surface of a tooth is proportional to bp, and the whole wearing surface of the pinion is proportional to $R_1 b$. Supposing the total wear to be proportional to the work expended, the depth worn away in the unit of time is proportional to

$$p\left(\frac{1}{R_1}+\frac{1}{R_2}\right) H \div R_1 b$$

Suppose the wheel to be worn out when the depth worn away is γp, where γ is some fraction varying in different circumstances, but constant for wheels in similar conditions. Then, for equal durability,

$$\gamma p \div \frac{p\left(\frac{1}{R_1}+\frac{1}{R_2}\right) H}{R_1 b} = \text{constant}$$

or

$$b = k_1 \frac{H}{R_1} \left(\frac{1}{R_1}+\frac{1}{R_2}\right) \qquad . \quad . \ (13)$$

where k_1 is a constant to be determined by experience. Since $P R_1 N_1$ is proportional to H, we have also

$$b = k_2 P N_1 \left(\frac{1}{R_1}+\frac{1}{R_2}\right) . \qquad . \ (13a)$$

or when the pinion is small compared with the wheel,

$$b = k_1 \frac{H}{R^2_1} = k_2 P \frac{N_1}{R_1} \text{ nearly} . \qquad . \quad . \ (13b)$$

Average values for these constants would be of little service, because the conditions in which wheels are employed are so variable. If k_1 or k_2 is deduced from a pair of wheels known to have worked well in given conditions,

the value so obtained may be applied to determine the minimum width of another pair of wheels which are to work in similar conditions.

CONSTRUCTION AND PROPORTIONS OF WHEELS.

198. *Rim of wheel.*—In iron wheels the teeth are cast on, and in mortice wheels they are tenoned into, a continuous

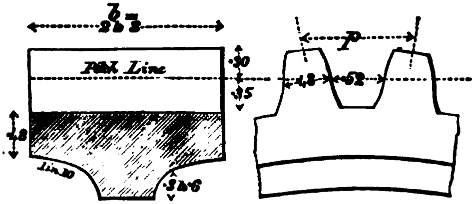


Fig. 196.

rim. Fig. 196 shows the section of a spur-wheel rim, and fig. 197 that of a bevil-wheel rim. The unit for the proportional figures is the pitch. The proportional figures for the

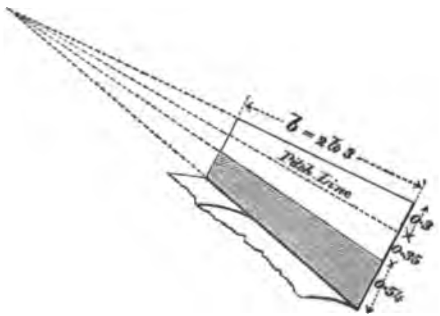


Fig. 197.

teeth are approximate only, more exact proportions having been already given in § 177.

Fig. 198 shows the section of a mortice spur-wheel rim, the

end elevations indicating two ways of forming the tenons. The mortice teeth are either fixed by wood keys, or by

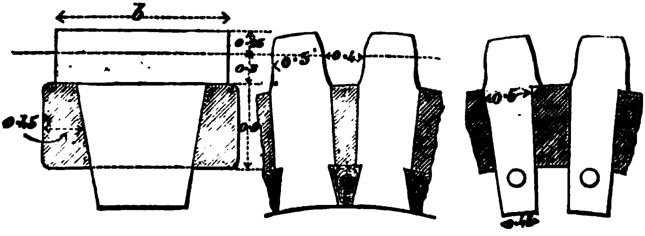


Fig. 198.

round iron pins driven in behind the rim of the wheel. Both methods are shown in fig. 198. In fig. 199 the cogs are fixed by bolts, iron plates about 2 ft. long being fitted to the inside of the rim of the wheel. Fig. 200 shows a mortice bevil wheel.

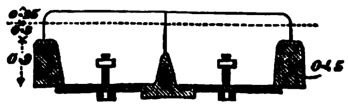


Fig. 199.

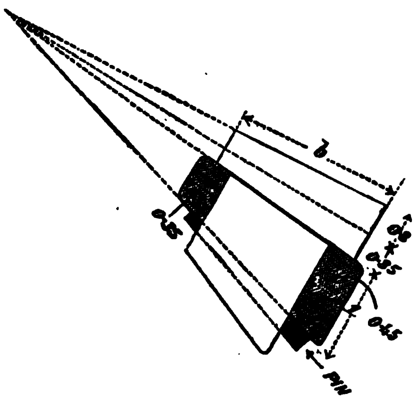


Fig. 200.

The radiating lines in the figures of bevil wheels meet at the intersection of the shafts on which the wheels are placed.

199. *Arms of wheels.*—The arms of wheels are most commonly cross-shaped in section for spur-wheels, and T-shaped for bevil-wheels. For machine-moulded wheels, the arms are often I-shaped, the spaces between the arms being cored out in casting with loam cores. The number of arms in wheels is fixed very arbitrarily. Usually there are four arms for wheels not exceeding four feet diameter; six arms for wheels of from four feet to eight feet; and eight arms for wheels from eight feet to sixteen feet diameter.

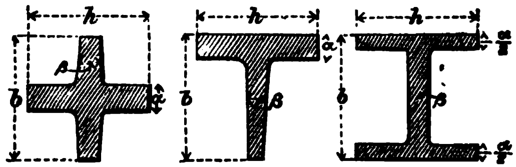


Fig. 201.

The arms are subjected to bending, and when the arms and rim are cast in one piece, they are fixed at both ends. If the arms are attached to the rim by bolts, they are free at the rim, and fixed at the nave. It will be assumed that the arms are equally loaded, and that they may in all cases be treated as if they were fixed at one end and free at the other. This will give a slight excess of strength when the arms are cast in one with the rim, but such arms are at the same time weakened by contraction in cooling.

Let ν be the number of arms,

R, the radius of the wheel,

P, the total pressure transmitted (§ 189)

Then the bending moment on each arm is $PR \div \nu$ nearly. The strength of the arm is almost entirely due to that part which is parallel to the plane of rotation. The ribs or feathers at right angles to this part add very little

to the resistance to the force acting on the wheel. They are necessary to give lateral strength and rigidity, and to resist accidental straining actions at right angles to the plane of rotation. Let h be the width, and a the thickness of the arm, exclusive of the feathers. The moment of resistance of that section is $\frac{1}{6} a h^2 f$. Equating this to the bending moment

$$a h^2 = \frac{6\,\text{P R}}{\nu f} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

In proceeding to design the arm either of the three following methods may be followed.

(1.) Given the ratio $\frac{h}{a}$, and the limiting stress on the arm.

If $\frac{h}{a}=5$, we get from eq. 14,

$$h = \sqrt[3]{\frac{30}{f}} \quad \sqrt[3]{\frac{\text{P R}}{\nu}}$$

The limiting stress must be taken at a low value, partly to allow for unequal distribution of load on the arms, and partly because of the initial stresses due to contraction in cooling. If $f=3{,}000$ lbs. per sq. in.,

$$h = \frac{0{\cdot}2154}{\sqrt[3]{\nu}} \sqrt[3]{\text{P R}} \quad . \quad . \quad . \quad . \quad (15)$$

$\nu=$	3	4	6	8	10	12
$\frac{0{\cdot}2154}{\sqrt[3]{\nu}}=$	·149	·136	·119	·108	·100	·094

(2.) Since the arm must be of equal strength with the teeth, we may replace P by its value in terms of the pitch in eq. 9, namely,

$$\text{P} = 0{\cdot}046\, b p f$$

Introducing this in eq. 14,

$$a h^2 = 0{\cdot}276 \frac{b p \text{R}}{\nu} \quad . \quad . \quad . \quad . \quad (16)$$

Let $a=0{\cdot}2\,h$

$$h=\frac{1{\cdot}113}{\sqrt[3]{v}}\sqrt[3]{(b\,p\,\mathrm{R})} \quad . \quad . \quad . \ (17)$$

$v=$	3	4	6	8	10	12
$\frac{1{\cdot}113}{\sqrt[3]{v}}=$	·772	·701	·613	·557	·517	·486

A comparison of some existing wheels shows that the arms are sometimes one-fifth wider than is given by this rule, this additional width being required to meet the stresses due to contraction.

(3.) Given the thickness of the arm. It is desirable to make the different parts of the wheel nearly uniform in thickness to secure regularity in cooling and contraction. Let $a=0{\cdot}48\,p$, so that the arm is the same thickness as the teeth. Introducing this in eq. 16,

$$h=\frac{0{\cdot}758}{\sqrt{v}}\sqrt{b\,\mathrm{R}} \quad . \quad . \quad . \ (18)$$

$v=$	3	4	6	8	10	12
$\frac{0{\cdot}758}{\sqrt{v}}=$	·438	·379	·309	·268	·240	·219

One-fifth may be added to the dimensions thus obtained to allow a margin against contraction, and for the unequal loading of the arms.

The dimensions given by the foregoing rules apply to the section of the arm produced to the centre of the wheel. Towards the rim the arm is usually tapered, the amount of taper being $\frac{1}{4}$ in. per foot of length on each side. The thickness of the arm a is constant.

The width of the cross feathers (marked b in fig. 201) may be b to $1\frac{1}{4}\,b$ at the centre, and $\frac{3}{4}\,b$ to $\frac{15}{16}\,b$ at the rim, where b=width of face of wheel. The thickness of the feathers may be $\beta=0{\cdot}3\,p$. The feathers must be slightly tapered at right angles to their length, so as to draw easily from the sand.

200. *Nave of the wheel.*—The thickness δ of the nave of the wheel may be taken at $0{\cdot}4 \sqrt[3]{(p^2\text{R})}+\frac{1}{2}$, and its length may be at least three times its thickness. Generally the nave length is not less than $b+0{\cdot}06\,\text{R}$ in iron wheels, and $b+p+0{\cdot}06\,\text{R}$ in mortice wheels, so that it may project a little beyond the rim. The key for fixing the wheel on the shaft may be $0{\cdot}4\,\delta$ wide and $0{\cdot}2\,\delta$ thick.

Mr. Heys uses the following rule for the nave thickness :—

$$\delta=\tfrac{5}{4}\sqrt[3]{\text{R}}+0{\cdot}7p+0{\cdot}1\,d-1,$$

where d is the actual diameter of the eye of the wheel.

Large wheels are usually fixed by four keys. Heavy wheels have the nave split to prevent fracture of the arms from contraction in casting. The nave is then gripped by two strong wrought-iron rings or hoops fitted over the nave on each side and shrunk on.

Screw Gearing.

201. In screw gearing the wheels have cylindrical pitch surfaces, like those of spur wheels, but the teeth are not parallel to the axes. The line in which the pitch surface intersects the face of a tooth is part of a helix drawn on the pitch surface. A screw wheel may have one or any number of teeth. A one-toothed wheel corresponds to a one-threaded screw; a many-toothed wheel to a many-threaded screw. In screw gearing the axes may be placed at any angle.

Screw-gearing with parallel axes.—Gearing of this kind was invented by Dr. Hooke. Let an ordinary spur wheel be cut into n slices by planes perpendicular to the axis. Let the slices be so arranged that, for example, in passing from left to right across the face of the wheel, each successive slice is $\frac{1}{n}$th of the pitch behind the previous one. Such a wheel is termed a stepped spur wheel. Two such wheels

will work together, and they have the advantage compared with ordinary wheels that one or other of the pairs of slices are always in contact at a distance not exceeding $\frac{1}{n}$th of the pitch from the pitch point.

As the slices come successively into gear, the motion of the wheels is very regular. Such wheels were at one time used for driving planing machine tables, and in other cases where regularity of motion was important. If the slices are infinitely numerous, then the front of the tooth intersects the pitch cylinder in a helical line, and we get the screw wheels shown in fig. 202. In two wheels of this kind which gear together the pitch measured circularly is equal; the obliquity is equal, but in opposite directions, and the velocity ratio is inversely as the radii of the wheels.

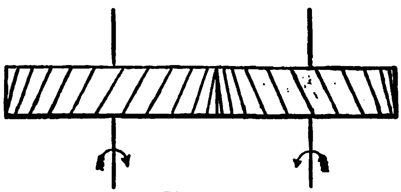


Fig. 202.

The wheels just described are open to the objection that from the obliquity of the teeth the wheels tend to thrust each other apart and thus produce prejudicial end thrust on the supports of the wheels. By combining two pairs of wheels constructed with right-handed and left-handed obliquity, this objection is obviated. Messrs. Jackson of Manchester appear to have overcome the difficulty of casting these wheels, both in the case of spur and bevil gearing. The double helical teeth are shown in fig. 203. Wheels of this kind work very smoothly, as the teeth have always two points touching in the plane of the axes. It is claimed for them also that the teeth from their form are of very great

strength. They work so noiselessly that they may be made in some cases to take the place of mortice wheels, and their cost is not very much greater than that of ordinary wheels.

Fig. 203.

202. *Screw gearing when the axes are not parallel.*— When the axes are not parallel the pitch cylinders touch at a single point, which may be termed the pitch point. Draw through that point a tangent to the pitch surfaces. If helices are traced on the pitch cylinders touching that tangent, they define the fronts of teeth which will drive each other.

The common tangent to the pitch surfaces and the teeth is termed the line of contact. It is shown at $a\ b$, fig. 205; the angles θ_1 θ_2 it makes with the axes are termed the angles of inclination of the teeth. The number of threads ν, in a screw wheel, is equal to the number of helices which intersect any plane perpendicular to the axis. Let fig. 204 represent a series of helices (in this case four), intended to mark out the teeth of a screw wheel. The same screw thread intersects a line $a\ b$, parallel to the axis at a and b. Then $a\ b$ is the *axial pitch* of the screw, and the distance $a\,c = p = \dfrac{a\,b}{\nu}$ is the *divided axial pitch*. Let a plane $d\ e$ perpendicular to

the axis intersect two successive threads in d and e. Then de is the circumferential pitch c, and is equal to $\frac{2\pi r}{\nu}$ where r is the radius of the pitch cylinder. Draw ef perpendicularly to the threads. Then the distance ef is the divided *normal pitch*, n. Let the tangent gh to a thread make the angle $g h a = \theta$ with the axis of the wheel. Then from the properties of helices the following relations obtain :—

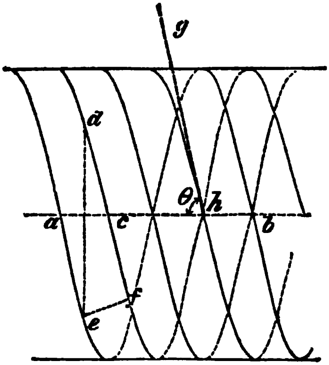


Fig. 204.

$$\left.\begin{array}{c} \tan\theta = \dfrac{2\pi r}{\nu p} \\ p : c : n :: 2\pi r \cot\theta : 2\pi r : 2\pi r\cos\theta \\ :: \cot\theta \quad : \quad 1 : \cos\theta \end{array}\right\} \quad (19)$$

Let fig. 205 represent two screw wheels projected on the common tangent plane to the two pitch cylinders. Let the angle between the axes $= i$, and let the tangent to the teeth $a\,b$ make with the axes the angles θ_1, θ_2, so that

$$\theta_1 + \theta_2 = i$$

Let a_1 a_2 be the angular velocities of the wheels; r_1 r_2 their radii; and ν_1 ν_2 the number of threads of each. Let c_1 c_2 be the circumferential, n_1 n_2 the divided normal, and p_1 p_2 the divided axial pitches. Then,

$$\frac{a_1}{a_2} = \frac{\nu_2}{\nu_1} \quad . \quad . \quad (20)$$

Let ν_1 and ν_2 be decided upon. The surface velocities of the wheels are a_1 r_1 and a_2 r_2, and these are proportional to the circumferential pitches, because each wheel rotates a

distance equal to the circumferential pitch in the same time. Hence,

$$\frac{c_1}{c_2}=\frac{a_1 r_1}{a_2 r_2} \quad . \quad . \quad . \quad (21)$$

If the circumferential pitches are chosen so as to satisfy this relation, then the axial pitch and inclination of the

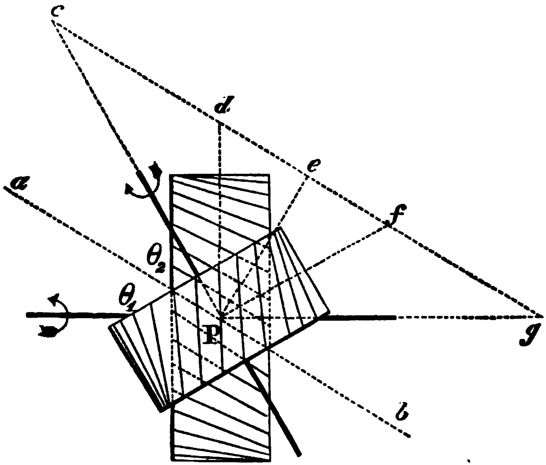


Fig. 205.

threads are determined by the condition that in two wheels which work together the normal pitches must be equal. Hence,

$$n_1=n_2$$

and using the proportions in eq. 19

$$c_1 \cos \theta_1=c_2 \cos \theta_2$$

$$\left.\begin{aligned} \therefore \cos \theta_1 &= \frac{c_2 \sin i}{\sqrt{(c_1^2-2c_1 c_2 \cos i+c_2^2)}} \\ \cos \theta_2 &= \frac{c_1 \sin i}{\sqrt{(c_1^2-2c_1 c_2 \cos i+c_2^2)}} \end{aligned}\right\} \quad . \quad (22)$$

$$\left.\begin{aligned} p_1 &= c_1 \cot \theta_1 \\ p_2 &= c_2 \cot \theta_2 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (23)$$

In fig. 205, set off from the pitch point P, the lines Pd, Pf perpendicular to the axes, in the directions the wheels are moving at the point P. Take Pd, Pf equal to the surface velocities $a_1\ r_1$ and $a_2\ r_2$ of the wheels; join df and produce it to meet the axes; then a P b parallel to cg is the line of contact, making angles $\theta_1\ \theta_2$ with the axes. Draw Pe perpendicular to cg. Then Pe is the common component of the surface velocities, and cg the velocity of transverse sliding of the teeth.[1]

$$c_1 : c_2 : p_1 : p_2 : n_1 \text{ or } n_2$$
$$:: \quad \mathrm{P}d : \mathrm{P}f : \mathrm{P}g : \mathrm{P}c : \mathrm{P}e$$

If Pd, Pf are set off equal to the circumferential pitches of the two wheels, and the same construction is made, then

$$p_1 = \mathrm{P}g;\ p_2 = \mathrm{P}c;\ \text{and } n_1 \text{ or } n_2 = \mathrm{P}e.$$

203. *Screw gearing when the shafts are at right angles. Worm and Wheel.*—If $i=90°$, then $\cos\theta_2 = \sin\theta_1$

$$\frac{c_1}{c_2} = \tan\theta_1$$

$$\frac{p_1}{p_2} = \cot\theta_1$$

Hence, $$p_1 = c_2 \text{ and } p_2 = c_1$$

or the axial divided pitch of one wheel is equal to the circumferential pitch of the other.

The most common form of screw gearing is that in which the shafts are at right angles, and a wheel of one thread, or sometimes of two or three threads, works with a wheel of many threads. Then the former is termed a *worm*, and the latter a *worm wheel*. With this arrangement, a high velocity ratio is obtained with a pair of small wheels. If $N_1\ N_2$ are the numbers of revolutions of the worm and wheel, $a_1\ a_2$,

[1] See Rankine's 'Millwork,' p. 160.

their angular velocities, and ν_1 ν_2 the number of threads on each,

$$\frac{N_1}{N_2}=\frac{a_1}{a_2}=\frac{\nu_2}{\nu_1}$$

Thus, if the worm has one thread and the wheel twenty-five, the velocity ratio is twenty-five. Spur wheels for that velocity ratio would have to be at least ten times larger in diameter. The disadvantage of screw gearing of this kind is that the friction and wear is excessive, hence it is rarely used for the continuous transmission of power. If the obliquity of the helices exceeds a certain amount, the wheels are no longer reciprocal; that is, one wheel will drive the other but the second will not drive the first. In that case the motion is prevented by the friction at the point of contact of the teeth. The worm and wheel are commonly so constructed that the worm will drive the wheel, but the wheel will not drive the worm. This is often advantageous, because the gearing remains stationary in any position after being moved.

204. *Friction of worm and wheel.*—Suppose the worm drives the wheel, and that a force P acts at the pitch line of the worm, in the plane of rotation, overcoming a resistance Q acting at the pitch line of the worm wheel in the plane of its rotation. Let θ_1, as before, be the inclination of the worm thread, μ the coefficient of friction, r_1 r_2 the radii of the worm and wheel, and $\nu_1 p_1$ the total axial pitch of the worm :—

$$\frac{P}{Q}=\frac{1+\mu\tan\theta_1}{\tan\theta_1-\mu} \quad . \quad . \quad . \quad (24)$$

or if ϕ is the angle of repose of metal on metal, so that $\mu=\tan\phi$,

$$\frac{P}{Q}=\cot(\theta_1-\phi) \quad . \quad . \quad (25)$$

When the worm drives the wheel this ratio must be positive. Hence θ_1 must be less than $90°+\phi$. The ratio of

the useful work done to the power expended, or the efficiency of the pair of wheels is

$$\eta = \frac{\cot\theta_1}{\cot(\theta_1 - \phi)} = \frac{1 - \mu\dfrac{\nu_1 p_1}{2\pi r_1}}{1 + \mu\dfrac{2\pi r_1}{\nu_1 p_1}} \quad . \quad . \quad (26)$$

For $\mu = 0\cdot15$, we get

$$\eta = \frac{\nu_1 p_1}{\nu_1 p_1 + r_1} \text{ nearly} \quad . \quad . \quad (27)$$

Hence the efficiency is greater the less the radius of the worm. Generally $r_1 = 1\cdot5$ to $3\,p_1$. For a one-threaded worm therefore the efficiency is only $\frac{2}{5}$ to $\frac{1}{4}$. For a two-threaded worm $\frac{4}{7}$ to $\frac{2}{5}$; for a three-threaded worm $\frac{2}{3}$ to $\frac{1}{2}$. Since so much work is wasted in friction it is not surprising that the wear is excessive.

The radius of the pitch surface of the worm is very variable in practice. The least value (which gives the greatest efficiency) is about $r_1 = p_1$. More commonly, especially if the worm is to be cast and keyed on the shaft, $r_1 = 1\frac{1}{4}$ to $1\frac{1}{2}\,p_1$. Sometimes for special reasons $r_1 = 4$ to $6\,p_1$.

Let $r_1 = x\,p_1$, where x may vary from $1\frac{1}{4}$ to any larger value. Then

$$\eta = \frac{\nu_1}{\nu_1 + x} \quad . \quad . \quad (27a)$$

where ν_1 is the number of threads in the worm. The following table gives the values of the efficiency η:

Number of threads ν_1	$x=$								
	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2	$2\frac{1}{2}$	3	4	6
1	·50	44	·40	·36	·33	·28	·25	·20	·14
2	67	·62	·57	·53	·50	·44	·40	·33	·25
3	75	·70	·67	·63	·60	·55	·50	·43	·33
4	·80	·76	·73	·70	·67	·62	·57	·50	·40

205. *Form of Worm Wheel Rim.*—Fig. 206 shows the forms adopted for the rims of worm wheels. The simplest

form to cast is shown at A, but this gives the weakest tooth, and as ordinarily constructed the most inaccurate. By cutting off the corners of the teeth radially as at B, both these evils are partially remedied, and the form C is still better. Good worm gearing is now always of these latter forms.

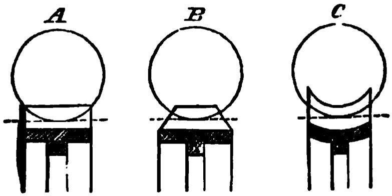


Fig. 206.

206. *Form of Worm Wheel Teeth and Threads of Worm.*—In the cases which most commonly occur the axes of the worm and wheel are at right angles, and the axial divided pitch of one wheel is equal to the circumferential pitch of the other. For this pitch the common symbol p will be used. In such cases it is convenient to design the teeth of the worm and wheel on a plane passing through the worm axes and normal to the wheel axis. Then it is evident (as Mr. Willis first pointed out) that if the section of the teeth of the worm wheel by that plane be made like those of a spur wheel of the same radius and pitch, and the threads of the worm like the teeth of a rack suited to gear with the spur wheel, the worm wheel and screw will gear together correctly. Any of the wheel and rack constructions given in the table above may therefore be taken for these sections of the worm wheel teeth and worm threads.

For these sections therefore we may take (p=axial pitch of screw or circumferential pitch of worm):

Thickness of tooth on pitch line . . .	$0{\cdot}48\,p$
Height outside pitch line	$0{\cdot}3\,p$
Depth below pitch line	$0{\cdot}4\,p$
Width of face of worm wheel usually	$1{\cdot}5$ to $2{\cdot}5\,p$
Length of worm	3 to $6\,p$
" " usually	$4\,p$

Fig. 207 shows a worm and wheel, the teeth of which are drawn in this way. The worm here shown is of wrought iron or malleable cast iron, formed in one piece with its shaft. Usually the worm is of cast iron, and when small may be fixed by a pin passing through both worm and shaft. When larger its rotation on the shaft may be prevented by a key, and its tendency to slide along the shaft by collars, one of which may be fixed and the other a loose collar fixed by a set screw. Sometimes the bearings which support the worm shaft are so arranged as to prevent the endways motion of the worm.

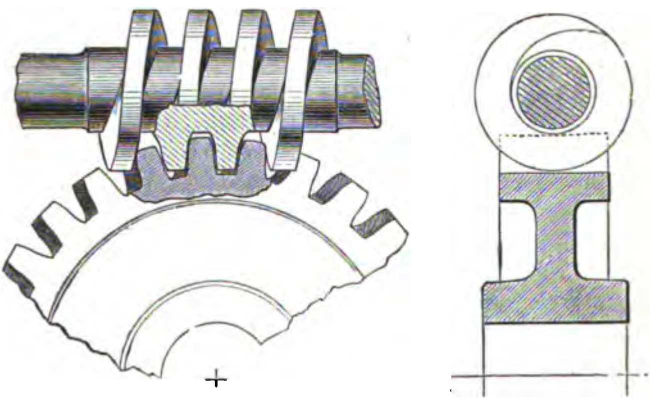


Fig. 207.

The statements in books about the form of the teeth and threads at other sections than that in the plane of the worm axis are very vague. In practice it has been common to make all radial sections of the worm threads similar to the one found as stated above, and all sections of the worm wheel teeth on planes perpendicular to the worm wheel axis similar to the sections found above. It is easy to see that this leads to an incorrect form. For the radial sections of the worm are not rightly placed with reference to the corresponding parallel sections of the worm wheel teeth.

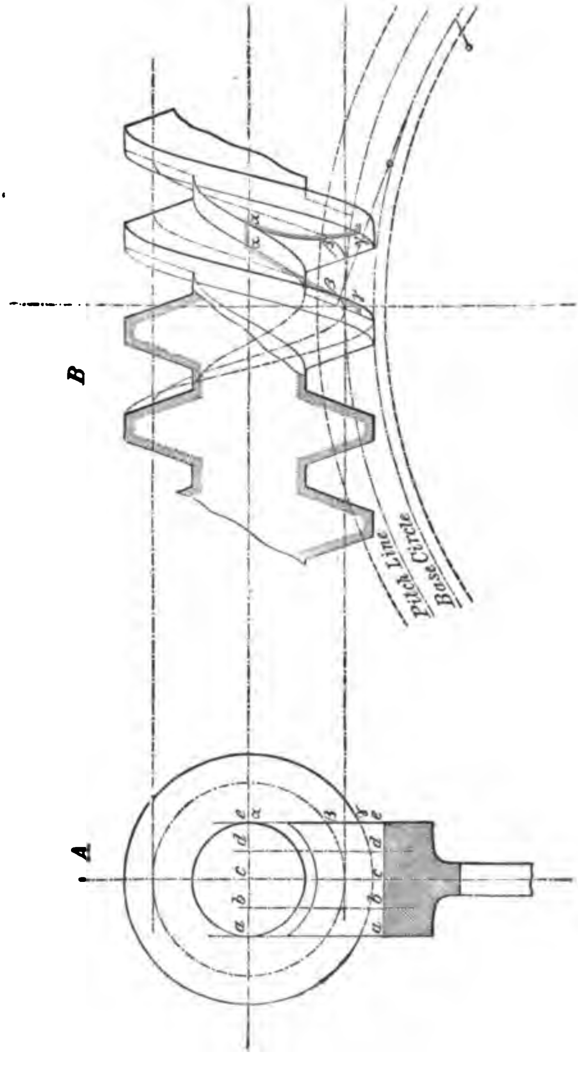


Fig. 208.

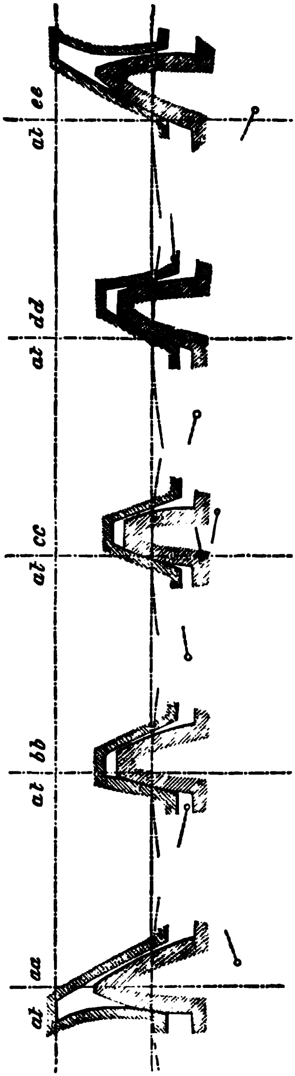


Fig. 209.

Mr. Briggs pointed out to the author that the ordinary mode of designing worm wheel teeth was erroneous, and mentioned that with rightly formed teeth working cranes he had found less friction and wear. This led to a re-examination of the mode of designing worm wheel teeth, and the following method is believed to be new. The case drawn has been so chosen as to show in a marked way the difference between the proper sections and those ordinarily adopted

To obtain correct forms of teeth, the rules applied to the sections in the plane passing through the worm axis normal to the wheel axis must be applied to all sections on planes parallel to that plane ; that is, all the sections of the worm threads and wheel teeth, on planes normal to the wheel axis, must be of forms suitable for a spur wheel and rack of the same pitch. To obtain these sections proceed as follows :

Draw first the two views, A, B, fig. 208, of the worm and wheel on planes passing through each axis normal to the other axis. With the proportions given above mark off the root

and addendum of the teeth, and design the section of the worm threads and wheel teeth on the plane *cc* in accordance with Prof. Willis's principle mentioned above. The sections chosen in the present example are shown at *cc* in fig. 209. The wheel teeth are involute teeth, and the worm threads are similar to those of a rack suitable to work with such teeth. They are bounded by straight lines normal to the base circle tangent. The section of the worm teeth is shown again in fig. 208.

Next in 208, B, draw helices corresponding to any points in the worm thread section. In the figure three helices are shown corresponding to the point, root, and pitch point of the thread. Then the true section of the worm thread on, for example, the plane *ee*, fig. A, is found in fig. B by projecting the points $\alpha\beta\gamma$ in fig. A to the corresponding helices in fig. B. We thus get the points $\alpha\beta\gamma$, $\alpha\beta\gamma$ marking the section in fig. B, and the same section has been transferred to *ee*, fig. 209. The other sections in that figure were obtained in the same way. It remains to find the corresponding worm tooth sections, of which only that for *cc* section is as yet determined.

In fig. 210, let *o* be the centre of the worm wheel, A A the worm pitch line, B B the worm wheel pitch line, and of the worm thread forms let *o* be the one corresponding to the section at *ee* in fig. 209. It will make no difference in the relative motion of the worm and wheel if we suppose the worm wheel pitch line to roll along the worm pitch line. On a piece of tracing paper mark off the centre *o*, and the pitch line B B. In order to roll this traced pitch line on A A, mark off equidistant positions of the centre of the worm wheel, o, 1, 2, 3, 4, 1*a*, 2*a*, 3*a*, 4*a*, and also the corresponding touching points P, 1′, 2′, 3′, 4′, ... along the worm pitch line. On the traced worm wheel pitch line take P 1″=P 1′; 1″ 2″=1′ 2′; and so on. The traced pitch line can then be easily placed in the successive positions shown by the dotted arcs touching the

worm pitch line. In each of these positions trace off on the tracing paper the worm thread form o. We shall thus ob-

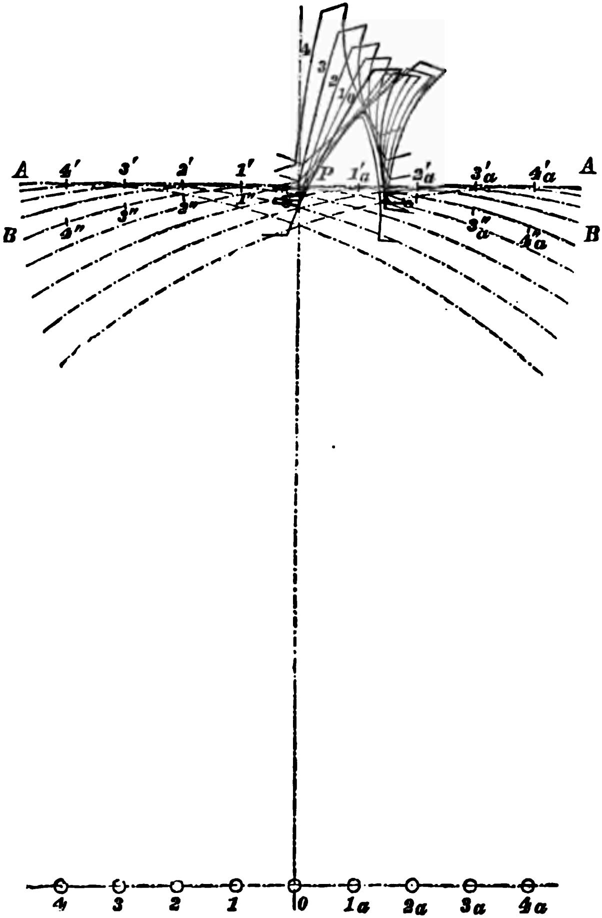


Fig. 210.

tain on the tracing paper the figure of the threads shown at o, 1, 2, 3, 4. The envelope of these positions is the proper

form of the worm wheel thread. In drawing this, however, the top and side clearance must be left as shown. The side clearance may be 0·04 of the pitch.

In fig 209, the result of applying this method to four sections other than the central one is shown. It may be remarked that a circular arc can be found by trial for each side of the worm tooth, which agrees accurately enough for any practical purpose with the required envelope of the worm threads. The teeth in fig. 209 are drawn with such circular arcs, the centres of the arcs being marked on the figure. The pattern of the wheel would have to be made by cutting the teeth to templets of the forms found by the construction. The worm wheel in the figure is shown with the front of the teeth concave. But they may be cut off parallel to the rim without altering the correct action of the teeth.

207. *Strength of worm wheels.*—The resultant pressure on the teeth (friction being neglected) is in the direction of the normal to the faces of the teeth at the point of contact, or in the direction in which the normal pitch is measured. Hence it is the tendency to break under the action of the force acting in that direction, which has to be considered in estimating the strength of the teeth.

The worm is usually at least as strong as the worm wheel, hence it is only necessary to consider the strength of the latter. Let n_2 be the normal divided pitch of the worm wheel, c_2 its circumferential pitch, r_2 its radius, and θ_2 the angle between the threads and the axis :

$$n_2 = c_2 \cos\theta_2 = \frac{2\pi r_2}{v_2}\cos\theta_2, \quad . \quad . \quad . \quad (28)$$

where $\cos\theta_2 = \frac{\text{P}e}{\text{P}f}$ in fig. 205. Let Q be the resistance to rotation at the circumference of the worm wheel. Then the pressure acting normally to the teeth is $Q_n = \frac{Q}{\cos\theta} = Q\frac{\text{P}f}{\text{P}e}$.
The worm wheel is equivalent to a spur wheel resisting the force Q_n at the pitch line, and having the pitch n_2. Hence

the normal pitch n_2 may be obtained by the rules for the teeth of spur wheels. Then $c_2 = \frac{n_2}{\cos\theta} = n_2 \frac{\mathrm{P}f}{\mathrm{P}e}$.

When the shafts are at right angles, the angle θ_2 is often small, so that $\cos\theta = 1$ nearly. Then the worm wheel is approximately equivalent to a spur wheel resisting the force Q, and having the pitch c_2. Hence, when θ_2 is small, the obliquity of the teeth may be neglected in calculating the pitch. The width of face of the worm wheel is about $1\frac{1}{2}$ times the pitch. In calculating the size of the worm shaft, from the resistance Q overcome, friction should not be neglected. The twisting moment acting on the worm shaft is $\frac{\mathrm{Q}\,r_2}{\eta}\;\frac{v_1}{v_2} = \mathrm{Q}\,r_2\,\frac{v_1 p_1 + r_1}{v_2 p_1}$ nearly.

208. *Weight of toothed gearing.*—Let p be the pitch, b the breadth of face, and n the number of teeth of a wheel. Then, its weight in lbs. is, approximately,

$$\mathrm{W} = k\,n\,b\,p^2, \quad . \quad . \quad . \quad (29)$$

where $k = 0{\cdot}38$ for spur wheels, and $0{\cdot}325$ for bevil wheels. The weight of a pair of wheels is independent of the radii, and depends directly on the H.P. transmitted and the numbers of revolutions of the wheels. The weight of a train of wheels is smaller when the number of pairs of wheels is as small as possible, and when all the pairs, except the quickest-running pair, have the greatest practicable velocity ratio.

Mr. D. K. Clark gives the following formula for the weight of cast-iron spur wheels per inch of breadth in lbs. :

$$\mathrm{W} = (5{\cdot}6 + 9p)d(1 + 0{\cdot}1d) \text{ Spur wheels,}$$
$$= (4 + 6{\cdot}3p)d(1 + 0{\cdot}1d) \text{ Bevil wheels,}$$

where d is the diameter in ft. and p the pitch in ins.

CHAPTER X.

BELT GEARING.

209. THE term belt, band, or strap is applied to a flexible connecting piece which drives a rotating piece termed a pulley, by its frictional resistance to slipping at the surface of the pulley. Such belts are most commonly of leather tanned with oak bark, cut into suitable strips, which are united in long lengths by cementing and lacing or riveting. Special kinds of leather are used where great strength is required. Various other materials have also been employed. Pure vulcanised india-rubber, or better india-rubber with plies of strong canvas interposed between its lengths, is often used. It is stronger than leather, and better in cases where the belt is liable to be wetted. Gutta percha has been used, but it stretches. The Americans are using cotton woven belts, and even paper belts have been tried. These belts are flat belts—that is, they are wide and thin, and they run on pulleys with nearly cylindrical surfaces. Round belts are also used, and at the present time their application is being greatly extended. Such round belts are of hemp rope, of cotton rope, of wire rope, or when small of catgut. The pulleys for round belts have usually V-shaped grooves, in which the belts are placed.

FLAT BELTS.

210. *Velocity ratio in belt transmission.*—A belt is not used in cases where a very exact velocity ratio is necessary. Hence it is generally accurate enough to regard the belt as inextensible. If also there is no slipping of the belt on the pulley, the velocity of the belt and the surface velocities of

the pulleys must all be equal. Let v be the velocity of the belt, d_1 d_2 the diameters of the pulleys, and N_1 N_2 their revolutions per minute—

$$\left.\begin{aligned}\pi d_1 N_1 = V\\ \pi d_2 N_2 = V\end{aligned}\right\} \therefore \frac{d_1}{d_2} = \frac{N_2}{N_1} \quad . \quad . \quad (1)$$

These equations are in strictness only true when the belt is infinitely thin. When the belt has a thickness δ, the effective diameters of the pulleys are $d_1+\delta$, and $d_2+\delta$. Then,

$$\frac{N_2}{N_1} = \frac{d_1+\delta}{d_2+\delta} \quad . \quad . \quad . \quad (1\ a)$$

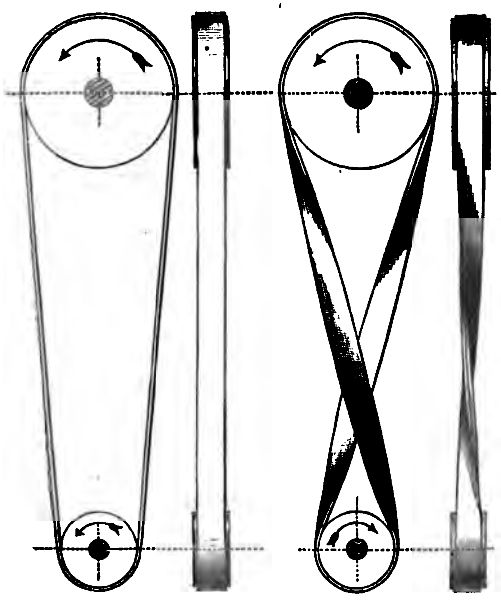


Fig. 211. Fig. 212.

As the belt thickness is generally small compared with the pulley diameter, δ may be neglected without any great

error, but it should be remembered that, in all questions of velocity ratio in belting, the virtual diameter of the pulley is the diameter measured to the centre of the belt.

211. *Endless Belt.*—When one shaft is driven from another, a pulley is placed on each shaft, and an endless belt is strained over the two pulleys. The belt may be an open belt (fig. 211) or a crossed belt (fig. 212). In the former case the two shafts rotate in the same direction. In the latter case they rotate in opposite directions.

212. *Length of belts.*—Let D and d be the diameters of the two pulleys in inches; c, their distance apart, from centre to centre; L, the length of the belt. Also, let $D+d=\Sigma$, and $D-d=\Delta$.

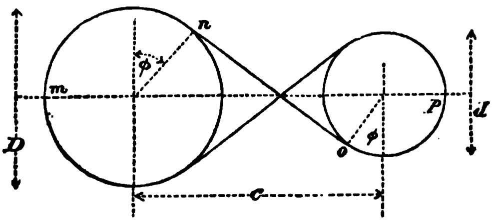


Fig. 213.

For a crossed belt (fig. 213) the total length—

$$=L=2(m\,n+n\,o+o\,p)$$

$$=\left(\frac{\pi}{2}+\phi\right)D+2\,c\cos\phi+\left(\frac{\pi}{2}+\phi\right)d$$

$$=\left(\frac{\pi}{2}+\phi\right)\Sigma+2\,c\cos\phi\quad .\quad .\quad .\quad (2)$$

$$\sin\phi=\frac{D+d}{2\,c}=\frac{\Sigma}{2\,c}\quad .\quad .\quad .\quad .\quad (3)$$

The length of the belt is obtained thus :—Calculate the value of sin ϕ. From a table of natural sines and cosines find the nearest values of cos ϕ and ϕ, the latter being ex-

pressed in circular measure. Then eq. (2) gives the belt length. If ϕ is found or measured off the drawing in degrees, the circular measure of the angle is obtained by multiplying by 0·0175.

With a crossed belt ϕ depends only on $D+d$. Hence, if Σ and c are constant for two or more pairs of pulleys, the same belt will run on any pair of pulleys of the set.

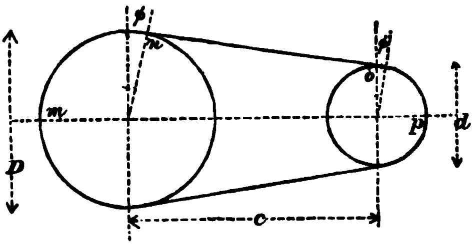


Fig. 214.

When the belt is an open one (fig. 214) the equations are rather less simple—

$$L=2\,(mn+no+op)$$

$$=\left(\frac{\pi}{2}+\phi\right)D+2\,c\cos\phi+\left(\frac{\pi}{2}-\phi\right)d$$

$$=\frac{\pi}{2}\Sigma+\phi\Delta+2\,c\cos\phi \quad . \quad . \quad . \qquad (4)$$

$$\text{Sin}\ \phi=\frac{\Delta}{2\,c}\,;\ \cos\phi=\sqrt{\left(1-\frac{\Delta^2}{4\,c^2}\right)} \quad . \qquad (5)$$

For an open belt ϕ is generally small, so that,

$$\phi=\sin\phi,\ \text{nearly}$$

$$\therefore L=\frac{\pi}{2}\Sigma+2\,c\left\{\frac{\Delta^2}{4\,c^2}+\sqrt{\left(1-\frac{\Delta^2}{4\,c^2}\right)}\right\}$$

$$=\frac{\pi}{2}\Sigma+2\,c\left\{1+\frac{1}{8}\,\frac{\Delta^2}{c^2}\right\}\ \text{nearly} \quad . \qquad (6)$$

Hence, if an open belt runs on a pair of pulleys, the sum and difference of whose diameters are Σ_1 and Δ_1 and the same belt is also to run on another pair of pulleys, the sum and difference of whose diameters is Σ_2 and Δ_2, since the length of the belt is the same in the two cases,

$$\frac{\pi}{2}\Sigma_1+2c\left\{1+\frac{1}{8}\frac{\Delta_1^2}{c^2}\right\}=\frac{\pi}{2}\Sigma_2+2c\left\{1+\frac{1}{8}\frac{\Delta_2^2}{c^2}\right\}$$

$$\Sigma_2=\Sigma_1+\frac{\Delta_1^2-\Delta_2^2}{4\pi c} \quad . \quad . \quad (7)$$

It is accurate enough for practical purposes to calculate the diameters D_2 and d_2 as if the belt were a crossed belt. Then, taking Δ_2 = the difference of these diameters, find the value of Σ_2. From that value of Σ_2 recalculate the diameters D_2 and d_2, using eq. (1) or eq. (1*a*).

213. *Speed cones.*—When a shaft running at a constant speed has to drive a machine at several different speeds, sets of pulleys are used which are termed stepped speed cones.

The speed cones (fig. 215) are placed opposite one another, so as to form a series of pairs of pulleys, and by shifting the belt from one pair to another the speed of the machine is altered. In designing these speed cones the ratio of the diameters of each pair depends on the speeds of the shafts, and the sum of the diameters should be so arranged that the same belt will work on any pair of the set without alteration of length.

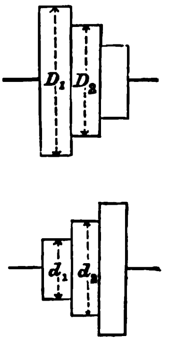


Fig. 215.

Let D_1 d_1 be the diameters of one pair; D_2 d_2 the diameters of another pair. Let N be the number of revolutions of the shaft on which D_1 and D_2 are placed; n_1 and n_2 the revolutions of the other shaft, when the belt is on d_1 and d_2 respectively.

If the belt is a crossed belt, from eq. (1)—

$$\frac{D_1}{d_1}=\frac{n_1}{N} \qquad \frac{D_2}{d_2}=\frac{n_2}{N}$$

also, $$D_1+d_1=D_2+d_2=\Sigma.$$

Hence,

$$\left.\begin{aligned} D_2&=\frac{n_2}{N+n_2}\Sigma \\ d_2&=\frac{N}{N+n_2}\Sigma \end{aligned}\right\} \quad . \quad . \qquad (8)$$

If the belt is an open belt, the diameters will be slightly different. Let $\Delta_1=D_1-d_1$; $\Delta_2=D_2-d_2$; $\Sigma_1=D_1+d_1$; $\Sigma_2=D_2+d_2$. If the belt were a crossed belt, we should have,

$$D_2=\frac{n_2}{N+n_2}\Sigma_1\ ;\ d_2=\frac{N}{N+n_2}\Sigma_1$$

and since the diameters for an open belt are but little different,

$$\Delta_2=D_2-d_2=\frac{n_2-N}{N+n_2}\Sigma, \text{ nearly.}$$

Then from eq. (7),

$$\Sigma_2=\Sigma_1+\frac{\Delta_1^2-\Delta_2^2}{4\pi c}$$

And from eq. (1),

$$\frac{D_2}{d_2}=\frac{n_2}{N}$$

Hence,

$$\left.\begin{aligned} D_2&=\frac{n_2}{N+n_2}\Sigma_2 \\ d_2&=\frac{N}{N+n_2}\Sigma_2 \end{aligned}\right\} \quad . \quad . \qquad (9)$$

Hence, the process of designing a set of speed cones is

this:—Having given the speed N of the driving shaft, decide on the speeds n_1, n_2, n_3 . . . . of the driven shaft. Choose a diameter for one of the pulleys of the first pair, and find the diameter of the other by equation (1). The values of Σ_1 and Δ_1 can then be found. From these the diameters of any other pair of pulleys can be found by the equations above.

214. *Resistance to slipping of a belt on a pulley.*—Let fig. 216 represent a belt strained over a pulley and on the point of slipping from T_1 towards T_2. Then the tension T_2 must be greater than the tension T_1, by the amount of the frictional resistance to slipping at the surface of the pulley.

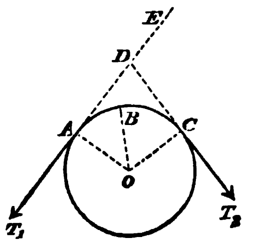


Fig. 216.

Let B be any point of contact, and let the tension at B=T. Let the angle A O B in circular measure be θ; the arc A B$=s$; the radius A O$=r$; the normal pressure of the belt on the pulley estimated per unit of arc $=p$; and the coefficient of friction $=\mu$.

Consider a small length, ds of the belt at the point B. The tensions at the ends of that small length are T and $T+dT$, so that $\frac{dT}{ds}$ is the increase of tension, per unit length of the arc of contact. But in unit length of belt the friction is μp,

$$\therefore \frac{dT}{ds}=\mu p$$

Let $d\theta$ be the small angle at the centre corresponding to the arc $a\,b=d\,s$ (fig. 217). Then the pressure R on the arc $a\,b$ is the resultant of the tensions T and $T+dT$ acting tangentially to the pulley at the extremities of the arc. Neglecting dT, we have

$$R=2\,T\sin\frac{d\theta}{2}=T\,d\theta \text{ nearly.}$$

But $\quad R = p\,d\,s.$

$$\therefore p = T\frac{d\theta}{d s} = \frac{T}{r}.$$

Combining these equations and remembering that

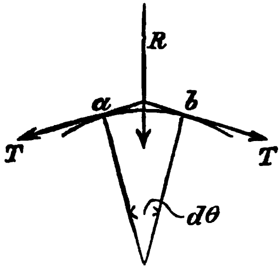


Fig. 217.

$$d s = r\,d\theta,$$

$$\frac{dT}{ds} = \frac{dT}{r\,d\theta} = \mu\frac{T}{r}\ .$$

$$\frac{dT}{T} = \mu\,d\theta$$

$$\int_{T_1}^{T_2}\frac{dT}{T} = \mu\int_o^{\theta} d\theta$$

$$hyp.\ log.\ \frac{T_2}{T_1} = \mu\,\theta\ . \quad . \quad . \quad (10)$$

where θ is the angle A O C, or, what is the same thing, the angle C D E, expressed in circular measure, or the arc A C÷radius A O. If the angle θ is measured in degrees, it can be reduced to circular measure, by multiplying by $\frac{\pi}{180}$ or by 0·0175.

This equation may be put in the form

$$k = \frac{T_2}{T_1} = \epsilon^{\mu\theta} \quad . \quad . \quad (11)$$

where $\epsilon = 2·71828$, the base of the system of natural logarithms ; and for simplicity k is put for the ratio of the tensions in the two parts of the belt. Hence $\mu\,\theta$ is the hyperbolic or natural logarithm corresponding to the number $T_2 \div T_1$. As common logarithms are more convenient,

Common log. $\frac{T_2}{T_1} = 0·434\ \mu\,\theta$ if θ is in circular measure.
$= 0·007578\ \mu\,\theta$ if θ is in degrees.
$= 2·729\ \mu\,n$ if n is the fraction of the circumference embraced by the belt.

Hence, if the right-hand member of either of these equations is calculated, the value obtained is the logarithm of $T_2 \div T_1$ or k. The natural number corresponding to that logarithm, found by means of a table of logarithms, is the value of

$T_2 \div T_1$ or k. This value is the greatest value of the ratio of the tensions consistent with the belt not slipping.

215. *The coefficient of friction.*—The value of μ for belts varies from 0·15 to 0·56 in different cases. For leather belting on iron pulleys, in an ordinary condition of working, $\mu = 0·3$ to 0·4. The experiments of Messrs. Briggs and Towne appear to show that the latter value may safely be taken.[1] For wire rope running on the bottom of a grooved pulley, $\mu = 0·15$, and if the pulley is bottomed with leather or gutta percha, $\mu = 0·25$.

The following table will give the values of $T_2 \div T_1$ or k for all cases likely to occur, with accuracy enough for most practical purposes :—

Greatest Value of the Ratio of Tensions on Tight and Slack Sides of Belting from eq. 11.

Angle embraced by belt $\theta =$			Ratio of Tensions $k =$			
In degrees	In circular measure	In fraction of circumference	$\mu = 0·2$	$\mu = 0·3$	$\mu = 0·4$	$\mu = 0·5$
30	·524	·083	1 110	1·170	1·233	1·299
45	·785	·125	1·170	1·266	1·369	1·481
60	1 047	·167	1·233	1·369	1·521	1·689
75	1·309	·208	1·299	1·481	1·689	1·924
90	1 571	·250	1·369	1·602	1·874	2·193
105	1·833	·319	1·443	1·733	2·082	2·500
120	2·094	·334	1·521	1 875	2·312	2·851
135	2·356	·375	1·602	2·027	2·565	3·247
150	2·618	·417	1·689	2·194	2·849	3·702
165	2 880	·458	1·778	2·372	3·163	4·219
180	3·142	·500	1·875	2·566	3·514	4·808
195	3·403	·541	1·975	2·776	3·901	5·483
210	3·665	·583	2·082	3·003	4·333	6·252
240	4 188	·666	2·311	3·514	5·340	8·119
270	4·712	·750	2·566	4·112	6·589	10·55
300	5·236	·833	2·849	4·808	8·117	13·70

216. *Tensions in an endless belt.*—Let an endless belt be strained over two pulleys with an initial tension T_0. At the

[1] 'Journal of Franklin Institute,' 1868.

moment the driving pulley begins to move the belt is stretched on the driving side and the tension increased, whilst the other side of the belt is shortened and the tension diminished. Since the lengthening of the tight and the shortening of the slack side must be equal in amount, the average tension remains unaltered. That is,

$$\frac{T_2+T_1}{2}=T_0 \quad . \quad . \quad . \qquad (12)$$

This process goes on till the force T_2-T_1, tending to rotate the driven pulley, is sufficient to overcome its resistance to motion. The driven pulley then rotates, the condition of the belt remaining permanent till the motion ceases again. It is necessary, however, that the initial tension should be sufficient to prevent slipping on either of the pulleys.

Let P be the resistance at the circumference of the driven pulley. Then

$$P=T_2-T_1 \quad . \quad . \quad . \quad . \quad . \quad . \qquad (13)$$

But if H is the number of horses power transmitted, V the velocity of the circumference of the pulley or of the belt,

$$P\,V=550\,H$$

$$P=\frac{550\,H}{V} \quad . \quad . \quad . \quad . \quad . \qquad (13a)$$

If N = number of revolutions of pulley per minute, and d=diameter of pulley in inches, then $V=\frac{\pi d N}{12 \times 60}$, and consequently

$$P=126{,}000\,\frac{H}{d\,N} \quad . \quad . \quad . \quad . \quad . \qquad (13b)$$

217. *Tensions in a belt transmitting a given horse-power.*— Suppose that the value of P is obtained from the equations just given, and the value of k from equation 11 or the table corresponding to it. Then from equation 13

$$\left.\begin{aligned} T_2&=P\frac{k}{k-1}=x\,P \\ T_1&=P\frac{1}{k-1}=y\,P \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \qquad (14)$$

Table to Facilitate the Calculation of the Belt Tensions.

$\theta =$			Values of x for				Values of y for			
In degrees	In circ. measure	In fractions of circumference	$\mu=0·2$	$\mu=0·3$	$\mu=0·4$	$\mu=0·5$	$\mu=0·2$	$\mu=0·3$	$\mu=0·4$	$\mu=0·5$
30	·524	·083	10·09	6·89	5·29	4·35	9·09	5·88	4·29	3·34
45	·785	·125	6·89	4·76	3·71	3·08	5·88	3·76	2·71	2·08
60	1·047	·167	5·29	3·71	2·92	2·45	4·29	2·71	1·92	1·45
75	1·309	·208	4·35	3·08	2·45	2·08	3·34	2·08	1·45	1·08
90	1·571	·250	3·71	2·66	2·14	1·85	2·71	1·66	1·14	·840
105	1·833	·319	3·26	2·37	1·93	1·67	2·26	1·36	·924	·667
120	2·094	·334	2·92	2·14	1·77	1·54	1·92	1·14	·762	·541
135	2·356	·375	2·66	1·98	1·64	1·44	1·66	·984	·649	·444
150	2·618	·417	2·45	1·84	1·54	1·37	1·45	·840	·541	·370
165	2·880	·458	2·29	1·73	1·47	1·31	1·29	·730	·462	·311
180	3·142	·500	2·14	1·64	1·40	1·26	1·14	·638	·398	·262
195	3·403	·541	2·03	1·56	1·35	1·22	1·03	·563	·345	·223
210	3·665	·583	1·93	1·50	1·30	1·19	·926	·499	·300	·190
240	4·188	·666	1·76	1·40	1·23	1·14	·763	·398	·230	·140
270	4·712	·750	1·64	1·32	1·18	1·10	·639	·322	·179	·105
300	5·236	·833	1·54	1·26	1·14	1·08	·541	·262	·140	·079

218. *Strength of the belt.*—The ultimate strength of the leather used for belting is from 3,000 to 5,000 lbs. per sq. in. of section. At the laced joints the strength is reduced to about 0·3 of the above values, or, say, to 900 to 1,500 lbs. per sq. in. At riveted splices the strength is about 0·5 that of the solid belt, according to the experiments of Messrs. Briggs and Towne. The greatest safe working tension (since the belt is subject to only one kind of stress) is about $\frac{1}{3}$rd of these values. Usually a belt has cemented and riveted joints made at the belting factory, and a laced joint, which is made when the belt is put in place, and which serves for tightening up the belt, if it wears slack. Hence, the greatest working tension is that corresponding to the laced joint, and is about 320 lbs. per sq. in. of the belt section.

The thickness of the belt varies from $\frac{3}{16}$ to $\frac{5}{16}$ inch if the belt is a single one, and from $\frac{3}{8}$ to $\frac{3}{4}$ inch if the belt is a double one. Hence, calling f the safe working tension per inch width of belt, and δ the belt thickness,

$$f = 320\,\delta \quad . \quad . \quad . \quad . \quad (15)$$

Thickness of Belt $=\delta=$

$\frac{3}{16}$	$\frac{7}{32}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$

Working Tension in lbs. per inch width $=f=$

60	70	80	100	120	140	160	180	200	220	240

Professor Karl Keller points out that generally thin leather is chosen for narrow belts and thick leather for wide belts, so that if β is the width of the belt, we have on the average,

$$\delta = 0{\cdot}1\sqrt{\beta},$$

and hence,

$$f = 32\sqrt{\beta}.$$

The rule is a good one for small belts, but would be unsafe if applied to very wide ones. For these a definite thickness should be calculated and provided.

Width of Belt in ins. $=\beta$							
2	3	4	6	8	10	12	15
Calculated thickness of Belt $=\delta$							
0·14	·17	·20	·24	·28	·32	·35	·39
Working Tension in lbs. per inch of width $=f$							
45	55	64	78	90	101	110	124

219. *Width of Belt for a given maximum tension.*—The greatest tension on the belt is the tension T_2 on the driving side. Then if f is the safe working tension obtained as above, the width of the belt is

$$\beta=\frac{T_2}{f}=\frac{P\,k}{f(k-1)}=\frac{P}{f}x \quad . \quad . \quad (16)$$

where the values of x are given in the preceding table, and P is obtained from eq. 13*a*.

220. *Horses power per inch width of belt.*—From (16) we get for the working tension per inch width of belt

$$P=\frac{f}{x}$$

and from eq. 13*a*, taking $f=70$ lbs.

$$\frac{H}{V}=\frac{7}{55\,x}.$$

Hence the horses power transmitted per inch width of belt at one foot velocity per second is

$$\frac{7}{55\,x} \quad . \quad . \quad . \quad . \quad (16a)$$

The following table gives values of this expression for different values of x. For any given pair of pulleys, look out the value of x in the preceding table. Then the number of horses power in the following table, opposite the

nearest value of x, multiplied by the velocity of the belt in feet per second, gives the horses power transmitted per inch width of belt. The most common rough value of x is 2.

Horses Power transmitted per inch width of belt at one foot per second.

$x=$	H=	$x=$	H=
1·0	·1273	2·75	·0462
1·25	·1020	3·00	·0424
1·50	·0730	3·25	·0392
1·75	·0636	3·50	0364
2·00	·0565	3·75	·0340
2·25	·0510	4·00	·0318
2·50			

221. *Rough calculations of the size of belts.*—In a great many cases in practice, the belt embraces about 0·4 of the circumference of the pulley on which it is most liable to slip,[1] and the coefficient of friction is at least 0·3. Then, $\frac{T_2}{T_1}=2$. When this is the case, the following simple rules may be used:

$$\left.\begin{array}{ll} \text{Driving force} & =P=\dfrac{550\,H}{V} \\ \text{Greatest tension} & =T_2=2\,P \\ \text{Initial tension} & =T_0=1\tfrac{1}{2}\,P \\ \text{Width of belt} & =\beta=\dfrac{2\,P}{f} \end{array}\right\} \quad . \quad . \quad . \quad (17)$$

The following approximate table gives the width of belt calculated by these rough rules, the belt being assumed to be $\frac{7}{32}$nds of an inch in thickness, and carrying safely 70 lbs. tension per inch of width.

[1] That is, the pulley having the smaller arc of contact.

Velocity of belt in ft. per sec.	Width of belt in ins. $\frac{7}{32}$ inch thick when the horses' power transmitted is									
	1	2	3	4	5	7½	10	15	20	25
1	15·7	31·4	47·0	63·0	...	..	...	...	...	...
2½	6·3	10·6	18·8	25·2	31·2	46·8	...	...	...	...
5	3·1	6·3	9·4	12·6	15·6	23·6	31·4	47·2	...	...
7½	2·1	4·2	6·3	8·4	10·4	15·6	21·0	31·2	42·0	52·4
10	1·5	3·2	4·7	6·4	7·8	11·8	15·7	23·6	31·4	39·2
12½	1·3	2·5	3·7	5·0	6·4	9·4	12·6	18·8	25·2	31·2
15	1·1	2·1	3·1	4·2	5·2	7·8	10·5	15·6	21·0	26·2
20	·79	1·6	2·4	3·2	3·9	5·9	7·9	11·7	15·7	19·6
25	·63	1·3	1·9	2·6	3·1	4·7	6·3	9·4	12·6	15·6
30	..	1·1	1·6	2·2	2·6	3·9	5·2	7·8	10·5	13·1
35	...	...	1·3	1·7	2·2	3·4	4·5	6·8	9·0	11·2
40	...	...	...	1·5	2·0	2·9	3·9	5·9	7·8	9·8
45	...	...	...	...	1·8	2·6	3·5	5·2	7·0	8·8
50	...	...	..	...	1·6	2·4	3·2	4·7	6·3	7·8
60	...	...	...	...	1·3	2·0	2·6	3·9	5·2	6·5
70	...	...	...	...	1·1	1·7	2·2	3·4	4·5	5·6
80	...	...	...	...	...	1·5	2·0	2·9	3·9	4·9
90	...	...	...	...	...	1·3	1·8	2·6	3·5	4·4
100	...	...	...	...	...	1·2	1·6	2·4	3·1	3·9

222. *American high-speed belting.*—In America belts are used for transmitting very great amounts of power, in conditions somewhat different from those described above. If the belt is very wide a partial vacuum is formed between the belt and pulley at a sufficiently high velocity. The pressure between the belt and the pulley is then greater than that calculated from the tensions in the belt, and the resistance to slipping is greater. This has the advantage not only of permitting a greater power to be transmitted by a given belt, but also of diminishing the strain on the shafting. For belts used in this way the speed is usually made from 4,000 to 6,000 feet per minute, by using sufficiently large pulleys. The belts are of single thickness, and as the roughness of the belt and pulley is disadvantageous rather than otherwise, the hair side of the belt is placed next the pulley. The widths which the Americans give to belts put up on this principle are such

that the driving pressure P is 50 to 67 lbs. per inch width of the belt Consequently the greatest strain in the belting T_2 is about 156 to 185 lbs. per sq. in. of section. The belts are generally not more than $3\frac{1}{2}$ to 4 feet wide. These details are taken from Achard, 'Proc. Inst of Mechanical Engineers,' 1881, p. 60.

223. *Influence of the elasticity of the belt on the velocity ratio.*—Let s be the length of belt which runs off either pulley in the unit of time, the belt being measured in its unstrained condition. In working, the length s is extended to s_2 by the tension T_2 on the driving side, and to s_1 by the tension T_1 on the slack side. Since the elongation is proportional to the straining force (§ 26),

$$s_2=(1+a\,T_2)\,s, \text{ and } s_1=(1+a\,T_1)\,s,$$

where a is the elongation of one foot of belt by one pound of tension. The driving pulley receives s_1 feet of belt in the unit of time, and the driven pulley s_2 feet. Hence the velocities of the pulley circumferences are not exactly the same (as assumed in § 210) but are equal to s_2 and s_1 respectively,

$$\left.\begin{array}{l}\pi\,d_1\,N_1=s_1\\ \pi\,d_2\,N_2=s_2\end{array}\right\}\;\therefore\;\frac{N_1}{N_2}=\frac{d_2}{d_1}\cdot\frac{s_1}{s_2}=\frac{d_2}{d_1}\left(\frac{1+a\,T_1}{1+a\,T_2}\right). \qquad (18)$$

where N_2 is the number of revolutions of the driving, and N_1 the number of revolutions of the driven pulley. According to M. Kretz, $\frac{1+aT_1}{1+aT_2}=0{\cdot}975$ for new, and $0{\cdot}978$ for old belts. Hence,

$$a=\frac{{\cdot}00011}{\beta\,\delta} \text{ for old belts}$$

$$=\frac{{\cdot}00015}{\beta\,\delta} \text{ for new belts.}$$

$\beta\,\delta$ being the area of section of the belt in sq. ins. The

velocity of the driven pulley is about 2 per cent. less than it would be if the belt were inelastic. If motion is transmitted through several belts, the loss of velocity due to this cause would become important. This loss of velocity may be termed the *slip* due to elasticity of the belt.

224. *Effect of centrifugal tension on the strength of belts.*—When belts run at high speeds, part of the belt tension is expended in deviating the belt as it passes over the curved surface of the pulley. Hence, a given belt tension produces a less normal pressure on the pulley, and less resistance to slipping, in consequence of the centrifugal force of the belt. The weight of belting is about $w = 0{\cdot}43\ \beta\,\delta$ lbs. per foot of length, where β and δ are in inches. The centrifugal force of one foot length of belting is $w\dfrac{\text{V}^2}{g r}$ lbs.

The normal pressure on the pulley is $p = \dfrac{\text{T}}{r} - \dfrac{w\,\text{V}^2}{g\ r}$ where the second term becomes unimportant at small velocities, as has been assumed above.

Hence the greatest tension in the belt is,

$$\text{T}_2 + \frac{w\,\text{V}^2}{g} \quad . \quad . \quad (19)$$

and the belt width must be calculated for that tension instead of for T_2.

Hence, if β is the width of the belt when the centrifugal tension is neglected, its width when centrifugal tension is allowed for will be

$$\beta_1 = \frac{743\,\beta}{743 - \dfrac{\text{V}^2}{g}} = \frac{23924\,\beta}{23924 - \text{V}^2} \quad . \quad (19\,a)$$

The influence of centrifugal tension was first pointed out by Professor Rankine ('Millwork,' p. 532).

225. *Single, double, and combined belting. Joints in belting.*—The leather used for belting is of ox-hide tanned

with oak bark, and only the best part of the hide, termed

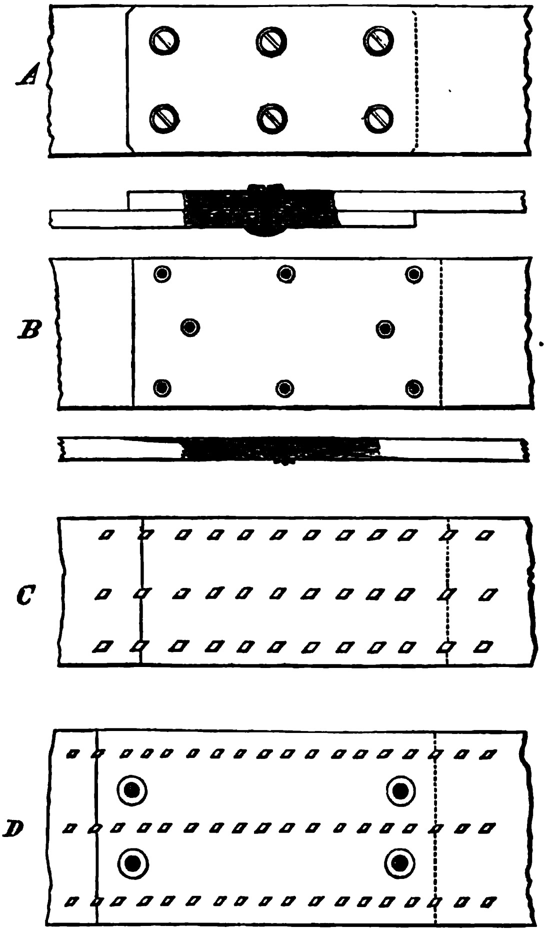


Fig. 218.

the butt, is used. The butts are cut into strips of the width

required, and joined together to form a belt of any required length. The joints are made by paring down the ends of the strip, overlapping them, and cementing them with glue. They are then either sewn, laced, or riveted as an additional precaution. Fig. 218 *C* shows a cemented and laced joint; the overlap is about 7 inches long, and the laces $1\frac{1}{2}$ inch apart, extending an inch beyond the overlap at each end. Sometimes a few rivets are used in addition to the lacing. Fig. 218 *B* shows a cemented and riveted joint, the overlap 6 to 7 inches long, and having about one rivet to $2\frac{1}{4}$ or $2\frac{1}{2}$ sq. ins. of overlap. Fig. 218 *D* shows a laced and riveted joint.

In an endless belt one joint must be uncemented, so that it can be easily broken when the belt requires to be tightened. This joint may be a laced joint, like that previously described, or it may be made with belt screws shown in fig. 218 *A*. These belt screws are of iron with a very flat nut. The length of overlap may be 6 ins., and there may be one screw to 6 or 8 sq. ins. of overlap. This joint is more clumsy than a laced joint, but is very easily broken or made. The laces commonly used are strips of white leather tanned with alum.

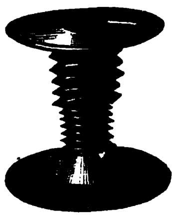


Fig. 219.

A very convenient belt screw with a right- and left-hand screw thread has been introduced lately (Sonnenthal's Patent), and is shown in fig. 219. The screws are made of steel

or gun-metal, and are less likely to work loose than ordinary screws.

Another convenient belt fastening intended to replace laces is that shown in fig. 220. The fastener is shown at *a*, the belt in process of fastening at *b*, and the belt in running condition at *c*.

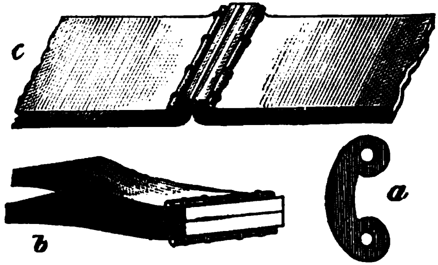


Fig. 220.

When a single belt would be of inconvenient width, a double belt is used. This is made by cementing two strips of leather together, and then sewing them or riveting them. There may be about one rivet to 3 to 4 sq. ins. of belt. The double belt is more rigid than a single belt, and does not work satisfactorily unless there is ample distance between the pulleys, and the pulleys are not less than 3 feet diameter.

When a very broad belt is required to connect two shafts which are not parallel (that is, when the belt has a half or quarter twist), it does not work well, because its rigidity prevents its lying down in contact with the pulleys. It comes in contact with the pulleys on one side only. Messrs. Tullis, of Glasgow, have in such cases employed several narrow belts instead of a single wide one. These run side by side on the same pulleys, and are kept parallel by cross strips of leather riveted to them. Thus, for instance, instead of a 12-inch belt, three four-inch belts may be used, connected by cross strips $1\frac{1}{2}$ inch wide, at intervals of about

12 inches. A combined belt of this kind runs quite parallel and comes much more perfectly in contact with the pulleys than an ordinary belt.

The inside of the leather is rougher than the outer surface, and belts should be so arranged that the rough side is always next the pulleys. Crossed belts and belts passing over guide pulleys require to be twisted in order to keep the same side of the belt next the pulleys.

Of late an excellent leather belting has been manufactured, under the name Victoria Belting. In making this the spongy and weak inside part of the leather is pared away by

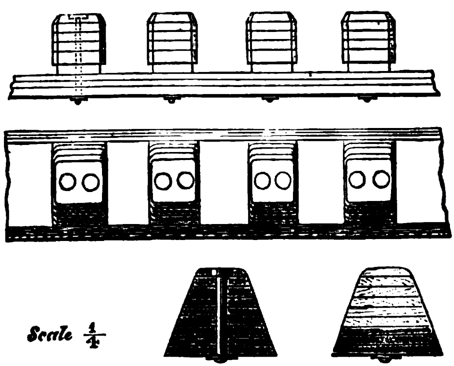


Fig. 221.

machinery, and the belt reduced to a uniform thickness. Although thinner and more flexible than ordinary belting, the Victoria Belting is equally strong. Usually two thicknesses of pared leather are cemented together and riveted with copper rivets. The belt is then virtually a double belt, though little thicker than ordinary single belting. With this belting the hair side of the belt runs in contact with the pulley.

Fig. 221 shows a peculiar leather belt introduced by

Messrs. Tullis, of Glasgow, and intended to work on pulleys having V-shaped grooves round their circumference. When the grooves have sides inclined at 45°, the adhesion of the belt to the pulley is increased about 2·6 times, so that the grooved pulley is equivalent to a cylindrical pulley with a coefficient of friction, $\mu = 0·8$ to 1·0. The V-shaped belt shown in fig. 221 has been used for some years in America. It is made of slices of leather riveted together. The continuous part of the belt consists of three strips about $\frac{5}{8}$ths of an inch in total thickness, and 2 ins. in average width. Hence the belt section is about $1\frac{1}{4}$ sq. in. Several of these belts may be used side by side, precisely in the same way as the rope belts which are described in the next chapter. Messrs. Tullis state that the driving power of the belt is considerably greater than that of an ordinary rope belt.

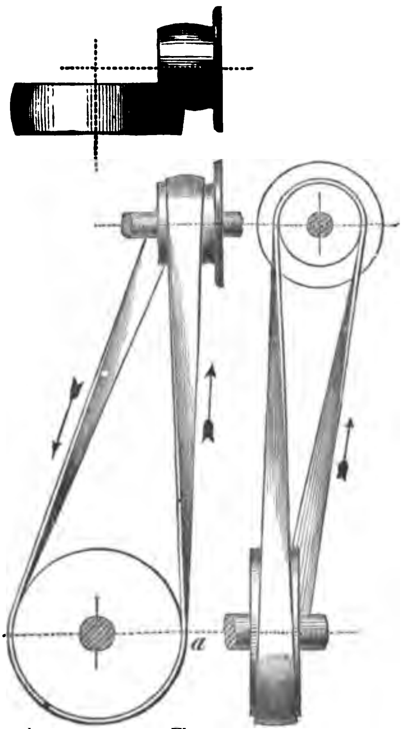


Fig. 222.

226. *Belts connecting shafts which are not parallel.*—When two shafts are not parallel and do not intersect, they may still be connected by an endless belt, provided the pulleys are properly placed. The single

and sufficient condition that the belt may run properly is this:—The point at which the belt is delivered from each pulley must be in the plane of the other pulley. This condition can only be fulfilled for a belt which always runs in one direction.

Fig. 222 shows three views of this arrangement of belting applied to two shafts at right angles. The arrows show the direction of the motion of the belt. If this be followed, it will be found that the point at which the belt runs off each pulley is in the plane passing through the centre of the other pulley. The belt would in this case be said to have a quarter twist.

227. *Guide pulleys.*—When two shafts are not parallel, and whether their directions intersect or not, they may be connected by a single endless belt if intermediate guide pulleys are used. These guide pulleys alter the direction of the belt without modifying the velocity ratio of the shafts. Fig. 223 shows an elevation and plan of an arrangement of pulleys and guide pulleys: *a b* is the intersection of the middle planes of the principal pulleys. Select any two points *a* and *b* on this line, and draw tangents, *a c*, *b d*, to the principal pulleys. Then *c a c* and *d b d* are suitable directions for the belt. The guide pulleys must be placed with their middle planes coinciding with the planes *c a c* and *d b d*. The belt will run in either direction.

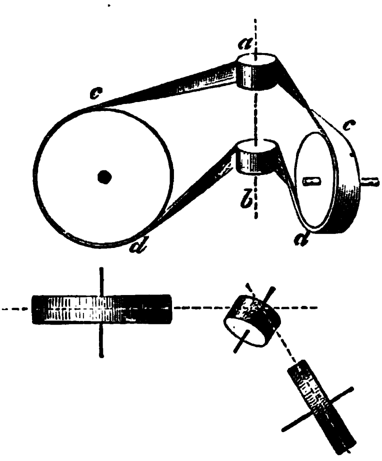


Fig. 223.

Guide pulleys are sometimes used merely to lengthen the belt between two shafts, which are too close together to be connected direct. Fig. 224 shows an arrangement of this kind. The middle planes of the guide pulleys are determined by the method just mentioned. It is, however, possible to place the guide pulleys with their axes parallel. Then the belt must be delivered from each pulley in the plane of the pulley on to which it is running. When this is provided for, it will be found that the belt will only run in one direction.

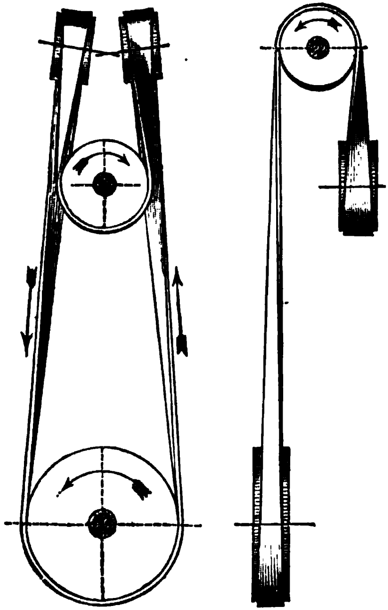


Fig. 224.

Figs. 225 and 226 show two arrangements of belting and guide pulleys for shafts at right angles. If the belts be traced round, it will be found that the rough side of the belt is always next the pulleys. It is to secure this that the belts have a quarter or half twist between the pulleys as shown. Contrary to the usual practice, it appears to be best to run the smooth or hair side of the leather next the pulley.

228. *Rounding of pulley rim.*—When a flat belt is placed on a conical pulley, it tends to climb towards the larger end. If the pulley is made of a double conical form, or still better with a rounded rim a little larger at the centre than at the sides, the flat belt keeps its place on the pulley and has no

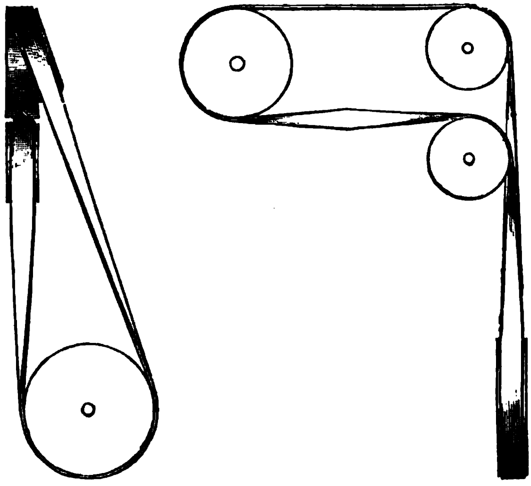


Fig. 225.

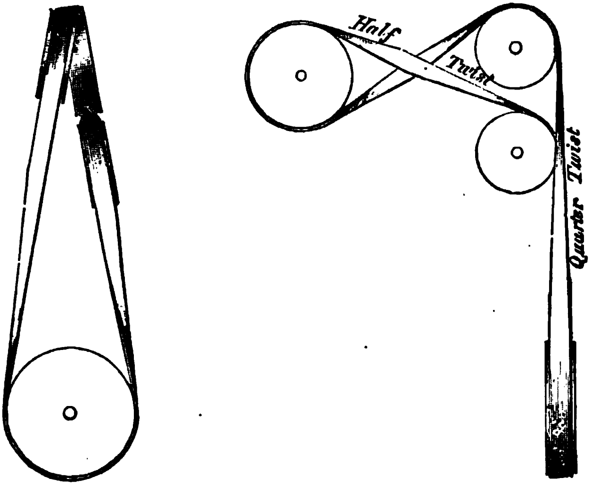


Fig. 226.

tendency to slip off. The rounding of the rim may be $\frac{1}{2}$ inch per foot of width of pulley, or the section of the rim may be struck with a radius equal to from three to five times the width of the rim.

229. *Proportions of pulley. Rim of pulley.*—The pulley rim is a little wider than the belt it is intended to carry. Let B=width of rim, β=width of belt. Then,

$$B=\frac{9}{8}(\beta+0{\cdot}4)$$

$\beta=2$	3	4	5	6	8	10	12
B=2·7	3·82	4·95	6·08	7·2	9·45	10·7	13·95
=$2\frac{3}{4}$	$3\frac{7}{8}$	5	6	$7\frac{1}{4}$	$9\frac{1}{2}$	$11\frac{3}{4}$	14

The form of the rim in section is shown in fig. 227; at the edge the thickness may be

$$t=0{\cdot}7\,\delta+{\cdot}005\,D,$$

where D is the diameter of the pulley and δ the thickness of the belt.

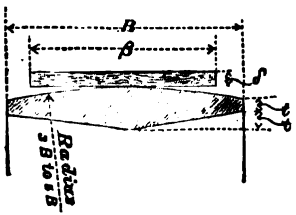


Fig. 227.

The diameter of pulleys should not be less than 6 to 8 times the diameter of a wrought-iron shaft suitable for transmitting the power transferred to the belt, and the diameter of the smaller of two pulleys should not be less than about 18 times the belt thickness.

230. *Centrifugal tension in rim of pulley.*—Pulleys run at high speeds are liable to burst from the tension in the rim. Let w = weight of a bar 1 sq. in. in section and 12 ins. long

($w = 3{\cdot}36$ lbs.). Then the weight of one foot length of a pulley rim of section ω sq. ins. is $w\,\omega$ lbs. If v = velocity in feet per second, and r = radius in feet of pulley rim, then centrifugal force of one foot length of rim $= \dfrac{w\,\omega\,v^2}{g\,r}$ (See p. 29.) Suppose the pulley divided by a diametral plane. Then the resultant centrifugal force of each half of the pulley rim, acting normally to the dividing plane, is

$$\frac{w\,\omega\,v^2}{g\,r} \times 2\,r = \frac{2\,w\,\omega\,v^2}{g}.$$

This force is balanced by the tensions on the two sections of the rim by the diametral plane. Consequently the whole tension in the rim is $\dfrac{w\,\omega\,v^2}{g}$, and the intensity of the stress is $\dfrac{w\,v^2}{g}$ lbs. per sq. in.

v in ft. per sec.	=	70	80	100	150	200
Centrifugal tension in lbs. per sq. in.	=	511	668	1043	2349	4175

The stresses due to the pull of the belt and those due to contraction in casting are additional to these stresses. Hence in practice the speed of pulley rims rarely is allowed to exceed 80 to 100 feet per second.

231. *Arms of the pulley.*—The arms of pulleys are of elliptical or segmental section, as shown in fig. 228. The latter form of section looks lighter than the elliptical section and is preferable. For a segmental arm the thickness $h_2 = \frac{1}{2}h_1$. For an elliptical arm the thickness $h_2 = 0{\cdot}4h_1$. The arms are either straight or curved. The curved arms are rather less liable to fracture from contraction in cooling, but in other respects the straight arms are preferable, being lighter and stronger. The section of the arms is diminished

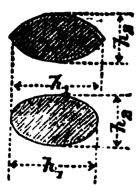


Fig. 228.

from the nave to the rim, so that if we put h_1 h_2 for the breadth and thickness of the arm, supposed produced to the

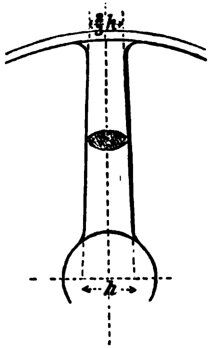


Fig. 229.

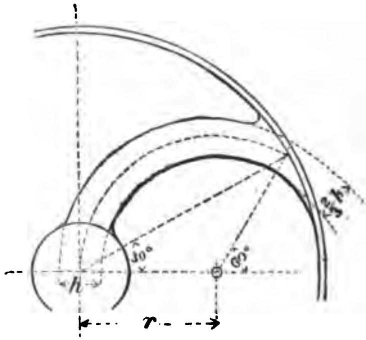


Fig. 230.

centre of the shaft, the breadth and thickness at the rim will be $\frac{2}{3}h_1$ and $\frac{2}{3}h_2$.

Fig. 229 shows an ordinary straight arm, fig. 230 a

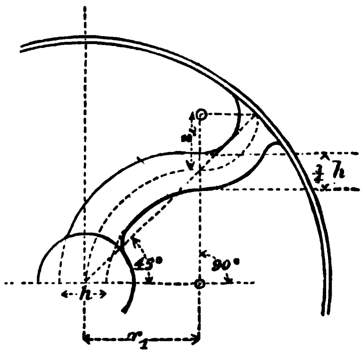


Fig. 231.

curved arm, and fig. 231 an S-shaped or doubly curved arm. The figures indicate sufficiently the way in which the centre line of the arm is drawn. Let R be the radius of the pulley

measured to the inside of the rim. Then in fig. 230, $r = 0{\cdot}577$ R; and in fig. 231, $r_1 = 0{\cdot}471$ R, and $r_2 = 0{\cdot}236$ R.

Let ν be the number of arms, B the breadth, and D the diameter of the rim. Then,

$$\nu = 3 + \frac{\text{B D}}{150}$$

the nearest whole number being taken.

Width of Pulley B	Diameter of pulley in inches when the number of arms is				
	4	5	6	8	10
3	50	100	150	...	...
6	25	50	75	125	175
12	12	24	36	62	87
18	8	16	24	42	58
24	6	12	18	31	44

The number of arms is really arbitrary, and may be altered if necessary. In calculating the strength of the arms it will be assumed that each arm is equally loaded, and also that each arm may be considered to be fixed at the nave and free at the rim. As these assumptions are only in a rough sense true, a large factor of safety must be allowed. Pulley-arms are also liable to be considerably strained by contraction in cooling. Hence a margin of strength must be allowed to meet this contingency. For these reasons the working stress on the cast-iron will be taken at $f = 2250$ lbs. per sq. in.

If P is the driving force transmitted by the belt, determined by eq. 13, and D is the diameter of the pulley, the greatest bending moment on each arm is—

$$\text{M} = \tfrac{1}{2}\,\frac{\text{P D}}{\nu}$$

For an elliptical section of width h (measured at the

centre of the pulley) and thickness 0·4 h, the section modulus (Table IV., § 29) is

$$\frac{\pi}{32} \times h^2 \times 0{\cdot}4\, h = 0{\cdot}0393\, h^3 \text{ nearly,}$$

and for a segmental section of width h and thickness 0·5 h, the modulus may be taken to be the same. Equating the bending moment and moment of resistance

$$\tfrac{1}{2}\,\frac{\text{P D}}{\nu} = 0{\cdot}0393\, f h^3$$

$$h = \sqrt[3]{\left(\frac{1}{{\cdot}0786 f} \cdot \frac{\text{P D}}{\nu}\right)}$$

and putting $f = 2250$

$$h = 0{\cdot}1781 \sqrt[3]{\frac{\text{P D}}{\nu}}. \quad . \quad (20)$$

Since in designing pulleys the driving force P will often be unknown, we may design the arms to resist the maximum driving force which is likely to be transmitted by a belt, the width of which is $\frac{4}{5}$ B. The driving force will be very often half the greatest tension in the belt, and will rarely exceed $\frac{4}{5}$th that tension, except when the belt embraces an unusually large arc. The greatest belt tension may be taken at 70 lbs. per inch width of the belt for single belting, and 140 lbs. for double belting. Hence, P will not exceed 56 and 112 lbs. per inch width of the belt, or 45 and 90 lbs. per inch width of the pulley.

$$\text{P} = 45\ \text{B for single belts.}$$
$$= 90\ \text{B for double belts.}$$

Inserting this value in the equations above,

$$\left.\begin{aligned} h &= 0{\cdot}6337 \sqrt[3]{\frac{\text{B D}}{\nu}} \text{ for single belts.} \\ &= 0{\cdot}798 \sqrt[3]{\frac{\text{B D}}{\nu}} \text{ for double belts.} \end{aligned}\right\} \quad (21)$$

These equations agree well with practice. If the arms are of wrought-iron f may be taken equal to 9000 lbs. per sq. in. If the section of the arms is different, the proper section modulus must be substituted for that assumed above.

232. *The nave of the pulley.*—The thickness of the nave may be

$$\delta = 0{\cdot}14 \sqrt[3]{\text{B D}} + \tfrac{1}{4} \text{ (single belt)}$$

$$= 0{\cdot}18 \sqrt[3]{\text{B D}} + \tfrac{1}{4} \text{ (double belt).}$$

The length of the nave, λ, should not be less than $2\frac{1}{2}\ \delta$, and is often $\frac{2}{3}$ B. The key is to be proportioned by the rules in § 87. When the pulley is to run loose on the shaft the nave should be bushed with brass, and the length of the nave should be equal to B.[1] Provision must also be made for lubrication. In large pulleys the nave may be strengthened by wrought-iron rings shrunk on.

233. *Split pulleys.*—When the pulleys are intended to be fixed on shafts which are bossed at the ends, they are often cast in halves. The two halves can then be bolted together on the shaft without dismounting the shaft and without having recourse to cone keys. Fig. 232 shows a pulley of this kind. The net section of the bolt at the rim should be about a quarter the section of the rim,

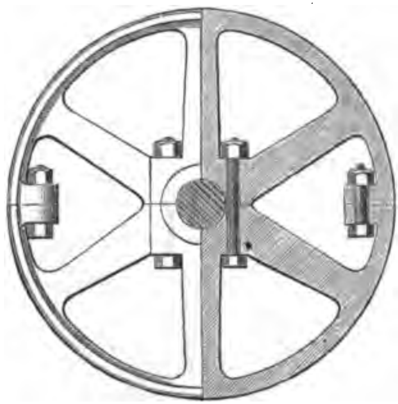


Fig. 232.

[1] Box, 'Millgearing,' p. 75.

plus $\frac{1}{4}$ sq. in. and that of bolt at the nave about $\frac{3}{4}$ sq. in. plus a quarter the section of the nave calculated as above. The two half-pulleys may be made to grasp the shaft so tightly that relative motion is prevented by friction, and no keys are necessary.

If it is undesirable to cast the pulley in halves, the eye of the pulley must be bored out large enough to pass over the bosses at the ends of the shaft and slightly conical. Then three cone keys, described in § 89, are fitted in the space between the pulley-eye and the shaft. Another plan is to use a conical sleeve, split on one side, like that shown in the drawing of the Sellers's coupling, fig. 137; this is drawn into the eye of the pulley by bolts. In either of these plans the pulley is fixed on the shaft by friction only.

Wrought iron pulleys of ordinary size and of exceptionally large dimensions are now made by Messrs. Hudswell, Clark and Rodgers. They are safer at high speeds because they are entirely free from strains due to contraction in cooling, and because if they should break, their toughness would prevent them from flying to pieces like cast iron.

Wrought-iron pulleys are generally made as shown in fig. 233. They are then virtually split pulleys. The split edges of the rim are joined by a lapping piece and screws.

Fig. 233.

Weight of pulleys in lbs. per inch of width.—The diameter being d in ft., Mr. D. K. Clarke gives

$$w = 7{\cdot}6\,d - 1{\cdot}5 \text{ to } 12\,d - 9{\cdot}5 \text{ for rough castings}$$
$$= 7\,d - 1{\cdot}75 \text{ to } 11{\cdot}6\,d - 9{\cdot}25 \text{ for finished pulleys.}$$

234. *Management of belting.*—In fixing, repairing, or splicing a belt, it must be thrown off the pulley, and it then rests chiefly on the upper of the two connected shafts. If the shaft on which it rests is the driven shaft, no great danger is incurred; but if the belt rests on the driving shaft there is danger of the belt getting entwined round the shaft and so causing injury to the machinery and perhaps to the workman.[1] The danger is greater the more flexible the belt, and depends to some extent on the direction of motion of the shaft. The danger is greatest when the lower side is the tight side, and the slack side is liable to rest on it, as in fig. 234. The shaft may then grip the belt and roll up the two sides together. In fig. 235, the arrow is placed on the driving pulley. A and C are comparatively safe, B and D dangerous arrangements.

Fig. 234.

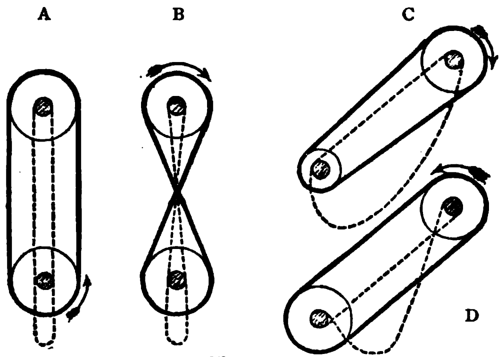


Fig. 235.

235. *Belt perch.*—The simplest way of preventing the entwining of the belt is to fix a light belt perch over the shaft on which the belt rests when unshipped. Fig. 236 shows a

[1] Thwaite, 'Factories and Workshops.'

simple perch of this kind. Where a pulley is placed close to a hanger it is desirable to fix a light guard to prevent the belt falling between the pulley and hanger, and it should be

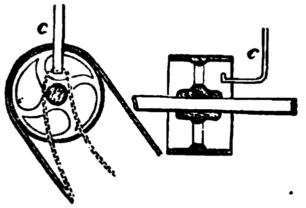


Fig. 236.

placed near the point where the belt advances towards the pulley in running.

Cotton belting can now be obtained, made of 4 to 10 thicknesses of American cotton duck stitched together. It is waterproof, and cheaper and stronger than leather. The ordinary widths are, for 4-ply, $1\frac{1}{2}$ to 6 ins.; for 6-ply, 3 to 12 ins.; for 8-ply, 6 to 30 ins.; and for 10-ply, 12 to 60 ins. According to a test made for the manufacturers, 8-ply cotton belting is twice as strong as double leather belting, the breaking stress being 1,135 lbs. per inch of width. The best way of making the joints is by butting the ends of the belts and using a special metal fastening. A test of this gave a breaking strength of 330 to 540 lbs. per inch of width (thickness of belt not stated). Ordinarily 4-ply is taken as equivalent to single leather, and 8-ply to double leather belting.

CHAPTER XI.

ROPE GEARING.

236. At the present time, ordinary hemp ropes are being used to replace leather belting and toothed gearing in the transmission of power. For special purposes, similar ropes, made of cotton, are also used. The pulleys for belts of this kind are made with V-grooves round their circumference, each groove having its own rope-belt. When only a small amount of power is to be transmitted, the rope rests on the bottom of the groove, but in most cases the rope rests against the sides of the groove, and is wedged between them, so that the frictional resistance to slipping is very great. The ropes most commonly used are patent ropes of three strands (fig. 237), white or untarred, and from 1 to 2 inches diameter. They are placed on the pulleys with very little initial tension, and the joint is made by splicing the rope. The pressure of the rope on the pulley is chiefly due to its weight. Hence, to secure sufficient frictional adhesion the pulleys should be large, and at a sufficient horizontal distance apart. If the pulleys are vertically over each other the rope must be strained more tightly, and its durability is impaired. Usually the horizontal distance between the pulleys is 20 to 60 feet, whatever their vertical distance may be. The ropes are never strained so tightly as to draw them nearly straight. They hang between the pulleys in catenary curves which approximate to parabolas. It is

Fig. 237.

advisable to have the driving side of the rope on the lower side of the pulleys and the slack side above. Then, in driving, the two sides approach each other, and the arc of contact on the pulleys is increased. The slacker the ropes are, consistently with obtaining sufficient frictional resistance to slipping at the pulleys, the better, because the ropes are less squeezed in the grooves and wear longer.

237. *Strength of ropes.*—The breaking strength of white or untarred rope varies from 7,000 to 12,000 lbs. per sq. in., and is to some extent dependent on the amount of twist given to the rope. The twist diminishes the strength of the rope, but makes it more solid and durable. The working strength may be taken at about $\frac{1}{8}$th of the breaking strength. Hence, the working strength is $f=875$ to 1500 lbs. per sq. in. In the following calculations it is assumed that $f=1200$ lbs. per sq. in.

Let δ be the diameter; γ, the girth; and G, the weight per lineal foot of the rope. The section of hawser-laid rope is about $\frac{9}{10}$ths of the area of the circumscribing circle. Hence,

$$\left.\begin{array}{l}\text{Area of section} = 0{\cdot}9 \times \frac{\pi}{4} \times \delta^2 = 0{\cdot}707\ \delta^2 = 0{\cdot}0716\ \gamma^2 \\ \text{Working strength} = 0{\cdot}707 f\ \delta^2 = 850\ \delta^2 = 86\ \gamma^2\end{array}\right\} \quad (1)$$

When the rope is wet or tarred, the strength is reduced by about one-fourth.

The weight of ropes, G, in lbs. per foot of length, is given by the following equations:[1]—

$$\left.\begin{array}{l}G = 0{\cdot}2812\ \delta^2 = 0{\cdot}0285\ \gamma^2 \ \text{ dry} \\ \quad = 0{\cdot}3376\ \delta^2 = 0{\cdot}0342\ \gamma^2 \ \text{ wet or tarred}\end{array}\right\} \quad . \quad (2)$$

Hence, for dry ropes, the weight of 3016 feet of rope is equal to the working strength.

[1] Karl Von Ott, 'Proc. Inst. of Civil Engineers,' xlv. p. 270.

238. *Ordinary driving force of rope belts.*—In order to ensure durability, the tension in the belt when at work is only a small fraction of the working strength. From data furnished by Messrs. Pearce Brothers, of Dundee, who have erected rope belting extensively, it appears that the difference of tension on the two sides of the belt, or driving force is:[1]—

$$T_2 - T_1 = P = 7\cdot 81\ \gamma^2 \text{ lbs.} \qquad . \qquad . \quad (3)$$

It will be shown presently that when the belt embraces 0·4 of the circumference of the smaller pulley, the greatest tension is $\quad T_2 = 1\cdot 208\ P = 9\cdot 43\ \gamma^2.$

Hence the greatest tension is less than $\frac{1}{9}$th of the working strength of the rope.

Table of Weight, Strength, and Driving Force of Rope Belts.

Girth of Rope in ins. γ	Diameter of rope in ins. δ	Weight per foot in lbs. G	Working strength in lbs.	Driving force in lbs.		
				P	K	K_1
$3\frac{1}{8}$	1	·279	842	$76\frac{1}{2}$	0·14	·00061
$4\frac{1}{4}$	$1\frac{5}{16}$	·513	1548	140	0·25	·00110
$4\frac{3}{4}$	$1\frac{1}{2}$	·643	1940	176	0·32	·00140
$5\frac{1}{4}$	$1\frac{11}{16}$	·862	2602	236	0·43	·00188
$6\frac{1}{2}$	$2\frac{1}{10}$	1·204	3633	330	0·60	·00262

239. *Work transmitted by rope belts.*—Since the power which any given rope will transmit is limited, and it is not convenient to use very large ropes, it is necessary in most cases to use several ropes. The pulleys have parallel grooves in which the ropes are placed, sometimes to the number of 20 or 25. Let n be the number of ropes on a pulley; V, the velocity of the rope in feet per second; d, the diameter of the pulley in inches; N, the number of revolutions of the pulley per minute. Then the work transmitted by each rope is

$$P\,V \text{ foot lbs. per second.}$$

[1] In different cases in practice, $P = 6\gamma^2$ to $8\gamma^2$.

Let H be the number of horses' power transmitted,

$$H = \frac{n\,P\,V}{550} = K\,n\,V \quad . \quad . \quad . \quad (4)$$

Also since $$V = \frac{\pi\,d\,N}{12 \times 60}$$

$$H = \frac{n\,P\,d\,N}{126100} = K_1\,d\,N\,n \quad . \quad (5)$$

where K and K_1 are constants, the values of which are given in the table above.

240. *Friction of rope belting.*—The coefficient of friction for a rope on a metal pulley is $\mu = 0{\cdot}28$. In rope transmission, however, the rope is wedged in the groove of the pulley, and the normal pressure between the rope and the sides of the groove is greater than the force pressing the rope into the groove, in the ratio of cosec. $\frac{\theta}{2} : 1$, where θ is the inclination of the sides of the groove. Hence, the resistance to slipping is the same as on a cylindrical pulley having a coefficient of friction,[1]

$$\mu = 0{\cdot}28 \text{ cosec. } \frac{\theta}{2}$$

In practice $\theta = 45°$, and then $\mu = 0{\cdot}7$.

From the equations in the previous chapter—

$$\left.\begin{array}{ll} \text{Greatest tension} & = T_2 = P\,x \\ \text{Tension on slack side} & = T_1 = P\,y \\ \text{Initial tension} & = \tfrac{1}{2}(T_1 + T_2) = P\,z \end{array}\right\} \quad . \quad (6)$$

And using the above value of μ :—

[1] Some recent experiments by Messrs. Pearce give $\mu = 0{\cdot}57$ to $0{\cdot}88$ for ropes on ungreased grooved pulleys, and $\mu = 0{\cdot}38$ to $0{\cdot}41$, when the pulleys were greased. The former values agree fairly with $\mu = 0{\cdot}7$ assumed above.

Fraction of Circumference of Pulley embraced by Rope.	0·1	0·2	0·3	0·4	0·5	0·6
$\frac{T_2}{T_1} =$	1·55	2·41	3·74	5·81	9·02	14·00
$x =$	2·82	1·71	1·73	1·21	1·13	1·08
$y =$	1·82	·71	·37	·21	·13	·08
$z =$	2·32	1·21	·87	·71	·63	·58

241. *Pulleys for rope belting.*—The pulleys are usually of cast iron, and when motion is taken from a steam-engine grooves are turned in the rim of the fly-wheel. The diameter of the smallest pulley should not be less than thirty times the diameter of the rope carried by it. The larger the pulley the less is the injury done to the rope by bending and unbending.

Fig. 174 shows the form of the grooves in the pulley-rim and the proportions adopted. The unit for the proportional

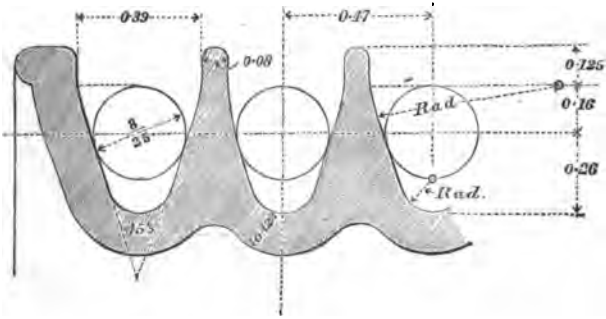


Fig. 238.

figures is γ, the girth of the rope. If the pulley is a guide-pulley merely, the rope should rest on the bottom of the groove. The sides of the groove are usually inclined at 45°. The pulleys are cast in one piece, when they are less than 8 feet diameter, unless they have to be fixed on shafts which

have bosses at the ends, or require to be fixed after the shafts are in position. When this is the case they are cast in halves, and they are also usually cast in halves when they are from 8 to 12 feet diameter. Larger pulleys are cast in segments and bolted together. The grooves in each pulley must be accurately turned to the same gauge, and of the same diameter. The splices in the rope should be 9 or 10 feet long.

Very great care must be taken to have the pulley grooves of the same form and the ropes of the same diameter. If these conditions are not secured, the ropes will be virtually running on pulleys of different diameters, and some of the ropes will be severely strained. All the ropes intended to work on a given pair of pulleys should be put on at the same time. They then stretch and decrease in diameter equally. The life of a rope is said to be from three to five years in ordinary cases. The velocity of rope belting is generally 3,000 ft. per minute or more.

The amount of power which may be transmitted by rope gearing is very great when the conditions are suitable. Thus, suppose the circumferential velocity of an engine fly-wheel is 5000 ft. per minute. Then 20 ropes of $6\frac{1}{2}$ inches girth placed on the fly-wheel would transmit 1000 indicated horses' power. The breadth of the fly-wheel rim in that case would need to be 5 feet. If the speed of the rim were reduced to 4000 feet, 25 ropes would be required. If $5\frac{1}{4}$-inch ropes were used, 20 ropes would transmit 666 horses' power at 5000 feet per minute.

Wire Rope Gearing.

242. Wire ropes have been occasionally used for the transmission of a greater amount of power than would be possible with a weaker material. Sometimes they have been used for direct haulage, and more rarely they have been used like ordinary belts to connect rotating pieces. In the latter case they often gave trouble from the fracture

of the whole rope or of individual wires. It may now be asserted that the proper conditions of using wire ropes in those cases were not fulfilled. During the last twenty years a method of wire rope transmission has been in use, on the Continent chiefly, which is perfectly successful, which combines economy of first cost with economy in maintenance, and by which large amounts of motive power can be transferred to great distances, with an efficiency impossible with any other mode of transmission. This mode of transmission was matured by M. G. A. Hirn, and received from him the name of telodynamic transmission.

In belt transmission, we may increase the amount of power transmitted in three ways: by increasing the frictional bite of the pulleys, by increasing the strength of the belt, and by increasing the velocity of the belt. The first principle is applied in ordinary rope transmission, by wedging the ropes in V-shaped grooves in the rim of the pulley. With wire ropes these wedge-grooves cannot safely be adopted, because of the injury done to the rope. On the other hand, wire ropes are enormously stronger than hemp ropes, and if in addition they are run at the highest practicable velocity, a very great amount of power can be transmitted, with comparatively light gearing. The principle of telodynamic transmission is, therefore, to use flexible belts of very great strength on ordinary pulleys, and to work them at very high velocities. Various expedients are necessary in the application of this principle and in securing the greatest possible efficiency, or the least waste of work in friction.

The pulleys are of large diameter, which tends to the preservation of the ropes by diminishing the bending action, and reduces the influence of the stiffness of the ropes and the loss of work in journal friction. If the distance to which power is transmitted is very great, the transmissions are divided into relays with a separate rope for each. The relays are separated by stations. Each station is provided with

a horizontal shaft upon which a double grooved pulley is fixed, which is the driven pulley as regards the relay terminating there, and the driving pulley as regards the succeeding relay. The stations are usually arranged on masonry pillars, more or less raised according to the configuration of the ground, for it is necessary that the rope should not touch the ground. Sometimes the power has to be partially distributed in its course; under these circumstances the shafts at the stations are made use of for the purpose. Frequently also intermediate pulleys are placed along a relay serving merely to support the rope. Occasionally a relay has been made 650 feet in length. Usually the length is 400 to 500 feet.

The system has proved so successful that power is now frequently transmitted to very great distances with comparatively little loss. That loss is estimated at only $2\frac{1}{2}$ per cent. + 1 per cent. in addition, for every 1,000 yards of distance. The method is not suitable when the distance to which the power is to be transmitted is short, and 130 feet has been fixed as the minimum distance for which transmission by wire rope is applicable.[1] At less distances the wire rope is subject to considerable oscillations, which, however, it is possible may be prevented.

243. *Form, strength, and weight of wire ropes.*—The rope used consists of six or more strands wound upon a hemp core. Each strand consists of six or more wires also twisted round a hemp core. The strands are wound in the opposite direction to the wires in each strand. Fig. 239 shows the section of a rope, the shaded circles being sections of the wires, and the unshaded portions hemp. The angles of twist are usually 8° to 15° for the strands, and 10° to 25° for the rope. The wire diameter varies usually from $\frac{1}{50}$ to $\frac{1}{12}$ of an inch.

The ropes most commonly used have six strands, each

[1] Vigreux, 'Proc. Inst. of Civil Engineers,' xlv. p. 266.

containing six wires and a hemp strand at the centre. For these ropes with 36 wires the diameter of the rope is nearly $9\frac{1}{4}$ times the diameter of the single wires. Ropes of 42 wires are used with the middle hemp core replaced by a strand of six wires, and their diameter is about $10\frac{1}{2}$ times the diameter of a single wire.[1] The number of strands and of wires in each strand is, however, arbitrary, and ropes of 8 strands, each of 10 wires, of 10 strands, each of 9 wires, and various other proportions, are adopted. The relation between the diameter of the rope Δ, the diameter of the wires δ, and the number of wires ν, is given very approximately by the formula

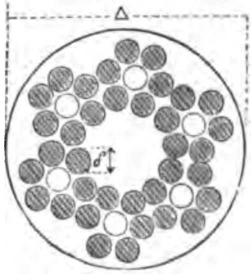


Fig. 239.

$$\frac{\Delta}{\delta} = \frac{\nu}{13} + 7 \quad . \quad . \quad . \quad . \quad (7)$$

The breaking strength of iron wire varies from 85,000 to 108,000 lbs. per sq. in., and the greatest working stress has been fixed at 25,600 lbs. per sq. in. Steel wire has a greater tenacity, and might be assumed to be capable of bearing a still higher working stress. At first steel wire ropes did not answer so well as ropes of iron wire. But according to M. Naville steel wire ropes are now preferred to those of iron. They are, however, worked only up to the same limiting stress, and in such conditions they last longer than iron. A rope running night and day lasts about 200 to 250 days, if of iron; and 250 to 300 days, if of steel. Moreover the steel ropes stretch less in working.[2] An iron rope

[1] According to Stahl, the ropes used in America consist of six strands having seven wires in each strand and no hemp core in the separate strands. The strands are wound round a central hemp core. In these ropes $\Delta = 9\delta$ and $G = 1\cdot43\ \Delta^2$.

[2] Achard, 'Transmission of Power,' Proc. Inst. of Mech. Eng., January 1881.

requires tightening once in 60 days; a steel rope only once in 120 days.

The weight of wire rope per lineal foot is very nearly

$$=G=3\cdot268\,\nu\,\delta^2=1\cdot341\;\Delta^2 \text{ lbs.} \quad . \quad . \quad (7a)$$

Splicing wire ropes.—The following directions are abbreviated from those given by Messrs. Roebling.[1] Overlap the ropes for a distance of 20 feet. Unlay the strands for a length of 10 feet of each rope, and cut away for that distance the central hemp core. Now let *a*, *b*, *c*, ... be the strands of one rope taken in order, and *a′*, *b′*, *c′* . . . . those of the other. Unlay *a* for a further distance of 10 feet, and lay into the spiral groove so formed *a′*, and cut off *a* and *a′* so as to leave two short ends about 6 inches long. Next unlay about 10 feet of *d′*, and lay in the corresponding strand *d*, cutting off as before. Proceed by unlaying *b* and laying in *b′*, and by unlaying *e′* and laying in *e*, stopping about 4 feet short of the previous cut ends. Lastly, unlay *c* and lay in *c′*, and unlay *f′* and lay in *f*, again stopping 4 feet short of the ends of the previously cut strands. To dispose of the cut ends, nip the rope about 6 inches on each side of the ends. Insert a stick and untwist the rope, cut out 6 inches of the hemp core, and force in the cut end into its place. Close the rope and hammer it even with a wooden mallet.

244. *Stresses in a wire rope belt.*—Used as a belt the wire rope is subjected to three different straining actions. (1) There is the longitudinal tension, due to the tightness with which the belt is strained over the pulleys, to the weight of the rope, and to the power transmitted. (2) There are stresses of tension and compression in the part of the belt which at any moment is bent to the curve of the pulley due to the bending. (3) There is a stress due to the centrifugal action of the part of the belt which is being bent. This last stress, though not insignificant, is sometimes left out of consideration.

Let f_t be the greatest working stress due to the longi-

[1] Stahl, 'Transmission of Power by Wire Ropes.'

tudinal tension of the belt, and f_b the stress due to bending. For those wires which lie on the stretched side of the belt in passing over the pulley, the total stress is

$$f=f_t+f_b \quad . \quad . \quad . \quad . \quad (8)$$

When a cylinder of diameter δ is bent to a radius R, the bending moment at any point is [1]

$$M=\frac{E\,I}{R}=\frac{E\,Z\,\delta}{2\,R} \quad . \quad . \quad . \quad . \quad (9)$$

where I is the moment of inertia, and Z the modulus of the section of the rope.

The moment of resistance to bending of a circular section of diameter δ (§ 28) is,

$$f_b\,Z.$$

Equating these values,

$$f_b=\frac{E\,\delta}{2\,R} \quad . \quad . \quad . \quad . \quad (10)$$

If T is the total longitudinal tension in a rope having ν wires, each of δ inches diameter,

$$f_t=\frac{T}{\frac{\pi}{4}\,\delta^2\nu} \quad . \quad . \quad . \quad . \quad (11)$$

Hence the total stress in the most strained wires is

$$f=\frac{E\,\delta}{2\,R}+\frac{T}{\frac{\pi}{4}\delta^2\nu};$$

hence

$$T=\left(f-\frac{\delta\,E}{2\,R}\right)\frac{\pi}{4}\,\delta^2\nu \quad . \quad (12)$$

For a given value of the limiting stress f, T will be a maximum for pulleys of a given radius, when δ is so chosen that

$$\frac{d\,T}{d\,\delta}=o,$$

or when

$$\frac{R}{\delta}=\frac{3\,E}{4\,f} \quad . \quad . \quad . \quad . \quad (13)$$

[1] Compare equation, p. 42, and the values of Z given at p. 46.

Putting $f=25{,}600$, and $E=29{,}000{,}000$ for wrought iron,

$$\frac{R}{\delta}=850.$$

That is, the longitudinal tension will be a maximum when the diameter of the wires is $\frac{1}{850}$th of the pulley radius.

When the ratio $\frac{R}{\delta}$ varies from these proportions, we have for the greatest safe working stress due to the longitudinal tension,

$$f_t=f-f_b=f-\frac{\delta E}{2R} \quad . \quad . \quad . \quad . \quad (14)$$

The deflection, when the rope is not working, should not be less than 18 inches. Sometimes when the pulleys are near together, the deflection of the rope will be too small with this tension. If this is the case, a lower value of the working tension should be adopted.

Direct Stress, and Stress due to bending in Wire Rope Belts of Iron or Steel.

Ratio $\frac{R}{\delta}$	Bending stress f_b	Longitudinal stress f_t	Total stress f
650	22,310	3,290	25,600
700	20,710	4,890	
750	19,330	6,270	
800	18,120	7,480	
850	17,060	8,540	
900	16,120	9,480	
950	15,270	10,330	
1,000	14,500	11,100	
1,100	13,180	12,420	
1,200	12,090	13,510	
1,350	10,740	14,860	
1,400	10,360	15,240	

The proportions most commonly adopted are those corresponding to $\frac{R}{\delta}=1350$, so that the working longitudinal

stress is only 14,860 lbs. per sq. in. The reason of this apparently low working stress will be obvious, if the bending action is considered.

245. *Total longitudinal tension of rope.*—Let T be the greatest tension in any part of the rope, exclusive of the bending stress. Then since the section of iron in the rope is $\frac{\pi}{4}\,\nu\,\delta^2$,

$$T = \frac{\pi}{4}\,\nu\,\delta^2 f_t.$$

Hence the size of wire for a given total tension is

$$\delta = \sqrt{\frac{4}{\pi f_t}}\;\sqrt{\frac{T}{\nu}} \quad . \quad . \quad . \quad (15)$$

$f_t=$	8000	9000	10000	12000	14000	16000
$\sqrt{\frac{4}{\pi f_t}}=$	·01262	·01190	·01128	·01030	·00954	·00892.

It has been stated already that δ is usually between $\frac{1}{10}$th and $\frac{1}{12}$th of an inch.

246. *Tension due to centrifugal force.*—The tension due to centrifugal force in a rope weighing G lbs. per foot, and travelling at v feet per second, is

$$C = G\frac{v^2}{g}.$$

Inserting the value found in eq. (7*a*) for G,

$$C = 3{\cdot}268\,\nu\,\delta^2\frac{v^2}{g} \quad . \quad . \quad . \quad (16)$$

and dividing this by the section of the rope, the intensity of centrifugal tension is—

$$c = 0{\cdot}1293\ v^2 \text{ lbs. per sq. in.} \quad . \quad . \quad (16a)$$

$v=$	50	60	70	80	90	100
$c=$	323	466	634	828	1047	1293

These stresses must be deducted from the stresses in the

table, § 244, p. 348, in order to find the safe tension due to the power transmitted.

247. *Driving force of belt, and power transmitted.*—The equations for the friction of a belt on a pulley given in Chapter X. are equally applicable for an iron wire rope, if proper values are taken for the coefficient of friction.

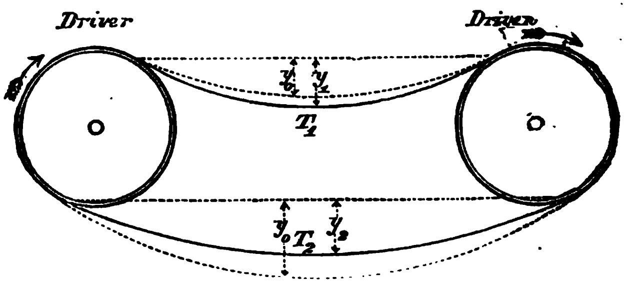


Fig. 240.

Taking $\mu=0{\cdot}24$, and supposing that the belt embraces nearly a semicircle of the pulley, so that $\theta=3$,

$$\epsilon^{\mu\,\theta}=2 \text{ nearly.}$$

The ratio of the tensions on the tight and slack sides of the belt due to the resistance to slipping is,

$$\frac{T_2}{T_1}=\epsilon^{\mu\,\theta}=2 \quad . \quad . \quad . \quad . \quad (17)$$

The driving force of the belt is the difference of the tensions, that is,

$$P=T_2-T_1$$

$$\left.\begin{array}{l} \therefore T_2=2\,P \\ T_1=P \end{array}\right\} \quad . \quad . \quad . \quad (18)$$

The work transmitted in foot pounds per second is P v, and if H is the horse-power transmitted,

$$P=\frac{550\ H}{v} \quad \text{or} \quad H=\frac{P\,v}{550} \quad . \quad . \quad (19)$$

We may put equation 15 in the form for calculating the size of rope from the driving force, instead of from the total tension. The tension, apart from the bending stress, must not exceed $f_t - c$ lbs. per sq. in., and the total tension due to the work transmitted and the initial tension is T_2 or $2P$. Hence,

$$\delta = \sqrt{\frac{2 \times 4}{\pi(f_t - c)}} \sqrt{\frac{P}{v}} \qquad . \quad . \ (20)$$

$f_t - c$	$\sqrt{\frac{8}{\pi(f_t - c)}}$	$f_t - c$	$\sqrt{\frac{8}{\pi(f_t - c)}}$
6000	·0206	10000	·0160
7000	·0191	12000	·0146
8000	·0178	14000	·0135
9000	·0168	16000	·0126

248. *Tightened belt.*—In some cases the diameter of the rope calculated by this rule will prove to be very small. Then it may be convenient to adopt a larger rope than is absolutely necessary. If this is done, either the size of the pulleys may be reduced, if desirable, the rope being capable of bearing a greater bending stress, or the tension in the rope may be increased beyond what is necessary to prevent slipping at the pulleys, with a view of reducing the deflection of the rope between the pulleys.[1] In this latter case, the tension T_2 may be calculated from the size of rope adopted; then T_1 is $T_2 - P$, and from these tensions the curves of the rope may be determined.

249. *Weight of ropes.*—The weight of wire ropes per lineal foot may be taken to be—[2]

$$G = 3{\cdot}268\, v\, \delta^2 = 1{\cdot}341\, \Delta^2 \text{ lbs.} \qquad . \quad . \ (21)$$

250. *The catenary curve.*—No important error is likely to

[1] Reuleaux, 'Der Constructeur,' p. 398.

[2] Karl Von Ott, 'Proc. Inst. of Civil Engineers,' xlv. p. 271.

be introduced by considering the rope to be perfectly flexible and of uniform section. In that case, the funicular curve in which the rope hangs between the pulleys is known to be the catenary, and the tensions in the rope are due to a distribution of load, vertical and constant per unit length of arc.

Let fig. 241 show the form of the curve in which the rope hangs, and let O be the lowest point of the curve. Take O for origin of co-ordinates, and let $x = O$ A, and $y =$ A B be the abscissa and ordinate of any point B of the curve. Since

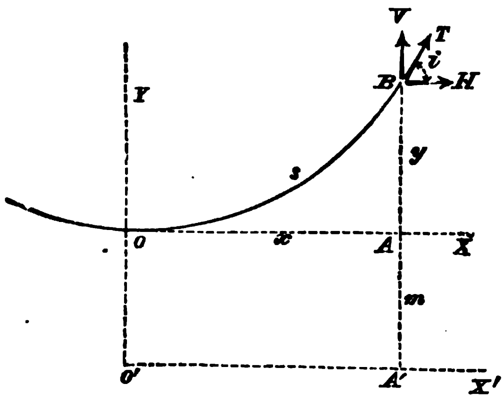


Fig. 241.

the rope is flexible, the tension at any point of the curve must be in the direction of the rope. Let T be the tension at B, and let V and H be its vertical and horizontal components. Let the length of the arc O B$=s$, and let the inclination of the curve at B to the horizontal be denoted by i.

Since G is the weight of a unit length of rope, G s is the weight of O B, and this is equal to the vertical component of the tension at B; hence, G $s =$ V. The other tensions H and T are equivalent to the weight of lengths m and n of rope, m and n being at present undetermined, so that H$=$G m and T$=$G n

The inclination of the rope at B is given by the equations—

$$\left.\begin{aligned}\cos i&=\frac{dx}{ds}\\ \sin i&=\frac{dy}{ds}=\sqrt{1-\frac{dx^2}{ds^2}}\\ \tan i&=\frac{dy}{dx}=\frac{\sqrt{\left(1-\frac{dx^2}{ds^2}\right)}}{\frac{dx}{ds}}\end{aligned}\right\} \quad . \quad (22)$$

$$\text{But } \tan i=\frac{\mathrm{V}}{\mathrm{H}}=\frac{s}{m} \quad . \quad . \quad . \quad (23)$$

Hence,

$$\frac{dx}{ds}=\frac{m}{\sqrt{m^2+s^2}}$$

Integrating, and putting $x=o$, when $s=o$,

$$x=m \text{ hyp. log.}\left\{\frac{s}{m}+\sqrt{\left(1+\frac{s^2}{m^2}\right)}\right\} \quad . \quad (24)$$

That is,

$$\frac{s}{m}+\sqrt{\left(1+\frac{s^2}{m^2}\right)}=e^{\frac{x}{m}}$$

$$s=\frac{m}{2}\left(e^{\frac{x}{m}}-e^{-\frac{x}{m}}\right) \quad . \quad . \quad . \quad . \quad (25)$$

$$\frac{s}{m}=\tan i=\frac{dy}{dx}$$

Inserting the value of $\frac{s}{m}$ in equation 25,

$$\frac{dy}{dx}=\frac{1}{2}\left(e^{\frac{x}{m}}-e^{-\frac{x}{m}}\right)$$

Integrating,

$$y=\frac{m}{2}\left(e^{\frac{x}{m}}+e^{-\frac{x}{m}}\right)+\mathrm{C}$$

The constant is determined by the condition that $y=o$, when $x=o$,

$$\therefore\ y=\frac{m}{2}\left(e^{\frac{x}{m}}+e^{-\frac{x}{m}}-2\right)=\sqrt{s^2+m^2}-m \quad . \quad . \quad (26)$$

This is the equation to the curve termed the catenary, and m is its parameter. For the relation between the tensions at B we have

$$\text{H}=\text{G}\,m \qquad\qquad \text{V}=\text{G}\,s$$

$$\text{T}=\sqrt{(\text{H}^2+\text{V}^2)}=\text{G}\sqrt{(m^2+s^2)}=\frac{\text{G}\,m}{2}\left(e^{\frac{x}{m}}+e^{-\frac{x}{m}}\right)$$

$$=\text{G}\,(y+m) \quad . \quad . \quad . \quad . \quad (27)$$

From this last equation it is seen that the tension, at any point of the rope, is equal to the weight of a length of the rope, $y+m$, equal to the ordinate y of the point added to the parameter m. If $O\,O'=m$ (fig. 241) and $O'\,X'$ is drawn horizontally through O', then the tension at any point B is the weight of a length B A$'$ of the rope.

At the vertex O of the curve the tension is horizontal, and equal to the weight G m of a length $O\,O'$ of the rope. But this is the same as the horizontal component of the tension at B. Hence the horizontal component of the tension at any point is equal to the horizontal tension at the vertex of the curve.

251. *Approximate equations.* — Introducing the value $\tan i=\frac{s}{m}$ in eq. 24, we get[1]

$$x=m \text{ hyp. log. }\left(\frac{\sin i+1}{\cos i}\right)$$

$$\frac{x}{m}=\text{hyp. log. }(1+\sin i)-\tfrac{1}{2}\text{ hyp. log. }(1-\sin^2 i)$$

$$=\tfrac{1}{2}\text{ hyp. log. }\frac{1+\sin i}{1-\sin i} \quad . \quad . \quad . \quad . \quad (28)$$

But hyp. log. $\frac{1+\sin i}{1-\sin i}=2\left(\sin i+\frac{1}{3}\sin^3 i+ \quad . \quad . \quad \right)$

[1] Keller's 'Treibwerke,' p. 201.

Or when i is small, neglecting the terms containing powers higher than the first—

$$\text{Hyp. log. } \frac{1+\sin i}{1-\sin i}=2 \sin i$$

$$\therefore x=m \operatorname{Sin} i \quad . \quad . \quad . \quad (29)$$

Using this value in equations 23 and 24,

$$s=\frac{x}{\cos i}=y \frac{\sin i}{1-\cos i} \quad . \quad . \quad (30)$$

$$y=x \frac{1-\cos i}{\sin i \cos i} \quad . \quad . \quad . \quad . \quad (31)$$

$$y+m=\frac{m}{\cos i} \quad . \quad . \quad . \quad : \quad (32)$$

$$\frac{2x}{y+m}=\sin 2i \quad . \quad . \quad . \quad . \quad (33)$$

These equations enable all problems relating to the form of the rope to be solved.

252. *Case I. Horizontal transmission.*—Let the supporting points of the rope be at the same level, and at a distance l apart, and let the total tension $T=T_2+C=2P+C$ be known.

From equation 27, we get $y+m=T \div G$ at the points of support. Since in this case the curve is symmetrical about its lowest point, $x=\frac{1}{2}l$. Hence at the points of support,

$$\left.\begin{aligned} &\operatorname{Sin} 2i=\frac{2x}{y+m}=\frac{Gl}{T} \\ &\text{The parameter}=m=(y+m)\cos i=\frac{T}{G}\cos i \\ &\text{The deflection}=y=\frac{T}{G}-m \\ &\text{The length of the rope}=2s=2y\frac{\sin i}{1-\cos i} \end{aligned}\right\} \quad . \quad (34)$$

Conversely if the deflection y at the centre is given in place of the greatest tension, and also the half span x,

$$y=\frac{m}{\cos i}-m=\frac{m}{\sqrt{1-\frac{x^2}{m^2}}}-m$$

$$m=\frac{x^2}{4y}+x\sqrt{\frac{x^2}{16\,y^2}+\tfrac{1}{2}}\ \text{nearly}\quad .\ (35)$$

Then the other values can be found as before.

253. *Case II. Inclined transmission. The points of support* B′ B″ *not at the same level.*—This case is best solved by approximation. We may assume the length $s'+s''$ of the rope to be the same as if the points B′ B″ were at the same level and the same distance apart.[1] Let T′, the tension at B′, be given, and also the length B′ B″ $=l$, and the difference of level $y'-y''=h$. Calculate first the length of rope $2\,s$ from the equations above, putting T[1] for T, and assuming the pulleys to be l feet horizontally apart.

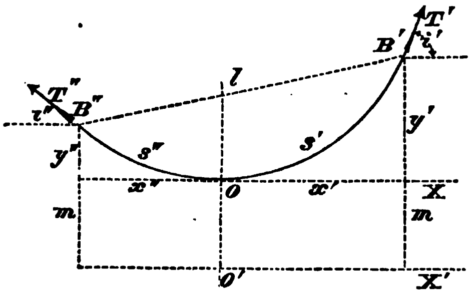


Fig. 242.

Then on the assumption above,

$$s'+s''=2\,s$$

$$y'+m=\frac{\mathrm{T}'}{\mathrm{G}}$$

$$y''+m=\frac{\mathrm{T}'}{\mathrm{G}}-h=\frac{\mathrm{T}''}{\mathrm{G}}$$

[1] Reuleaux, 'Der Constructeur'; Keller, 'Treibwerke.'

By equation 26,

$$(y'+m)^2=m^2+s'^2$$

$$(y''+m)=m^2+s''^2$$

$$s'^2-s''^2=(y'+m)^2-(y''+m)^2$$

$$s'-s''=\frac{(y'+m)^2-(y''+m)^2}{2\,s}$$

Having now obtained $s'+s''$ and $s'-s''$, it is easy to find s' and s''. Let $i'\ i''$ be the inclinations of the ropes at the points of support. The vertical forces at B′ B″ are

$$V'=G\,s' \text{ and } V''=G\,s''$$

$$\text{Sin } i'=\frac{V'}{T'} \text{ and } \sin i'=\frac{V''}{T''} \quad . \quad . \quad (36)$$

The value of T″ being given above. Hence i' and i'' can be found,

$$\left.\begin{aligned} m&=\frac{s'}{\tan i'}=\frac{s''}{\tan i''}\\ x'&=m\sin i'\\ y'&=\frac{m}{\cos i'}-m \end{aligned}\right\} \quad . \quad . \quad (37)$$

From these values of x' and y' the position of the vertex of the curve can be found.

254. *Deflection for which the longitudinal tension is a minimum.*—From the equations $y=\frac{m}{\cos i}$ and $\sin i=\frac{x}{m}$ we get

$$y=\tfrac{1}{2}\,\frac{x^2}{m} \text{ nearly}$$

But

$$T=G\,(y+m)=G\left(y+\frac{x^2}{2y}\right) \text{ nearly.}$$

This will be a minimum for different values of the deflection, when,

$$\frac{d\,\mathrm{T}}{d\,y}=1-\frac{x^2}{2\,y^2}=0$$

or when,

$$y=\frac{x}{\sqrt{2}}=0{\cdot}7\,x$$

The tension increases to infinity for $y=0$ and for $y=\infty$.

255. *Tensions in the sloping wire rope.*—Let $\mathrm{T}'_2\ \mathrm{T}'_1$ be the tensions on the tight and slack sides of the rope at the upper pulley, T''_2 and T''_1 those at the lower pulley,

$$\left.\begin{aligned}\mathrm{T}'_2&=2\,\mathrm{P}+\mathrm{C}\\ \mathrm{T}'_1&=\ \ \mathrm{P}+\mathrm{C}\\ \mathrm{T}''_2&=2\,\mathrm{P}+\mathrm{C}-\mathrm{G}\,h\\ \mathrm{T}''_1&=\ \ \mathrm{P}+\mathrm{C}-\mathrm{G}\,h\end{aligned}\right\} \quad . \quad . \quad (38)$$

where h is the difference of level of the pulleys.

255*a*. *To draw the curve of the rope.*—In drawing the curve of the rope, which is often necessary to determine the space it will occupy, it is sufficiently accurate to substitute for the catenary curve a common parabola. Divide the abscissa O A and the ordinate A B, of any point, into an equal number of equal parts. Join $\mathrm{O}_1\ \mathrm{O}_2\ \mathrm{O}_3$, and from $1'\ 2'\ 3'$ draw verticals. These verticals will intersect the corresponding sloping lines, in points situated in a parabola.

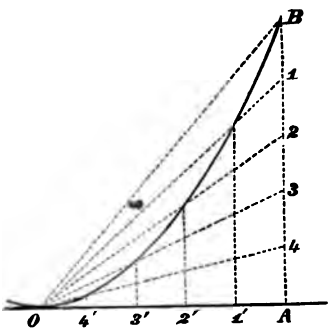


Fig. 243.

256. *Approximate simpler equations.*—M. Achard has pointed out that a parabola may be substituted for the cate-

nary curve in calculating the tensions, without introducing any serious error. Suppose the two pulleys on the same level, then the lowest point of the curve is midway between them, and the tension at that point is

$$H=\frac{G\,l^2}{8\,v} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

G being the weight of the rope per unit of length; l its horizontal projection, which is approximately equal to the distance between the pulleys; and v the versed sine of the curve in which the rope hangs or the deflection of the rope at the centre of the span. From the property of the catenary curve described above, the difference of tension at any two points of the curve is equal to the weight of a portion of rope of length equal to the difference of level of the two points. Consequently the tensions at the ends of the rope will be

$$T=H+G\,v=\frac{G\,l^2}{8\,v}+G\,v \quad . \quad . \quad . \quad (2)$$

Now let T_2 and T_1 be the tensions at the ends of the rope in the driving and slack sides of the belt, and v_2, v_1, the corresponding deflections. From equation 17 we have already $T_2=2\,P$ and $T_1=P$. Putting these values in eq. 2 above, we get for the deflections,—

$$v_2=\frac{P}{G}\pm\sqrt{\left\{\frac{P^2}{G^2}-\frac{l^2}{8}\right\}} \quad . \quad . \quad (3)$$

$$v_1=\frac{P}{2\,G}\pm\sqrt{\left\{\frac{P^2}{4\,G^2}-\frac{l^2}{8}\right\}} \quad . \quad . \quad (4)$$

The deflection common to the two portions of the rope when not transmitting power is

$$v_0=\sqrt{\{\tfrac{1}{2}(v_1{}^2+v_2{}^2)\}} \quad . \quad . \quad . \quad (5)$$

These equations determine the deflections of the span; the driving force and the size of rope are given.

The greatest tension in the rope is

$$H_2+G\,v_2=\frac{G\,l^2}{8\,v_2}+Gv_2. \quad . \quad . \quad (6)$$

Let $f_t - c$ be the greatest permissible working stress, from the table p. 348, and the values of the centrifugal stress on p. 349; then

$$(f_t - c)\frac{\pi}{4}\nu\delta^2 = \frac{G\,l^2}{8\,v_2} + G\,v_2. \quad . \quad . \quad (7)$$

But $G = 3{\cdot}268\,\nu\,\delta^2$, and hence

$$f_t - c = 4{\cdot}16\left(\frac{l^2}{8\,v_2} + v_2\right). \quad . \quad . \quad (7a)$$

This gives $f_t - c$, if l and v_2 are assumed, or conversely determines v_2 in terms of the stress, if $f_t - c$ is assumed. Commonly δ is so chosen that $f_t - c = 14{,}000$ lbs. per sq. in. Then

$$\frac{l^2}{8\,v_2} + v_2 = 3364$$

$$v_2 = 1682 - \sqrt{\left(1682^2 - \frac{l^2}{8}\right)}. \quad . \quad . \quad (8)$$

This gives for

$l =$	420	500	600 feet
$v_2 =$	7	9	14 feet.

257. *Efficiency of wire rope transmission.*—The experiments of M. Ziegler on the transmissive machinery erected at Oberursel give for the efficiency of a single relay

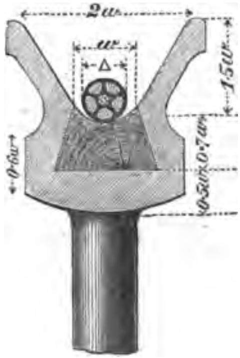
$$\eta_1 = 0{\cdot}962.$$

Hence if there are m intermediate stations, the efficiency is approximately

$$\eta = \eta_1^{\frac{m+2}{2}} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

No. of intermediate stations . .	0	1	2	3	4	5
Efficiency . .	0·962	0·944	0·925	0·908	0·890	0·873
Proportion of work wasted in per cent. . . .	3·8	5·6	7·5	9·2	11·0	12·7

258. *Pulleys for wire rope transmission.*—Wire ropes will not support without injury the lateral crushing which occurs when the rope rests against the sides of V-shaped grooves. Hence, it is necessary to construct the pulleys with grooves so wide, that the rope rests on the rounded bottom of the pulley. It was found by Hirn that the wear of the rope was greatly diminished, and at the same time the frictional resistance to slipping was increased, by lining the bottom of the groove of the pulley with gutta-percha or wood. The gutta-percha is softened and hammered into the groove, which is dovetailed in section. The wood may be inserted in short blocks, through a lateral opening, which is afterwards covered by a metal plate.



$w = \Delta + \frac{1}{8}$

Fig. 244.

More recently, leather has been found to succeed better than either wood or gutta-percha. The leather is cut into pieces the shape of the notch, and placed in it edge upwards. When these pieces are filled in all round, the pulley is placed in the lathe, and the bottom of the groove is turned to the section required. This lagging of leather lasts on the average three years.

Fig. 244 shows the section of a pulley rim. The unit for the proportional figures is $w = \Delta + \frac{1}{8}$, where Δ is the diameter of the rope.

The pulleys are often of cast iron, with cross-shaped arms, which may be calculated in the same way as the arms of toothed wheels. Sometimes they have oval curved arms like those of ordinary pulleys, and sometimes the arms are of round bar iron. These are cut to the right length and tinned at the ends. They are then placed in the sand mould, and the rim and nave cast round them. Such arms are usually

placed sloping in the plane of the axis of the pulley, the slope being alternately in opposite directions. The pulley is thus rendered rigid enough to resist accidental lateral forces.

It has already been proved (eq. 14) that the radius of the pulley must not be less than

$$R = \frac{2(f-f_t)}{\delta E}.$$

Or, when $f=25{,}000$, and the tension f_t, due to the work transmitted and the centrifugal force, does not exceed 8,000 lbs. per sq. in.,

$$R = 900\,\delta \text{ nearly.}$$

The pulleys commonly used are 12 to 15 ft. diameter.

When the distance to which the power is transmitted is great, intermediate guide or supporting pulleys are introduced to lessen the deflection of the rope. The supporting pulleys for the tight side of the belt must be of the same size as the principal pulleys, those for the slack side may be smaller, in the ratio

$$\frac{R'}{R} = \frac{f - \frac{1}{2}f_t + \frac{3}{2}c}{f - f_t}$$

where f is the total stress in the rope, f_t the stress due to the longitudinal tension, including centrifugal force, c the stress due to centrifugal force.

The pulleys are supported on shafts which rest in pedestals on masonry piers or timber trestles.

The weights of the most ordinary sizes of pulleys employed, including their shafts, are on the average as follows (Achard):

Diameter	Weight in lbs	
	Single pulley	Double pulley
18 ft. 0 in.	6,232	8,267
14 ,, 9 ,,	5,180	6,988
12 ,, 4 ,,	2,425	4,078
7 ,, 0 ,,	798	1,164

259. *Velocity of the rope.*—The rope is run at the highest safe velocity. That velocity is determined by the liability of the pulleys to burst, under the action of the centrifugal force, when the speed exceeds a certain limit. Let G be the weight of a cubic foot of cast iron, v the velocity of the pulley rim in feet per second, a its sectional area in square feet, R its radius in feet, and f its tensile strength in lbs. per sq. ft. The tension in the rim due to centrifugal force is,

$$\frac{G a v^2}{g}$$

The resistance of the rim is $f a$. Equating these,

$$v = \sqrt{\frac{g f}{G}}$$

Thus, putting $G=450$ lbs., $f=4500 \times 144$,

$$v=215 \text{ ft. per sec.}$$

The actual speed is never as high as this, a larger margin of safety being necessary. Usually the speed of the rope is from 60 to 100 feet per second.

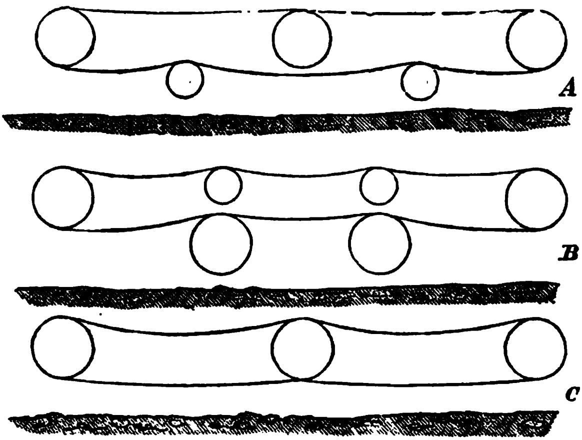


Fig. 245.

Fig. 245 shows three arrangements of a wire-rope transmission.

In A and B guide or supporting pulleys are used. The upper part of the rope is the driving side in A, and the lower part in B; C is the arrangement adopted by Ziegler at Frankfort for transmitting 100 H P a distance of 984 metres.

260. *Duration of ropes.*—The ropes appear to last about a year. To preserve them from oxidation and improve their adhesion, they are coated with a mixture of grease and resin applied hot.

The following table contains data taken from a paper by Achard :[1]—

Wire Rope Transmission.

Locality	Rope			Pulleys		H.P. transmitted	Total distance		Velocity of belt ft. per sec.
	Diam. Δ ins.	Diam. of wires δ in ins.	No. of wires ν	Diam. 2 R in ft.	Distance apart in ft.		Horiz.	Vert.	
Oberursel	0·59	·06	36	12·3	394	94	3,153	146	73·8
Schauffhausen [2]	0·95	·072	80	14·75	333 to 456	326	1,997	...	61·87
Fribourg	0·97	·070	90	14·75	502	300	2,510	269	65

261. *Change of direction of the rope.*—When the direction of a rope requires to be changed either at a right angle or otherwise, two plans may be adopted. A horizontal pulley may be used, in which case the pulley must have the same diameter as the other pulleys used in order that the bending stress may not be increased. More commonly, however, two vertical guide pulleys, in the required directions in plan, are connected by bevil gearing. The splitting of the power

[1] 'Annales des Mines,' viii. p. 229; 'Proc. Inst. of Civil Engineers,' xlv. p. 267.

[2] There are two cables. If one breaks, the other is capable of transmitting the power.

transmitted to different points of application may be effected in the same way.

Vertical rope.—In the case of a vertical rope the initial tension on the lower pulley due to the weight of the rope would vanish, and that on the upper pulley would generally be insufficient unless special devices were used for producing the initial tension of the rope. Then tension or tightening pulleys may be used, like those for leather or rope belts.

Stretching of ropes.—In course of time the ropes stretch, and especially in summer sag so much that they become incapable of transmitting the required power. This may be remedied by resplicing the ropes, but this must be avoided as long as possible because it injures the ropes. Of late a mode of laterally compressing and stretching the ropes before use has been adopted which diminishes the stretching while working. According to Stahl the simplest way of neutralising the stretching which occurs in the working of the ropes, is to increase a little the diameters of the pulleys by nailing wood strips to the material filling the bottom of the groove. Poplar or willow is used in pieces 1¼ inches thick and 45 to 70 inches in length. They are half cut through on one side with saw cuts, and steeped in water for two days to render them flexible. They are then nailed to the groove filling by wrought nails, long enough to pass through it and clinch themselves against the iron.

Cost of wire rope transmission.—The cost is estimated in France at about £330 per mile, exclusive of the terminal stations. These cost about £1 per H. P. transmitted.

Lateral swaying of the ropes.—When a rope transmission is running well there should be little lateral swaying of the ropes, except the unavoidable motion produced by the wind. If swaying occurs it may be due to the pulleys being unbalanced or untrue, or to their not being in the plane of the rope; or it may be due to the pulley filling, or the rope being too much worn, or to bad splicing of the rope.

The following short table, abbreviated from one calcu-

lated by Roebling and given by Stahl, may serve as a general guide to the size of ropes and amount of power transmitted by wire ropes :—

Power Transmitted by Wire Ropes.

Diam. of Rope	Diam. of Pulleys	No. of Revs. per min.	Breaking Strength of Rope	Horses Power Transmitted	Velocity of Belt in ft. per sec.
ins.	ft.		lbs.		
$\frac{7}{16}$	5	100	4260	8·6	26
$\frac{15}{32}$	6	100	5660	13·4	31
$\frac{1}{2}$	7	100	8200	21·1	36
$\frac{5}{8}$	8	100	11600	27·5	42
$\frac{5}{8}$	8	120	11600	33·0	50
$\frac{5}{8}$	9	100	11600	51·9	47
$\frac{5}{8}$	9	120	11600	62·2	56
$\frac{11}{16}$	10	100	15200	73·0	52
$\frac{11}{16}$	10	120	15200	87·6	62
$\frac{11}{16}$	10	140	15200	102·2	73
$\frac{11}{16}$	12	100	15200	116·7	63
$\frac{3}{4}$	12	120	17600	148·9	75
$\frac{3}{4}$	12	140	17600	173·7	87
$\frac{3}{4}$	14	100	17600	185·0	73
$\frac{3}{4}$	14	120	17600	222·0	87
$\frac{3}{4}$	15	120	17600	300·0	94

The ropes have each 42 wires.

CHAPTER XII.

CHAINS.

262. CHAINS are used both as flexible transmitters of energy, like belts, and as simple fastenings. As, however, the two modes of use are more or less connected, it is convenient to treat chains generally without respect to their special application. Chains may be divided into :

Round iron chains, open links.
" " stud links.
Flat bar chains.
Gearing chains.

Round iron chains are those most commonly used, and they are forged out of round iron bars of the best quality. When a tension is applied to such a chain, each link is subjected to a bending action additional to the tension, the bending being greatest at the extremities of the longer diameter of the link. Hence, on purely theoretical grounds the link should be stronger at the ends of the link. On the other hand, it would involve excessive expense to vary the section of the link, and the question of the best theoretical section is complicated by the uncertainty as to the strength at the weld. Welds in bars have been found to be from 10 to 40, or on the average 20, per cent. weaker than the bars themselves. Further chains are constantly used on pulleys and may then be subjected to bending action greatest at the ends of the smaller diameter. In designing a chain the links should be as small as possible, (1) because the

greater the number of links in a given length the more flexible is the chain; (2) because the less the transverse dimensions of the link the less is the bending action. The inside radius at the ends of the link must be a little greater than the radius of the iron of which the chain is made. Let *d* be the diameter of the iron. Then 2·6*d* and 1·5*d* are about the smallest possible internal diameters of the chain link, and 4·6*d* and 3·5*d* the least outside diameters.

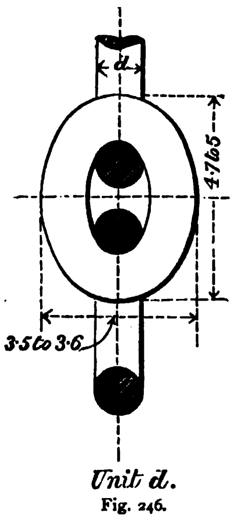


Fig. 246.

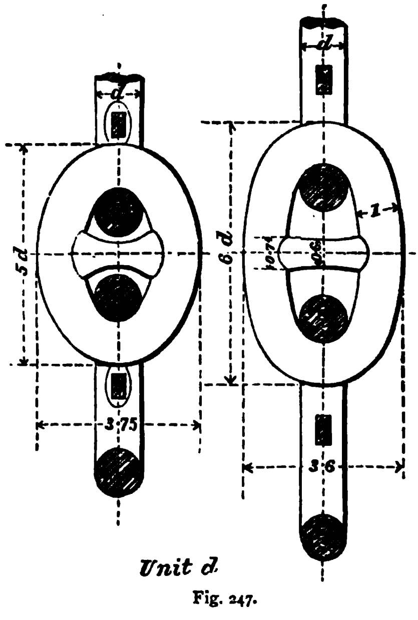


Fig. 247.

263. Very common proportions for ordinary chains are given in fig. 246. Such chain is termed close-link chain. Cheaper but weaker chain is made with longer links. Such chain may have the inside diameters 1·5*d* and 4*d*, and consequently the outside diameters 3·5*d* and 6*d*. In fig. 247, two forms of studded chain are shown. The stud resists the tendency of the link to collapse, and renders the chain less

liable to kink. The proportions of studded chain cables differ a good deal. The following are probably extreme proportions:

Length of link, outside . .	$5d$ to $6\frac{1}{2}d$
" " inside. . .	$3d$ " $4\frac{1}{2}d$
Width of link, outside. . .	$3\frac{1}{2}d$ " $3\frac{3}{4}d$
" " inside . . .	$1\frac{1}{2}d$ " $1\frac{3}{4}d$
Stud, diameter at ends . .	$0{\cdot}7d$ " d
" " centre . .	$0{\cdot}6d$

The end links of a length of cable are usually made of iron of $1{\cdot}2d$ in diameter; they are a little larger than the other links, say $6\frac{1}{2}d$ in length and $4{\cdot}1d$ in width outside. Chain cable is often made in lengths of $12\frac{1}{2}$ or 25 fathoms. The lengths are joined by swivels and shackles.

A convenient method of drawing the elliptical form of chain links is shown in fig. 248. Set off *a c*, *c b*, the semi-diameters. Take the radius *d a* a little greater than $1{\cdot}5d$, and draw the circular curve *a f e* for the end of the link. Draw *d e* parallel to *c b*. Join *b e* and produce it to meet the arc in *f*. Join *f d* and produce it to meet the smaller diameter of the link in *g*. Then *g* will be the centre for the arc *f b*, and *g b* or *g f* will be its radius.

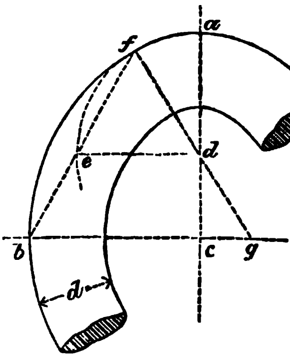


Fig. 248.

264. *Strength of chains.*—The strength of the iron of which chains are made is about 27 tons or 60,000 lbs. per sq. in. Experiments at Woolwich in 1842–3 showed that studded chain cable broke with a mean tension of $15{\cdot}9$ tons per sq. in., and crane chain (two sizes only were tested) with a mean tension of $17\frac{1}{2}$ tons per sq. in. The reduction of

strength of the chain compared with the bar iron of which it is manufactured is partly due to the welds and partly to the bending stress.

The Admiralty rule for the proof stress of studded chain cables is—

$$\text{Test load in tons} = 18d^2,$$

corresponding to a stress of $11\frac{1}{2}$ tons per sq. in. of section. For close-link crane chains without studs,

$$\text{Test load in tons} = 12d^2,$$

corresponding to a stress of 7·7 tons per sq. in. of section.

It is difficult to assign the proper working load for chains because the circumstances in which they are used vary so much. Generally where they are subjected to some vibration and shock, and looking to the fact that, after allowance is made for welding and bending action, their ultimate strength can hardly be taken higher than 16 tons per sq. in. it would seem to be necessary to limit the working stress to 4 tons to the sq. in. Then the working load is

$$P = 2 \times \frac{\pi}{4} d^2 \times 4 = 6{\cdot}28 d^2 \text{ tons.}$$

In some cases, where the maximum load is only occasionally applied and where handiness is of importance, a stress of $5\frac{1}{3}$ tons might be allowed. Then the greatest working load would be

$$P = 2 \times \frac{\pi}{4} d^2 \times 5\tfrac{1}{3} = 8{\cdot}4 d^2 \text{ tons.}$$

The greatest working load is given by manufacturers at half the test load, or $6d^2$ for close-link crane chain and $9d^2$ for studded-link cable.

265. *Weight of chains.*—The weight of chains in lbs. per fathom (of six feet) is :

$$w = 54d^2 \text{ to } 58d^2.$$

The stowage room required for chains is about $35d^2$ cubic feet for each 100 fathoms.

In all the above equations d is to be taken in inches.

Strength and Weight of Close-Link Crane Chains, and Size of Equivalent Hemp Cable.

Diameter of iron *d* in inches	Weight of chain per fathom	Breaking strength in tons	Testing load in tons	Girth of equivalent rope in inches	Weight of rope in lbs. per fathom
1/4	3·5	1·9	0·75	2	1 3/8
5/16	6·0	3·0	1·10	2 1/4	1 1/2
3/8	8·5	4·3	1·6	3 1/4	2 1/2
7/16	11·0	5·9	2·3	4	3 3/4
1/2	14·0	7·7	3·0	4 3/4	5
9/16	18·0	9·7	3·8	5 1/2	7
5/8	24·0	12·0	4·6	6 1/4	8 1/2
11/16	28·0	14·6	5·6	7	10 1/2
3/4	31·5	17·3	6·8	7 1/2	12
13/16	37·0	20·4	7·9	8 1/4	15
7/8	44·0	23·1	9·1	9	17 1/2
15/16	50·0	26·1	10·5	9 1/2	19 1/2
1	56·0	29·3	12·0	10	22
1 1/8	71·0	36·3	15·3	11 1/4	27 3/4
1 1/4	87·5	44·1	18·8	12 1/4	34 1/2
1 3/8	105·8	52·8	22·6	13 3/4	41 1/4
1 1/2	126·0	62·3	27·0	15	49 1/2

Strength and Weight of Studded Link Cable.

Diameter of iron *d* in inches	Weight in lbs. per fathom	Breaking strength in tons	Test load in tons	Girth of equivalent rope in inches	Weight of rope in lbs. per fathom
5/8	24	9·5	7	6 1/2	9
11/16	28	11·4	8 1/2	7 1/2	12
3/4	32	13·5	10 1/2	8	14
7/8	44	20·4	13 3/4	9 1/2	19 1/2
1	58	24·3	18	10 1/2	22 1/2
1 1/8	72	29·5	22 3/4	12	30 1/4
1 1/4	90	38·5	28 1/2	13 1/2	39 1/4
1 3/8	110	48·5	34	15	48 1/4
1 1/2	125	59·5	40 1/2	16	55
1 5/8	145	66·5	47 1/2	17	62
1 3/4	170	74·1	55 5/8	18	68 1/4
1 7/8	195	92·9	63 1/4	20	86
2	230	99·5	72	22	104
2 1/8	256	112	81 1/4	24	124
2 1/4	285	126	91 1/8	26	145

The breaking strength is calculated from the Woolwich experiments. But the values (at least for crane chains) are somewhat doubtful.

266. *Chain pulleys and chain barrels.*—Chain pulleys have a continuous groove, or properly placed depressions, to receive the edges of the links which lie in the plane of rotation, and a broad rim for the alternate links which are at right angles to the plane of rotation. Such pulleys have sometimes rims to prevent the chain surging sideways. The barrels of cranes should have a spiral groove, just wide enough to receive the edges of the links, and so deep that the alternate links lie flat on the cylindrical portion of the barrel. The diameter of a chain barrel should be at least $24d$ to $30d$.

267. *Crane hook.*—A crane hook is most often required to receive a rope sling. The opening of the hook should therefore have a width δ=the diameter of the rope. The inside of the hook should have a broad rounded surface which will not injure the rope; and the section of the metal of the hook should be as deep as convenient in those parts where the bending is greatest. Fig. 249 gives proportions for such a hook. The hook is forged of round bar flattened, as shown in the section on the left, to deepen the section where the bending is greatest. Taking δ as the opening of the hook, or diameter of the rope sling, the unit for the proportional numbers is $1 \cdot 3\ \delta$.

The diameter at the bottom of the screw thread (of external diameter d^1) which supports the hook is $\frac{3}{8}\ \delta$ very nearly. Taking the limiting stress at 3 tons per sq. in., the hook will support a load

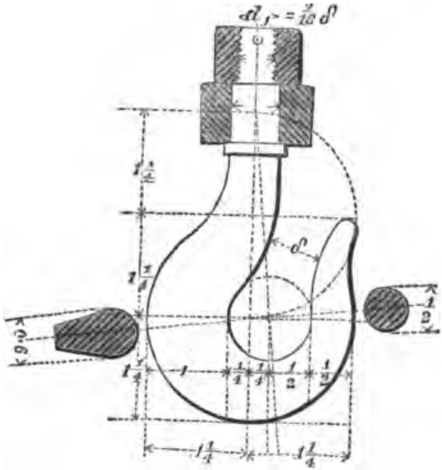
$$P = \frac{\pi}{4} \left(\tfrac{3}{8}\ \delta\right)^2 \times 3 = 0 \cdot 33\ \delta^2 \text{ tons.}$$

If, then, P is given instead of δ, we may design the hook from the value

$$\delta = 1 \cdot 74 \sqrt{P},$$

where δ is in inches and P in tons.

268. *Plate-link chains.*—Chains of this kind are used when a great load is to be supported, as in the case of suspension



Unit = 1·3 δ

Fig. 249.

bridges, patent slips, &c., or as gearing chains when the work transmitted is very heavy, as in the turning gear of

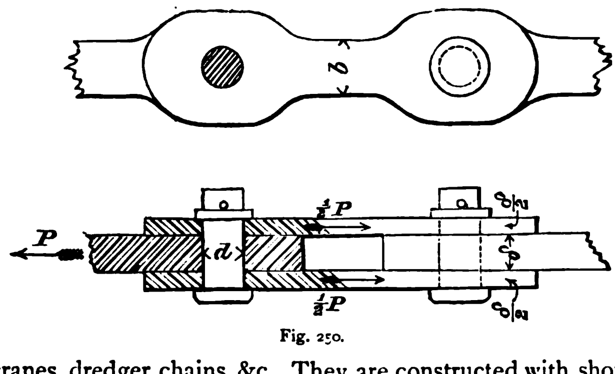


Fig. 250.

cranes, dredger chains, &c. They are constructed with short or long links, according to the amount of flexibility required.

Let d be the diameter of the pin in the eyes of the links. Then the shortest convenient length of link is about $2{\cdot}9\,d$. On the other hand, links are sometimes made 24 feet in length. Fig. 250 shows a simple flat-link chain with one and two links alternately, the double links being half the thickness of the single link.

Let b be the width of the link, δ the thickness of the single link, d the diameter of the pin, P the load on the chain. Then the stress on the link is given by the equation:

$$f_t\, b\, \delta = \text{P} \quad . \quad . \quad . \quad . \quad . \quad (1).$$

The shearing stress on the pin is given by the equation:

$$\frac{\pi}{4} d^2 f_s = \tfrac{1}{2}\text{P} \quad . \quad . \quad . \quad . \quad . \quad (2).$$

And the bending stress on the pin is given by the equation:

$$\tfrac{3}{8}\text{P}\,\delta = f\,\frac{\pi}{32}\,d^3 \quad . \quad . \quad . \quad . \quad (3).$$

Suppose $f_t = 5$ tons per sq. in.; $f_s = 4$ tons per sq. in.; and $f = 5$ tons per sq. in. Then the equations become:

$$b\,\delta = 0{\cdot}2\,\text{P} \quad . \quad . \quad . \quad . \quad . \quad (1\,a)$$
$$d^2 = 0{\cdot}159\,\text{P} \quad . \quad . \quad . \quad . \quad (2\,a)$$
$$d^3 = 0{\cdot}764\,\text{P}\,\delta \quad . \quad . \quad . \quad . \quad (3\,a).$$

Equation $(3\,a)$ will in general give a greater value of d than $(2\,a)$. Equations $1\,a$ and $3\,a$ give

$$d^3 = 3{\cdot}82\, b\, \delta^2.$$

Experiment shows that if d is less than $\tfrac{2}{3}\,b$, the link crushes in the eye and is weakened. Hence if

$$\delta < 0{\cdot}2785\, b,$$

d must be taken not less than $\tfrac{2}{3}\,b$, and its bending resistance will be in excess.

Fig. 251 shows the forms of link eyes found by experiment to be strongest. A is the form arrived at by Mr. G.

Berkley, and B that arrived at by Sir C. Fox. If links are short, they will not generally be made of the most economical form. They should then have a form which includes the shape here indicated.

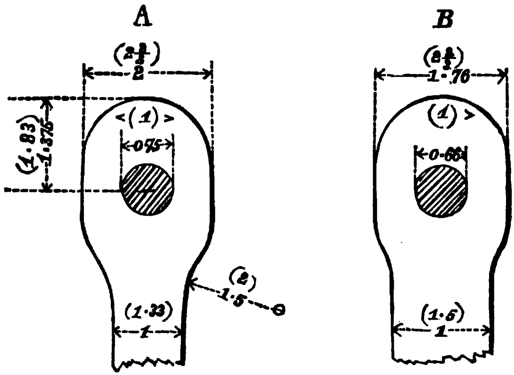


Fig. 251.

Three modes of fastening the pin in flat-link chains are shown in fig. 252. At *a*, the pin is simply riveted over the outside link. At *b*, a washer-plate is interposed, which secures greater freedom of motion in the links. At *c*, a

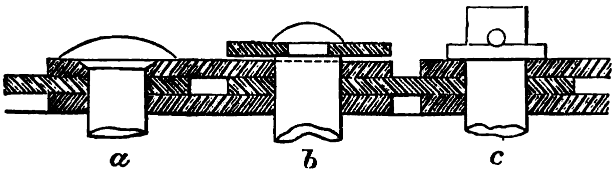


Fig. 252.

washer and split pin are used; and this probably would be the best plan but for the possibility of the split pin falling out when the chain is in use.

Gearing Chains.

269. In cases where a considerable amount of work has to be transmitted between two shafts at a slow speed, the

tension in a flexible transmitter may easily be much greater than ordinary belts can sustain. In such cases metal chains may be used, so formed that the links fit into the projections of toothed wheels on the shafts. There can then be no slipping of the belt on the toothed wheels; and as the chains may have almost any strength, an extremely great force can be exerted through the chain. Such chains are used on a small scale in some forms of reaping machinery, and on a large scale in cranes and lifting gear. The chains carrying the buckets of large dredgers act in the same way. Such chains forming a class of transmitting organs intermediate between belting and gearing, are termed gearing chains or pitch chains. The chief objection to their use is, that however well they fit the toothed wheels at first, they are liable from stretching and wear to become of slightly greater pitch than the toothed wheel, and they then work very badly. To obviate this as far as possible, the links should be short.

The commonest form of such a gearing chain is a flat-linked chain, having two systems of links spaced some distance apart. The toothed wheel acts in the space between the two systems of links, and the teeth gear with the pins passing through the link ends. A mode of construction in some respects better than this is to divide the toothed wheel into two parts, between which the chain is placed. The teeth of the wheel then gear with the alternate link ends on each side. With a chain of this kind, however, a larger toothed pulley is required than for the ordinary form. Long-linked chains are sometimes used on a polygonal pulley without teeth. The polygonal pitch line of such a pulley has sides equal to the lengths of the links between the centres of the pins; and it has usually five or six sides. With four sides the twisting moment is too variable and the motion too irregular.

Fig. 253 shows the ordinary form of a flat-link gearing chain and its toothed pulley. The side views also show

chains with one set of links in each system and with two sets.

270. Let $2\,T$ be the total tension on the loaded span of the chain, and $2\,i$ the whole number of links in the width of the chain. Then the tension in each half of the chain, having i links, is T. Let δ be the thickness, b the breadth of a link, and d the diameter of the pin in the link eye. If

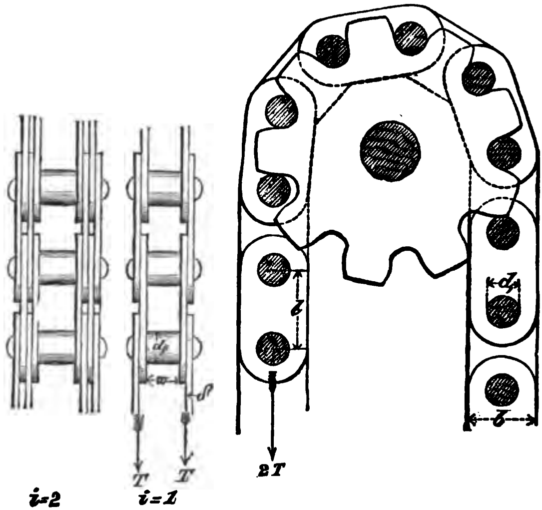


Fig. 253.

$n\,T$ is the greatest tension in any one link, then the stress f_t on a section through the link eye is given by the equation

$$n\,T=(b-d)\,\delta f_t.$$

Usually $b=2{\cdot}5\,d$, and then

$$n\,T=1{\cdot}5\,d\,\delta f_t \quad . \quad . \quad . \quad (1)$$

The average bearing pressure of the pin in the link eye, f_c, is given by the equation

$$n\,T=d\,\delta f_c \quad . \quad . \quad . \quad . \quad (2)$$

The pins are subjected to bending action which will increase with the total stress T and with the thickness δ of the links. Hence, whatever the distribution of the tension T amongst the links, the greatest bending moment on the pin will be

$$M = m\,T\,\delta,$$

and the stress due to bending will be given by the equation

$$M = m\,T\,\delta = \frac{\pi}{32} d^3 f \quad . \quad . \quad . \quad (3)$$

where f is the greatest intensity of bending stress, and m a coefficient depending on the distribution of the tension in the links.

Consider a part of the chain hanging freely between the toothed pulleys. For such a portion of chain the links may be assumed to carry equal portions of the load, and $n = \frac{1}{i}$. Supposing the tension in each link to act at the centre of the link, we get for the greatest bending moment on the pin

$$M = \frac{T\,\delta}{i}\left(\tfrac{1}{2} - \tfrac{3}{2} + \tfrac{5}{2} - \ldots\ldots - 4\frac{i-1}{2}\right)$$

$$= T\,\delta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Hence, equations 1, 3 and 4 give

$$1{\cdot}5\, d\,\delta\, i\, f_t = \frac{\pi}{32}\,\frac{d^3}{\delta} f;$$

and supposing the stress in the link and pin equal, and taken at $f = 10{,}000$ lbs. per sq. in., we get

$$\frac{\delta}{d} = \frac{\cdot 256}{\sqrt{i}}$$

$$d = 0{\cdot}01614\sqrt{\frac{T}{\sqrt{i}}} \quad . \quad . \quad (5)$$

and if $f = 14{,}000$

$$d = 0{\cdot}01365\sqrt{\frac{T}{\sqrt{i}}} \quad . \quad . \quad (5a)$$

Prof. K. Keller [1] has pointed out that the links in gear with the toothed pulley are subjected to a very different straining action. Consider the link just leaving the driven or coming into gear with the driving pulley on the loaded side of the chain, and suppose for the present that the whole tension 2 T in the chain acts through the pin A on the first tooth of the wheel. Then if t_1 t_2 t_3 are the tensions in the links,

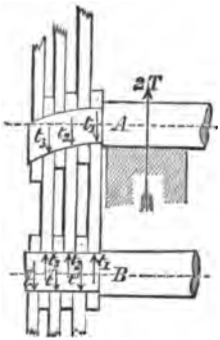


Fig. 254.

$$t_1+t_2+t_3 \quad . \quad . \quad . \quad . \quad =\mathrm{T};$$

under the action of the tensions the pin A will bend, and its flexure will be much greater than that of the second pin B in the free part of the chain, which is subjected to two sets of tensions in opposite directions. Consequently the inside links will be more stretched than the outside links, and $t_1 > t_2 > t_3$ . . . .

Professor Keller has worked out the stresses in the links and the bending action on the pin on the assumptions: (1) That the whole tension comes on the first tooth of the wheel; (2) that the bending of the second pin B may be neglected. If we put Δ_1 Δ_2 Δ_3 for the deflections of the pin at the centres of the links, l for the length of the link when unstrained, ω for the section of a link, λ_1 λ_2 λ_3 for the elongations of the links; then

$$\lambda_1=t_1\frac{l}{\omega\,\mathrm{E}}; \quad \lambda_2=t_2\frac{l}{\omega\,\mathrm{E}} \; . \; . \; . \; .$$

$$\lambda_1+\Delta_1=\lambda_2+\Delta_2=\lambda_3+\Delta_3 \; . \; . \; . \; .$$

equations which, with the known equation for the neutral axis of a bent bar, suffice to determine the tensions t_1 t_2 t_3, and the bending moment on the pin. The calculation is too long to be given here, but it leads to the result that if

[1] Zeitschrift des Oester. Ing. Vereins, 1878.

the links are numerous, the outermost links may happen to be in compression instead of in tension, and thus do not add anything to the strength of the chain, but on the contrary weaken it. In designing the chain the condition may be introduced that the tension in the outermost link should not fall below $\frac{1}{10}$ T. This condition fixes a relation between d and δ. Using this condition Prof. Keller gets:

$$\text{for } i=1 \quad n\,T=T$$

$$\left.\begin{array}{lll} =2 & =\cdot 9\,T \text{ instead of} & \cdot 5\,T \\ =3 & =\cdot 55\,T & \cdot 33\,T \\ =4 & =\cdot 38\,T & \cdot 25\,T \end{array}\right\} \text{for equal distribution of the tension;}$$

and for the greatest bending moment on the pin:

$$\text{for } i=1 \quad M=1\cdot 5\,T\,\delta$$

$$\left.\begin{array}{lll} =2 & 1\cdot 7\,T\,\delta \text{ instead of} & 2\cdot 5\,T\,\delta \\ =3 & 2\cdot 6\,T\,\delta & 3\cdot 5\,T\,\delta \\ =4 & 3\cdot 6\,T\,\delta & 4\cdot 5\,T\,\delta \end{array}\right\} \text{for equal distribution of the tension.}$$

$$\text{Taking } \omega=2\cdot 5\,d\,\delta$$

$$l=2\cdot 9\,d$$

The value required for δ to satisfy the condition above is

$i=$	2	3	4
$\frac{d}{\delta}\,\overline{>}$	2·85	4·90	7·1

and the values of m and n in the equations above are:

$i=$	1	2	3	4
$m=$	1·5	1·7	2·6	3·6
$n=$	1·0	0·9	0·55	0·38

Comparing the results of the application of these rules with experience in the use of large gearing chains, Prof. Keller is led to the conclusion that the whole tension 2 T is not carried by a single tooth of the pulley, but must be distributed to two teeth. He assumes that the one tooth takes $\frac{2}{3}$ and the other tooth takes $\frac{1}{3}$ of the whole tension. If this correction is made $\frac{T}{3}$ must be substituted for T in cal-

culating the bending moment on the pin. The correction, however, only requires to be made if $i=4$ or more. For less values of i the bending moment on the pin is greater in the free part of the chain than on that in gear with the pulley. The following table gives the proportions of gearing chains calculated by Prof. Keller's rules. The general proportions are:

Breadth of link . . . . $b=2\cdot5\,d$
Thickness of middle part of pin . . . $d_1=1\cdot2\,d$
Width of thickened part of pin . . . $w=1\cdot7\,d+0\cdot2$
Width of eye of link between hole and end of link . . . . $=0\cdot85\,d$
Length of link, centre to centre of pins . $=2\cdot9\,d$

Table of Proportions of Flat-Link Gearing Chains (Keller). Dimensions in inches.

Load on chain $2T$ lbs.	No. of links $2i$	Pin diameter at ends d	Pin diameter at centre d_1	Length of middle part of pin w	Link thickness δ	Link breadth b	Link length l	Tooth thickness $a=l-d_1$ $=1\cdot7\,d$
220	2	0·16	0·20	0·43	0·060	·39	·55	0·35
550	2	0·26	0·32	0·67	0·098	·65	·83	0·51
1100	2	0·37	0·47	0·83	0·118	·93	1·10	0·65
1650	2	0·45	0·55	0·94	0·158	1·12	1·42	0·75
2200	4	0·49	0·59	1·02	0·158	1 24	1·42	0·83
3300	4	0·61	0 75	1·22	0·177	1·52	1·77	1·04
4400	4	0·69	0·83	1·38	0·216	1·79	2·01	1·18
5500	4	0·85	1·02	1·61	0·256	2·11	2·44	1·42
8820	6	0·85	1·02	1·61	0·177	2·11	2·44	1·42
12100	6	0 94	1·14	1·81	0·197	2·36	2·76	1·61
16500	6	1·08	1·30	2·05	0·216	2·70	3·15	1·85
22050	6	1·26	1·54	2·32	0·256	3·15	3·66	2·14
33070	8	1·44	1·73	2·60	0·197	3·60	4·17	2·44
44100	8	1·65	1·97	3·00	0·236	4·13	4·80	2·72
55100	8	1·85	2·24	3·31	0·256	4·61	5·35	3·13
66100	8	2·03	2·44	3·62	0·295	5·05	5·87	3·43

271. *Toothed pulley.*—A gearing chain runs on a pulley the pitch line of which is strictly a polygon. Usually it is desirable to make the pulley as small as possible, and the minimum number of teeth adopted in practice is about as given in the following table :

2 T not greater than		1,000	7 teeth
"	"	4,000	8 "
"	"	18,000	9 "
"	"	70,000	10 "

If l is the length of the links from centre to centre of

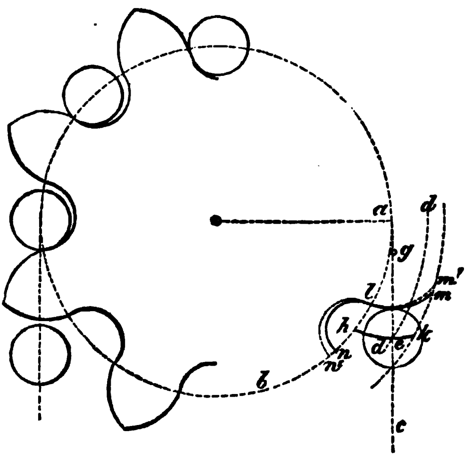


Fig. 255.

pins, Z the number of teeth in the wheel, R the radius of the wheel,

$$l = 2 R \sin \frac{\pi}{Z}$$

$$R = \frac{l}{2 \sin \frac{\pi}{Z}}.$$

The diameter of the part of the pin which acts on the toothed wheel may be $d_1 = 1 \cdot 2\, d$.

272. *Form of teeth of wheel.*—As the chain unwinds from the wheel, the centre of the pin in the chain describes the involute of the pitch circle. Let $a\,b$, fig. 255, be the pitch line of the wheel, $a\,c$ that of the chain. Draw the circle $d\,d$ at $\frac{2}{3}$ the intended height of the wheel tooth, cutting $a\,c$ in e. Take $a\,g = \frac{1}{4}\, a\,e$. Then a circle $h\,e\,k$ struck from g with radius $g\,e$ will sensibly coincide with an involute of the pitch line, and a circle $l\,m$ struck from g touching the circumference of the pin will be a suitable form for the wheel tooth. The part of the tooth below the pitch line may be a circle struck from h with radius equal to the radius of the pin. As, however, gearing chains are not quite so exact as toothed wheels, it will be better to take a circle for the tooth form such as $l\,m'$ lying a little within the curve $l\,m$, and a circle $l\,n'$ for the bottom of the tooth lying a little outside of $l\,n$. If the wheel teeth are tenoned into the wheel, the parts below the pitch line may be tangents to $l\,m$.

Karl von Ott, comparing the weight, cost, and strength of the three materials, hemp rope, wire rope, and chains, arrives at the conclusion that for equivalent strength the cost is proportional to the numbers 2 : 1 : 3. That is, hemp rope is twice as costly and chains three times as costly as wire rope of the same tensional strength.

The following rules are added for comparison with the rules for chains. They give the weight and strength of hemp ropes and wire ropes used for supporting a simple tension. Let G = weight of rope in lbs. per fathom ; γ = girth of rope in inches ; Δ = diameter of rope in inches ; P = breaking weight of rope in tons,

$$G = \alpha\gamma^2 = \beta\Delta^2$$

	$\alpha =$	$\beta =$
Tarred Hemp Ropes . . .	0·25	2·47
White ,, ,, . . .	,,	,,
Iron Wire Ropes . . .	0·785	8·6
Steel ,, ,, . . .	,,	,,

$$P = \kappa\gamma^2 = n\Delta^2$$

	$\kappa =$	$n =$
Tarred Hemp Ropes . . .	·36 to ·42	3·5 to 4·2
White ,, ,, . . .	·49 to ·69	4·8 to 6·7
Iron Wire Ropes . . .	1·75	17·28
Steel ,, ,, . . .	3·00	29·61

CHAPTER XIII.

LINKWORK.

CRANKS AND LEVERS.

273. CRANKS are levers fixed on shafts which rotate continuously or through a limited angle. The simplest form of crank is the ordinary winch handle, used for driving rotating pieces by hand. Most commonly the crank forms part of a linkwork arrangement for changing reciprocating into rotative motion.

HAND LEVERS AND WINCH HANDLES.

Fig. 256 shows an ordinary straight lever for working machinery by hand. The part grasped by the hand may be $1\frac{1}{4}$ inch in greatest and $1''$ in smallest diameter, and 5 inches long. Let P be the force exerted at the handle, and l the length of the lever. Then P l (nearly) is the greatest bending moment on the arm. Let b be the width and h the thickness of the arm at its largest part. Then, § 28,

$$\tfrac{1}{6}\, b^2\, h f = \mathrm{P}\, l$$

$$b^2\, h = \frac{6\, \mathrm{P}\, l}{f}$$

Let the greatest force, P, exerted by a man be taken at 84 lbs.; and let f=9,000 lbs. per sq. in. for wrought-iron. Then,

$$b^2\, h = \tfrac{1}{18}\, l \text{ nearly} \quad . \quad . \quad . \quad (1)$$

If $h=\frac{3}{4}$ inch, $b=0{\cdot}27\sqrt{l}$. If the flat part of the lever is of

uniform thickness, its least width should be half its greatest width, the case corresponding with Case I. Table VI. Let d= diameter of shaft on which the lever is keyed ; n=dis-

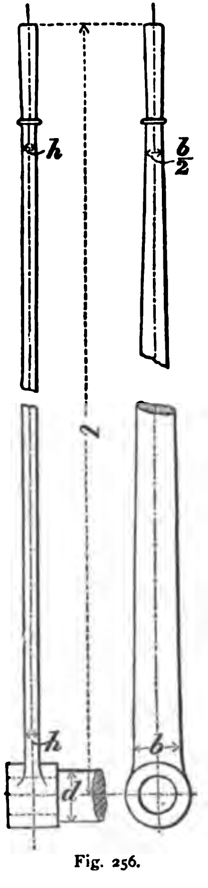


Fig. 256.

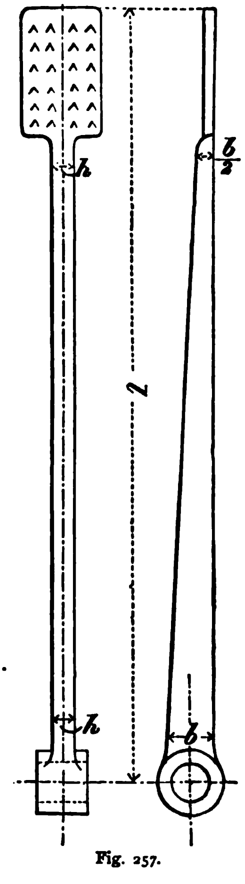


Fig. 257.

tance from centre of lever to centre of nearest bearing of shaft. Then the shaft is subjected to a twisting moment P l and a bending moment P n, and its strength is determined

by the rules in § 44 and § 126. The equivalent bending moment is P $(0{\cdot}7\,n+0{\cdot}48\,l)$ nearly. Hence,

$$d=0{\cdot}0947\sqrt[3]{\{P(1{\cdot}4\,n+0{\cdot}96\,l)\}}$$
$$=0{\cdot}42\sqrt[3]{(1{\cdot}4\,n+0{\cdot}96\,l)} \quad . \quad . \quad . \quad (2)$$

The part in the eye of the lever may have a diameter $=0{\cdot}42\sqrt[3]{l}$. The eye of the lever may have a thickness $=0{\cdot}3\,d$ and a length$=1$ to $1\frac{1}{4}\,d$.

Fig. 257 shows a foot lever. The foot plate is about 8 ins. by 5 ins., and $\frac{5}{8}$ in. thick. In designing this lever P may be taken at 180 lbs. Then,

$$\left.\begin{array}{l} b^2\,h=\frac{1}{8}\,l \\ d={\cdot}54\sqrt[3]{(1{\cdot}4\,n+0{\cdot}96\,l)} \end{array}\right\} \quad . \quad (3)$$

274. Fig. 258 shows a winch handle or cranked lever. When this is intended to resist the full force of one man, P

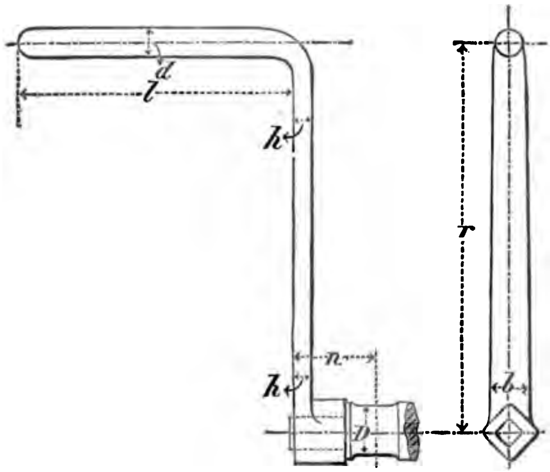


Fig. 258.

may be taken at 84 lbs., and if worked by two men, P$=168$ lbs. The mean effort per man in continuous work

is only 15 to 30 lbs. The radius r is usually 16 or 17 ins., and the height of the shaft from the ground may be 3 ft. to 3 ft. 3 ins. The length of handle l may be 10 or 12 ins. for one man, and 20 ins. for two men. The pressure on the handle may be taken to act at $\frac{2}{3}$rds of the length. The greatest bending moment at the handle is $\frac{2}{3}$ P l. Then, its diameter should not be less than

$$d = 0{\cdot}0947 \sqrt[3]{\tfrac{2}{3} \text{P}\, l} = 0{\cdot}1042 \sqrt[3]{\text{P}\, l} \quad . \quad (4)$$

or say $1\frac{1}{16}$ inch for one man and $1\frac{1}{2}$ inch for two men. The journal of the shaft is subjected to a twisting moment P r, and a bending moment P ($\frac{2}{3}\, l + n$). The equivalent bending moment (§ 44) is P ($0{\cdot}6\, l + 0{\cdot}9\, n + 0{\cdot}4\, r$) nearly. Then,

$$\left.\begin{aligned} \text{D} &= 0{\cdot}0947 \sqrt[3]{\{\text{P}\,(1{\cdot}2\, l + 1{\cdot}8\, n + 0{\cdot}8\, r)\}} \\ &= 0{\cdot}42 \sqrt[3]{(1{\cdot}2\, l + 1{\cdot}8\, n + 0{\cdot}8\, r)} \text{ for one man} \\ &= 0{\cdot}54 \sqrt[3]{(1{\cdot}2\, l + 1{\cdot}8\, n + 0{\cdot}8\, r)} \text{ for two men} \end{aligned}\right\} (5)$$

For the part in the eye of the crank, the term $1{\cdot}8\, n$ may be omitted. The greatest bending moment on the arm is P r, and the twisting moment $\frac{2}{3}$ P l nearly. Hence the equivalent bending moment is P ($0{\cdot}9\, r + {\cdot}27\, l$). If b is the breadth and h the width of the arm at the larger end,

$$\left.\begin{aligned} b^2 h &= \frac{6\,\text{P}}{f} (0{\cdot}9\, r + 0{\cdot}27\, l) \\ &= {\cdot}056\, (0{\cdot}9\, r + 0{\cdot}27\, l) \text{ for one man} \\ &= {\cdot}112\, (0{\cdot}9\, r + 0{\cdot}27\, l) \text{ for two men} \end{aligned}\right\} \quad (6)$$

Either b or h may be selected and the other obtained from the formula. If the arm is of uniform thickness, its least breadth should not be less than $\frac{1}{2}\, b$, or less than $2\sqrt{\frac{\text{P}\, l}{f\, h}}$ or $0{\cdot}19\sqrt{\frac{l}{h}}$ for one man, and $0{\cdot}27\sqrt{\frac{l}{h}}$ for two men. Thickness of eye of crank, $0{\cdot}3$ D; length of eye, $1\frac{1}{2}$ D.

ENGINE CRANKS.

275. Engine cranks are of cast- or wrought-iron. A single crank consists of a nave bored to receive the crank shaft, an arm, and a crank pin. If the crank pin is a separate piece, it is fitted into an eye formed at the small end of the crank. Disc cranks have plain circular discs, instead of the ordinary crank arm, and they have the advantage of being nearly balanced with respect to the crank shaft. A double crank is used when the crank pin cannot be placed at the end of the crank shaft. An eccentric is a crank of peculiar form. It is essentially a crank, with a crank pin, the radius of which is greater than the sum of the crank and crank shaft radii.

276. *Crank and connecting rod. Forces acting on the crank pin.*—Usually the crank is driven by the pressure on a piston, transmitted to it through a connecting rod. The path of the crank pin is $2 \pi R$ in one revolution, while the path of the piston is $4 R$. Hence the mean driving pressure on the crank pin is less than the mean pressure on the piston, in the ratio of $4 : 2 \pi$, or $2 : \pi$. The resultant pressure on the crank pin is, however, at times, much greater than the mean pressure.

In designing a steam engine, the steam pressure on the piston is known or can be assigned with sufficient accuracy. That pressure may be a constant pressure throughout the stroke, or much more commonly a pressure diminishing in consequence of the expansion of the steam. If a line S S is taken equal on any scale to the stroke of the engine, and at each point in the stroke an ordinate A C is erected (fig. 261) representing on any scale the whole steam pressure P, on the piston, or perhaps more conveniently the pressure p per sq. in. of piston area, we obtain a diagram such as S D D S termed an indicator diagram. The variations of the ordinates of this diagram exhibit the variation of the

steam pressure in the cylinder during the stroke, and the area of the diagram is proportional to the work done by the steam on the piston.

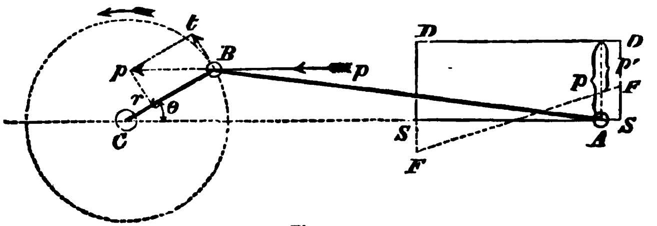


Fig. 259.

277. *Variation of pressure on crank pin when the steam pressure is uniform and the obliquity of the connecting rod is neglected.*—Suppose first that, for simplicity, the steam pressure is taken constant throughout the stroke; then if S S represents (fig. 259) the path of the crosshead, the rectangle S D D S (in which S D is taken equal to the steam pressure p per sq. in. of piston area) is the indicator diagram, for a single stroke. Let C B, B A be the position of the crank and connecting rod at any instant. To simplify the problem a little more, sup-

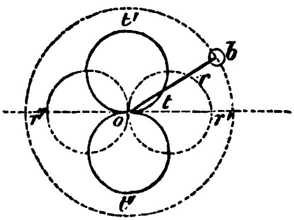


Fig. 260.

pose that the obliquity of the connecting rod is neglected, so that the pressure p acting along the connecting rod is taken to be acting in the direction of the arrow at B parallel to A C. The pressure p at B is balanced by the tangential resistance t of the crank to rotation, and by the radial thrust

r acting along the crank. If therefore B p is taken equal to p, and the parallelogram completed, it is seen that

$$t = p \sin \theta$$
$$r = p \cos \theta$$

where θ is the angle through which the crank has turned, measured from the beginning of the stroke. The values of t and r vary as the crank moves. If in fig. 260 we take any pole O, and draw any radius vector O b parallel to the crank and take O t equal to B t, and repeat the construction for a sufficient number of points, all the points t will lie on the circumference of the full circle O t t'. Similarly the values of the radial component B r laid off along O b will give points on the dotted semicircles r' r O r''. The tangential and radial pressures during the return stroke give the circles in the lower half of the figure. Once these circles are drawn, the radial and tangential components of the pressures on the crank pin for any position of the crank can be found by drawing a radius vector from O parallel to the crank, and measuring the values of O r and O t.

278. *Influence of the inertia of the piston, piston rod and crosshead.*—In actual engines, especially in those running at high speed, the inertia of the parts connected with the crosshead and reciprocating with it greatly modifies the distribution of the crank-pin pressure during the stroke. During the earlier half of the stroke the velocity of all the horizontally moving pieces is accelerated, and during the latter half retarded. Part of the steam pressure is used in accelerating the heavy reciprocating parts in the earlier half of the stroke, and during the later half the pressure required to retard them is added to the steam pressure.

If the obliquity of the connecting rod is neglected, the horizontal velocity of the crosshead and piston is V $\sin \theta$, where V is the velocity of the crank pin B, which is absolutely or nearly constant. Then the acceleration of the crosshead, piston, &c., is

$$\frac{d(\mathrm{V} \sin \theta)}{dt} = \frac{d(\mathrm{V} \sin \theta)}{d\theta} \cdot \frac{d\theta}{dt}$$

$= \text{V} \cos \theta \times a$, where a is the angular velocity of the crank; or putting $a = \frac{\text{V}}{\text{R}}$, the acceleration is $\frac{\text{V}^2 \cos \theta}{\text{R}}$, where R is the crank radius and V the velocity of the crank pin. Now let W be the weight of the horizontally moving parts in lbs., $\frac{\text{W}}{g} \cdot \frac{\text{V}^2 \cos \theta}{\text{R}}$ is the whole force required to accelerate them. Or if ω is the area of the piston in square inches, the accelerating force, in lbs. per unit of piston area, is—

$$\frac{\text{W}}{g} \cdot \frac{\text{V}^2 \cos \theta}{\omega \text{R}},$$

which for $\theta = 0°$ or $180°$ becomes

$$\pm \frac{\text{W}}{g} \cdot \frac{\text{V}^2}{\omega \text{R}}.$$

For any position A of the crosshead, when the crosshead has moved a distance,

$$x = \text{A S} = \text{R} (1 - \cos \theta)$$

the ordinate of a curve representing the accelerating force is

$$y = \frac{\text{W}}{g} \cdot \frac{\text{V}^2 \cos \theta}{\text{R} \omega}$$
$$= \frac{\text{W}}{g} \cdot \frac{\text{V}^2 (\text{R} - x)}{\omega \text{R}^2}$$

the equation to a straight line. Hence, if S F, S F′ are taken equal to $\pm \frac{\text{W}}{g} \cdot \frac{\text{V}^2}{\omega \text{R}}$ on the same scale as that used for the steam pressure, and F F′ is joined, then the effective force at the crosshead driving the crank is the vertical ordinate p' of the figure F D D F′. In this case the curves of tangential and radial pressure are no longer circles.

279. *Forces acting on the crank pin when the steam pressure is variable and the obliquity of the connecting rod is taken into account.*—Let C B, B A, fig. 261, represent as before any position of the crank and connecting rod, and let S D D D S be an indicator diagram, S S being the stroke of the cross-

head, and S D the initial steam pressure. Produce C B, and a perpendicular at A to S S, to meet in I. Then I is the

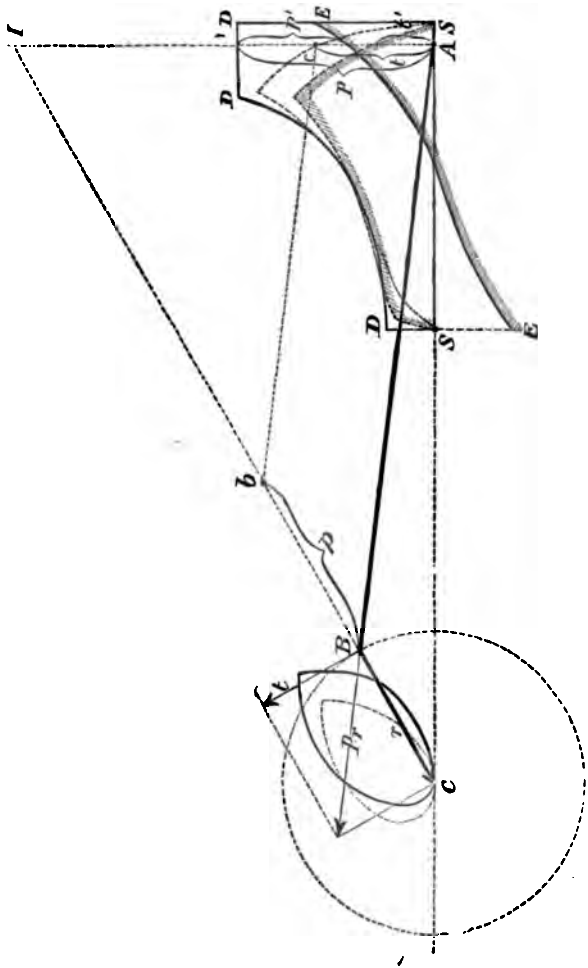


Fig. 261.

point about which the link B A is rotating at the instant

considered. If V is the velocity of B and v the velocity of A,

$$\frac{v}{V}=\frac{AI}{BI}.$$

Let p, the ordinate of the indicator diagram, be the pressure acting at A, and t the tangential pressure at B; then

$$p\,v=t\,V$$

$$\frac{t}{p}=\frac{AI}{BI}.$$

Take B $b=p$; draw $b\,c$ parallel to B A,

$$\frac{t}{p}=\frac{AI}{BI}=\frac{cA}{bB};$$

$$\therefore \qquad t=c\,A.$$

Values of t found for all points of the stroke give the dotted curve S c S. If the values of t are laid off on the corresponding positions of the crank, we get the oval curve shown by a full line on the left, which corresponds to one of the full circles in fig. 260. The return stroke would give a similar curve below the horizontal line.

Set off t tangential to the crank pin circle at B, and complete the parallelogram of forces. We thus get the radial component r of the crank-pin pressure, and the resultant pressure p_r along A B on the crank pin.

If this construction is made for several positions of the crank, it will be seen that the tangential component t vanishes when the crank is at the dead point, and, except when the pressure on the piston diminishes before half-stroke, it reaches its maximum when the crank and connecting rod are at right angles. Its maximum value, if R=crank radius and L=connecting rod length, is,

$$t_{max}=p\,\frac{\sqrt{R^2+L^2}}{L} \quad . \quad . \quad . \quad (7)$$

Since L is usually 4 to 5 times R,

$$t_{max}=1{\cdot}02 \text{ to } 1{\cdot}03\,p \quad . \quad . \quad . \quad (7a)$$

The radial component N vanishes when the crank and con-

necting rod are at right angles, and is greatest and equal to p when the crank is at the dead point.

280. *Determination of the acceleration when the obliquity of the connecting rod is taken into account.*—If acceleration needs to be taken into account, suppose a curve E E drawn, the ordinates of which are the horizontal accelerating forces acting on the parts connected to the crosshead, estimated per unit of piston area. Then the effective horizontal thrust or pull at the crosshead is equal to the ordinate p' measured between

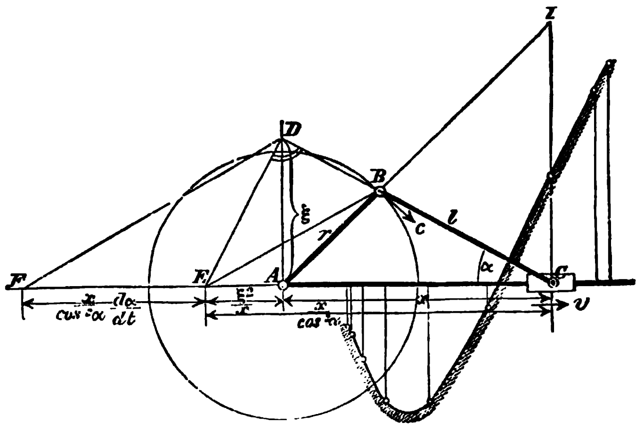


Fig. 262.

D D and E E. Treating this precisely as before, the values of the tangential effort at the crank pin, t', give the shaded curve S S. The same values set off on the corresponding crank positions give the polar curve of twisting efforts shown by the dotted curve on the left of the figure.

By far the simplest construction for determining the acceleration of reciprocating parts of steam engines is that of Rittershaus ('Civilingenieur,' xxv. 461). Let A B be the crank, B C the connecting rod of an ordinary horizontal engine, fig. 262. Draw A Z perpendicular to B's direction of motion, C Z perpendicular to C's direction of motion.

Then Z is the instantaneous axis of the link B C, and, as is well known, putting V for B's velocity and v for C's velocity,

$$\frac{v}{V}=\frac{CZ}{BZ}.$$

Produce C B to meet a perpendicular A D to the line of stroke A C. Call R the crank radius a B, ξ the intercept A D. Then from similar triangles

$$\frac{v}{V}=\frac{\xi}{R}$$

$$v=\left(\frac{V}{R}\right)\xi.$$

Draw D E perpendicular to D C, and D F parallel to B E. Let A C$=x$ and the angle A C B$=\alpha$. Then

$$\xi=x\tan\alpha,$$

differentiating with respect to the time

$$\frac{d\xi}{dt}=\left(\frac{R}{V}\right)\frac{dv}{dt}=\frac{x}{\cos^2\alpha}\ \frac{d\alpha}{dt}+\tan\alpha\ \frac{dx}{dt}.$$

But

$$\frac{dx}{dt}=v=\left(\frac{V}{R}\right)\xi$$

and the acceleration f is

$$f=\frac{dv}{dt}=\frac{V}{R}\ \frac{x}{\cos^2\alpha}\ \frac{d\alpha}{dt}+\left(\frac{V}{R}\right)^2\xi\tan\alpha.$$

But if to the actual velocities we add a velocity $-v$, so as to bring C to rest, we have for the angular velocity of the connecting rod round C,

$$\frac{d\alpha}{dt}=-\frac{v}{\xi}\ \frac{DB}{BC}=-\frac{V}{R}\ \frac{DB}{BC};$$

consequently

$$f=\frac{dv}{dt}=-\left(\frac{V}{R}\right)^2\left\{\frac{x}{\cos^2\alpha}\ \frac{DB}{BC}-\xi\tan\alpha\right\},$$

or for the scale which makes V$=$R

$$\frac{dv}{dt}=-\left\{\frac{x}{\cos^2\alpha}\ \frac{BD}{BC}-\xi\frac{\xi}{x}\right\}.$$

But by construction

$$\text{E C} = \frac{x}{\cos^2 a}$$

$$\frac{\text{F E}}{\text{E C}} = \frac{\text{D B}}{\text{B C}}$$

$$\text{F E} = \frac{x}{\cos^2 a} \; \frac{\text{B D}}{\text{B C}};$$

also

$$\frac{\text{E A}}{\xi} = \frac{\xi}{x}$$

$$\text{E A} = \frac{\xi^2}{x}$$

consequently,

$$f = -\{\text{F E} - \text{E A}\}.$$

For the dead points, at which the construction fails, since $a=0$, we have for the inner dead point

$$x = l + \text{R} \text{ and } \frac{\text{D B}}{\text{B C}} = \frac{\text{R}}{l};$$

for the further dead point

$$x = l - \text{R} \text{ and } \frac{\text{D B}}{\text{B C}} = -\frac{\text{R}}{l}.$$

Hence for the inner dead point

$$f_1 = \frac{dv}{dt} = -(l+\text{R})\frac{\text{R}}{l} = -\text{R}\left(1 + \frac{\text{R}}{l}\right)$$

and for the outer

$$f_2 = \frac{dv}{dt} = (l-\text{R})\frac{\text{R}}{l} = \text{R}\left(1 - \frac{\text{R}}{l}\right).$$

Let W be the weight of the horizontally moving parts, ω the area of the piston. The whole accelerating force, per unit of piston area, is

$$\frac{\text{W}f}{g\,\omega};$$

we have therefore only to measure the values of f on a scale for which $\text{V}=\text{R}$, and multiply them by $\frac{\text{W}}{g\,\omega}$ to get the values of the ordinates of the acceleration curve E E in fig. 261.

281. *General case. Straining action on crank arm.* Let fig. 263 represent a crank in any position, and let P be the total pressure on the crank pin. Resolve P into a tangential component T, and a radial component N. Let $a\,b$ be any section of the arm at a distance r from the centre of crank pin, and let m be the distance between centre lines of crank pin and crank arm. Then the straining actions at $a\,b$ which require to be considered are :—

(*a*) A direct pressure (or tension) equal to N.
(*b*) A bending moment N m in the plane of the arrow B.
(*c*) A bending moment T r in the plane of the arrow A.
(*d*) A twisting moment T m.

To take into account all these straining actions in several positions of the crank would be laborious. Generally it is sufficient to estimate the strength of the crank in two positions, when the crank is at the dead point and when the crank and connecting rod are at right angles. In the former case, T vanishes and N becomes equal to the greatest piston pressure, or to $N''=P''+\frac{W\,V^2}{g\,R}$, which will also for simplicity be denoted by N simply. In the latter case N vanishes and T is equal to 1·02 or 1·03 times the piston pressure.

282. *Strength of the crank.*—Let N be the radial pressure when the crank is at the dead point, and T the tangential pressure when the crank and connecting rod are at right angles. Let further

d, l=diameter and length of crank pin.
D, L=diameter and length of crank-shaft journal.
d', l', t'=internal diameter, length and thickness of small eye of crank.
$d''\ l''\ t''$=internal diameter, length, and thickness of large eye of crank.
h, b=thickness and width of arm at any section $a\,b$; the same letters with one accent referring to the section of the arm supposed produced to the centre

of small eye, and with two accents the section produced to the centre of large eye.

R = crank radius.

m = distance from centre line of crank pin to centre line of crank arm.

n = distance from centre line of crank pin to centre line of crank-shaft journal.

The crank pin and crank-shaft journal are first designed by the rules in Chapter VII. For the section of the crank

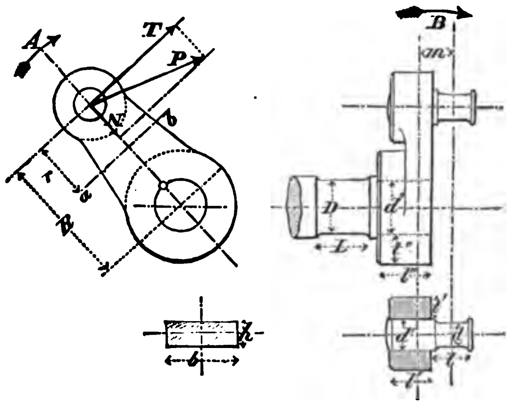


Fig. 263.

arm we have, when the crank is at the dead point, a direct tension or pressure N and a bending moment N m. Then the greatest stress is (§ 43),

$$f = \text{N}\left(\frac{1}{h\,b} + \frac{6\,m}{b\,h^2}\right)$$

$$b = \frac{\text{N}}{f}\left(\frac{1}{h} + \frac{6\,m}{h^2}\right) \quad . \quad . \quad . \quad (9)$$

or,

$$h = 2{\cdot}65\sqrt{\frac{\text{N}\,m}{f\,b}} \text{ nearly} \quad . \quad . \quad (9a)$$

If this straining action only were considered, the crank arm would be of uniform section throughout. Hence this equation is chiefly useful for determining the breadth and thickness of the arm at the small end, where the straining action due to the force T is least important.

When the crank and connecting rod are at right angles, there is a bending moment $\text{T}\,r$, and a twisting moment $\text{T}\,m$. Combining these, the equivalent bending moment is (§ 44),

$$=0{\cdot}91\,\text{T}\,r+0{\cdot}41\,\text{T}\,m \text{ nearly.}$$

The bending is parallel to the plane of rotation, so that the modulus of the section is $\frac{1}{6}\,b^2\,h$. Then

$$\tfrac{1}{6}\,b^2\,hf=0{\cdot}41\,\text{T}\,r+0{\cdot}41\,\text{T}\,m$$

$$b=\sqrt{\left\{\frac{6\,\text{T}}{fh}(0{\cdot}91\,r+0{\cdot}41\,m)\right\}} \quad . \quad (10)$$

$$\text{or, } h=\frac{6\,\text{T}}{fb^2}(0{\cdot}91\,r+0{\cdot}41\,m) \; . \quad . \quad . \; (10a)$$

Select a value for h or b. Then the greater of the values given by equations 9 and 10, or 9*a* and 10*a*, is the proper value for the remaining dimension. It will often be sufficient to use equation 9 or 9*a* to determine b' or h'; equation 10 or 10*a* to determine b'' or h''; and the sides of the crank arm may be drawn as planes.

When the crank is of cast-iron the arm may be trough-shaped (fig. 265), and is then somewhat lighter than when it is rectangular. Let b and h be the width and thickness of a rectangular arm, and let $b_1\,h_1$ and $b_2\,h_2$ be the dimensions of an arm of trough-section of equivalent strength. Then if flexure is in the plane of rotation,

$$b^2\,h=\frac{b_1^3\,h_1-b_2^3\,h_2}{b_1}$$

Let $$b_1=b$$

$$b^3\,h=b^3\,h_1-b_2^3\,h_2$$

Let $$b_2 = x\,b$$

$$h_2 = y\,h_1$$

Then $$h_1 = \frac{h}{1 - x^3 y} = c\,h \quad . \quad . \quad . \quad (11)$$

where c has the following values :—

		$x=$		
$y=$	0·6	·65	·7	·75
·6	1·15	1·20	1·26	1·34
·65	1·16	1·22	1·29	1·38
·7	1·18	1·24	1·32	1·42
·75	1·19	1·26	1·35	1·46

At the section at the centre of the small eye of the crank the flexure is at right angles to the plane of rotation, and the

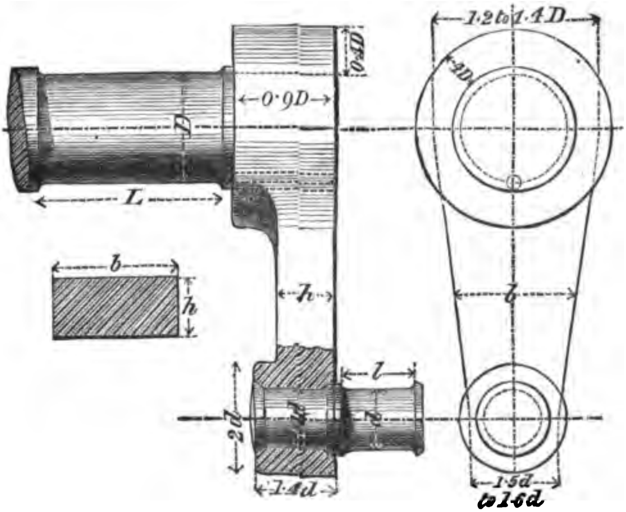


Fig. 264.

feathers strengthen the section very little. Hence the section there may remain unchanged, the feathers being allowed to diminish towards that end.

When a crank has a T-form of section, it is but little strengthened by the feather, but it is more easily cast.

283. *Proportions of cranks.*—The crank is shrunk on to the crank shaft, and the crank pin is also fixed in the same way and riveted cold. The key in the crank shaft may have a breadth$=\frac{1}{3}$ D, and a thickness $\frac{1}{6}$ D, for small cranks, and $\frac{1}{4}$ D and $\frac{1}{8}$ D for large cranks.

Fig. 264 shows a wrought-iron crank with a section of the arm. The arm is sometimes tapered and the back face of the

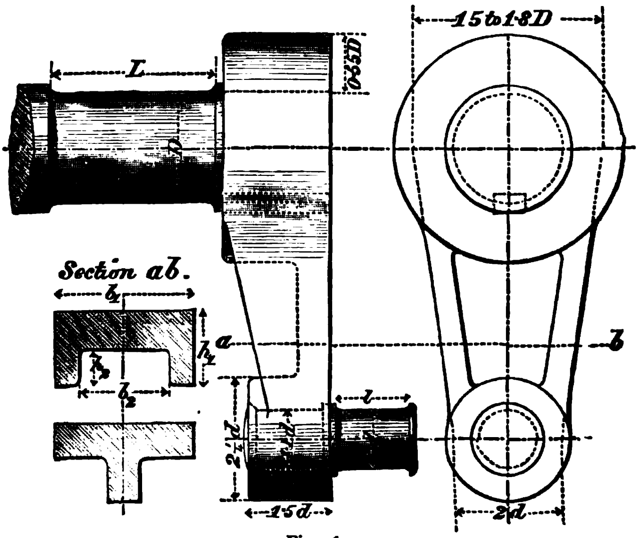


Fig. 265.

arm is then rounded, so that it forms, in fact, part of a slightly conical surface, turned in the lathe. Fig. 265 shows a cast-iron crank, with sections of arms both trough-shaped and T-shaped.

In quick-running engines it is desirable to balance as directly as possible the weight of the crank and connecting rod. The crank then takes a disc form, as shown in fig. 266. By hollowing the part of the disc on the crank side and leaving the opposite side full, a surplus weight is obtained which balances the crank pin and connecting-rod end.

The question of the amount of balance weight required involves some difficulty, because part of the weights attached to the crank pin reciprocate without rotating, and part rotate with the crank pin. Let w_1 be the weight of the balance weight, and ρ the radius to its centre of gravity; w_2 the weight of the crank pin and half the weight of the connecting rod, which may be taken as rotating with the crank pin at radius r; w_3 the weight of the piston, piston rod, cross-head, and the other half of the weight of the connecting rod. Then for balance of the forces at right angles to the line of stroke

$$w_1 \rho = w_2 r.$$

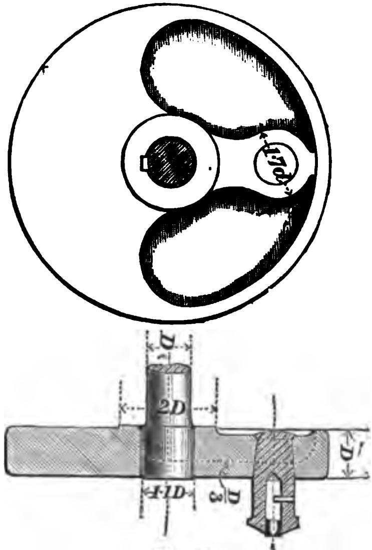


Fig. 266.

But for balance of the forces parallel to the line of stroke

$$w_1 \rho = (w_2 + w_3) r.$$

Hence the axial and normal disturbing forces cannot both be balanced. If the balance weight is sufficient for the axial forces, it over corrects the normal forces and introduces a new unbalanced force perpendicular to the line of stroke. In ordinary practice for vertical engines, in which the forces at right angles to the line of stroke are most injurious,

$$w_1 = w_2 \frac{r}{\rho}.$$

But for horizontal engines, where the horizontal forces are most injurious,

$$w_1 = \tfrac{2}{3} (w_2 + w_3) \frac{r}{\rho} \text{ to } \tfrac{3}{4} (w_2 + w_3) \frac{r}{\rho}.$$

In locomotives, when the balance weight is made as large as in the latter case, the vertical unbalanced forces are considerable, and act dangerously in tending to throw the engine off the rails, or, at all events, tend to damage the wheels and rails. Consequently for locomotives,

$$w_1 = w_2 \frac{r}{\rho}.$$

284. *Built-up steel cranks.*—The difficulty of forging large double cranks has led to the use of built-up cranks like that shown in fig. 267, which shows the cranks used in the

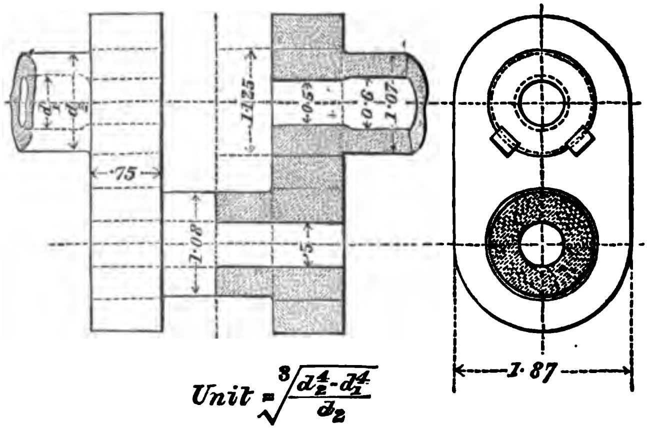


Fig. 267.

City of Rome, S.S.—A double collared hollow steel shaft, formed as described in Chapter VII. § 147, is cut in half to form the single collared pieces. The crank cheeks or webs are first forged solid in the form of slabs, and then a small hole is bored at each end, and enlarged by being forged on a mandril placed on suitable supports; thus insuring that the metal is thoroughly worked in the most important part. The cheeks are afterwards shrunk and keyed on the cut ends of the half lengths of shaft. The hollow crank pin is drawn to length by forging, and is shrunk in but is not keyed into the cheeks.

Eccentrics.

285. An eccentric is a modified crank, chiefly employed to drive the slide valve of steam-engines. It is really a crank and connecting rod, with a crank pin enlarged, so as to include the crank shaft within its section, the radius of the eccentric being greater than the sum of the crank and crank-shaft radii. The eccentric consists of a sheave, which is virtually a crank pin, and a strap and rod which is virtually equivalent to a connecting rod. The sheave is most commonly of cast iron, and is often cast in two parts connected by bolts. In very hard-worked eccentrics the sheave may be of wrought iron, case-hardened. When the sheave is in two parts, the smaller may be of wrought and the larger of cast iron. The strap is in two parts, and is prevented from slipping sideways by a flange or flanges, or it has internally a spherical surface fitting on the sheave. The strap is of brass, of cast iron lined with brass, or of wrought iron lined with brass or with white metal. The friction of the eccentric is much greater than that of a crank, and it is therefore not used where ordinary cranks can be applied.

The distance between the centres of the crank shaft and eccentric sheave is termed the eccentricity, the radius, or the half stroke of the eccentric. Let this be denoted by r, and let d be the diameter of the shaft on which the eccentric is fixed. Then the least diameter suitable for the eccentric sheave is about

$$= \mathrm{D} = 1{\cdot}2\, d + 2\, r + \tfrac{3}{4}.$$

Professor Reuleaux has pointed out that the width of the eccentric sheave, or virtual length of the crank pin, should be the same as the length of an ordinary crank pin for the same work. The width b can then be decided by the rules for journal lengths in § 115. There is, however, great difficulty in applying these rules, because the force which the eccentric has to overcome cannot be very accurately ascertained.

286. *Friction of the slide valve.*—Let a be the area of the back of the valve subjected to the steam pressure, and p the steam pressure in lbs. per sq. in., reckoned above atmospheric pressure in the case of non-condensing engines, and above zero in the case of condensing engines.

Then the frictional resistance is ordinarily taken to be

$$F=\mu p a,$$

where μ is about 0·15 for smooth surfaces, such as slide-valve surfaces, not well lubricated. But it is possible that the steam may insinuate itself partially, or over the whole extent of the faces of the valve, which are in contact with the surface of the valve chest; and in that case the downward pressure of the steam on the back of the valve at those parts would be neutralised by the upward pressure of the layer of steam between the surfaces, and the friction would be due to the pressure on the remainder of the valve only. According to some experiments of Mr. Thomas Adams, steam does so insinuate itself, so long as the intensity of pressure between the valve and steam chest faces is less than the steam pressure, but when the pressure is greater, as must be the case with ordinary slide valves, this layer of steam is squeezed out, and then the coefficient of friction is found to have a much higher value, so that in the formula above we ought to take $\mu=0·2$ to 0·35, the value being greater as the pressure and temperature of the steam is greater.

The friction which the eccentric has to overcome may be assumed to be proportional to $p\,a$, and the unit for the following proportional dimensions will therefore be taken,

$$=k=c\sqrt{p\,a} \quad . \quad . \quad . \quad (12)$$

where c appears to have on the average the value $\frac{1}{80}$ to $\frac{1}{100}$ for stationary and marine engines, and $\frac{1}{100}$ to $\frac{1}{120}$ for locomotives.

287. *Radius of eccentric.*—Let w be the greatest width of port opened to steam; l, the lap of the valve; r, the radius of the eccentric,

$$r=w+l,$$

w is in some cases the whole width of the steam port, but in quick-running engines the opening to steam is less than the opening to exhaust. This is secured by making w about $\frac{2}{3}$rds of the width of the port. The external lap l may vary from $\frac{1}{5}$th of the width of the port to the whole width of the port, according to the amount of expansion required, § 328.

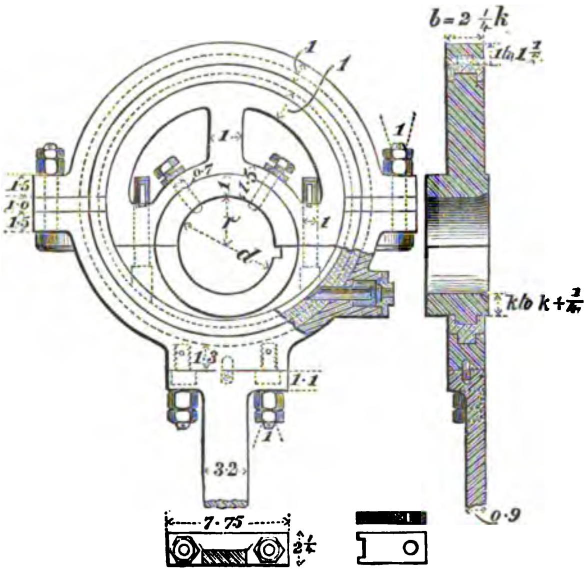


Fig. 268.

288. *Proportions of sheave.*—The width b of the bearing surface of the sheave may be taken equal to $2\,k$, or $2\frac{1}{4}\,k$. The diameter of the bolts connecting the two parts of the sheave may be $0{\cdot}85\,k$ to k, and the cotter in these bolts may have a width equal to their diameter, and a thickness equal to $\frac{1}{4}$ of their diameter. The set screws may

be 0·7 k diameter, or if there is only one it may be k in diameter. If the sheave has flanges to retain the strap, their projection may be 0·4 k, and their thickness 0·3 k. The smaller part of the sheave, when it is in two parts, is

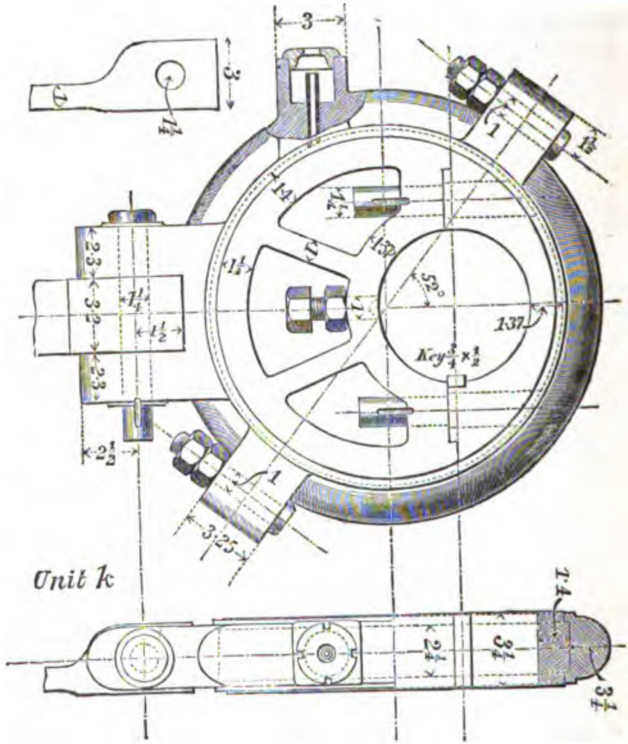


Fig. 269.

generally of wrought iron in small eccentrics. The least thickness of this part, where the shaft comes near the edge of the sheave, should be at least k for wrought iron and $1\frac{1}{4}$ k for cast iron.

289. *Proportions of strap.*—The strap thickness varies very much. For wrought iron it may be from $\cdot 5\, k$ in large to k in small eccentrics. For gun-metal it should be from $\cdot 625\, k$ to $1\frac{1}{4}\, k$. For cast-iron from $\cdot 75\, k$ to $1\cdot 5\, k$. The brass lining may be about $\frac{1}{5}$th the strap thickness in large eccentrics, and in other cases its thickness may be $\frac{D}{40}+\frac{1}{8}$.

When the strap is recessed to fit a projection on the sheave, the depth of the recess may be $\frac{4}{7}$ths of the thickness of the brass, and its width $0\cdot 3b$. The corresponding recess in the brass may be of the same depth, and its width $b-\frac{5}{8}$ to $b-\frac{7}{8}$.

Proportions of eccentric-rod.—The eccentric-rod is very commonly attached to the eccentric-strap by a T-end, and has at the other an eye to receive the pin of the valve rod. At its smaller or eye end, it may have a width of $1\cdot 8\, k$, and a thickness $0\cdot 55\, k$. It tapers in width about $\frac{1}{2}$ inch per foot of length to the T-end, the thickness being constant. The bolts in the T-end may be of the same size as the strap-bolts.

Fig. 269 shows a link motion eccentric for a locomotive having both sheave and strap of cast iron, and so arranged as to be easily taken apart.

In both fig. 267 and fig. 268 the eccentric sheave is shown divided into two parts for convenience of fixing. When the eccentric can be put on from the end of the shaft this is not necessary.

Friction of eccentric.—Let R be the radius of the eccentric sheave, in ins., P the resistance of the slide valve or other part moved by the eccentric, in lbs., N the number of rotations per minute, μ the coefficient of friction. Then the frictional resistance at the surface of the sheave is about μ P lbs., and the work expended in friction is

$$\frac{2\,\mu\,P\,\pi\,R\,N}{12\times 60}\ \text{ft. lbs. per sec.}$$

Or, putting $\mu = 0{\cdot}12$,

$$\cdot 00104 \; P R N \text{ ft. lbs. per sec.}$$

This is so large that in some cases it amounts to 40 or even 50 per cent. of the whole work transmitted to the eccentric.

CHAPTER XIV.

LINKWORK.

Connecting Rods, Crossheads, and Slides.

Connecting Rods.

290. Connecting rods are the pieces which connect a rotating crank with a reciprocating piece such as a piston or pump plunger. A link connecting two cranks is generally termed a Coupling-rod. In the older engines the connecting rod was often of cast iron. Now it is almost always made of wrought iron or steel. Cast-iron rods are cross-shaped in section to secure stiffness as well as strength. Wrought-iron and steel rods are circular in section in most cases, but for engines running at high speeds the depth in the plane of rotation may be greater than the breadth; they are then stronger to resist the bending action due to the inertia of the rod. Such rods may have a rectangular or approximately rectangular section. An I-shaped section is adopted in some wrought-iron locomotive rods. The ends of a connecting rod are fitted with brass steps and adjusting arrangements for receiving journals.

In consequence of the varying obliquity of the connecting rod, the velocity of the piston is different at corresponding positions of the crank in the two halves of the stroke, and with a very short connecting rod the action of the engine is rendered irregular and the pressure on the guide block becomes excessive. Usually in engines the connect-

ing rod length is from four to seven times the crank radius. For engines running at high speed the connecting rod should be not less than six cranks in length.

A connecting rod may be subjected to tension or to compression only; and in that case, when it is open to choice, it is preferable to arrange the machine so that the rod may be in tension. Rods in compression, which have a journal at each end, are in the position of the rod in Case II., Table VII., and their strength is to be calculated by the rule there given, if the ratio of length to diameter exceeds the value given in § 39. Most connecting rods are subjected to reciprocating stress, alternately compressive and tensile. Then their least section must be sufficient for the tension, and their greatest section, near the middle of their length, must be sufficient for the pressure.

291. *Forces to which connecting rods are subjected.*—Let P_1 be the force transmitted to the end of a connecting rod, due either to steam pressure on a piston or to any other kind of load. Let θ be the angle between the direction of P_1 and the axis of the rod. Then the force acting along the axis of the rod is $P = P_1 \sec \theta$. Hence, if a connecting rod is n times the length of the crank (usually $3\frac{1}{2}$ times to 6 times), the thrust along the connecting rod, at its greatest obliquity, is $\frac{\sqrt{n^2+1}}{n} P_1$, or about $1 \cdot 03$ to $1 \cdot 08\ P_1$. In addition to this load, the connecting rod is subjected to straining actions due to the inertia of the parts connected with it, and to its own weight and inertia. In quick-running engines these straining actions become of importance. In slow-moving machines they may be neglected.

Suppose the connecting rod to be an ordinary engine connecting rod, attached at one end to a rotating crank, of radius R in ft., and at the other to a reciprocating cross-head. The crank pin moves, nearly uniformly, with a velocity V, under the control of the fly-wheel, and the cross-head and parts connected to it (piston, piston-rod, slide

blocks, etc.), have, in consequence, a varying velocity. Let W be the total weight of the parts which reciprocate, inclusive of half the weight of the connecting rod itself. When the crank is near the dead points, the resistance to acceleration is $\pm W \frac{V^2}{g R}$. Hence the thrust in the connecting rod will be $P_1 - W \frac{V^2}{g R}$ at the beginning, and

$$P_1 + W \frac{V^2}{g R} \quad . \quad . \quad . \quad (1)$$

at the end of the stroke, where for P_1 is to be put in each case the corresponding piston pressure.[1] The greater of the two values is to be taken. In a non-expansive engine the greatest thrust will be at the end of the stroke, but in an expansive engine it is not always so.

When the connecting rod and crank are nearly at right angles, the former is subjected to a transverse bending action due to its resistance to acceleration in a direction perpendicular to that of the motion of the piston. According to Grashoff, the bending action is greatest at $\frac{6}{10}$ths of the length of the rod from the cross-head end, and consequently, that is the point at which the rod should have the greatest diameter. The rod is sometimes tapered uniformly from the cross-head end to the crank-pin end, and in quick-running engines this is better than making the diameter greatest at the centre of the rod. Let w be the average weight of the rod in lbs. per inch of length; l, the length of the rod between the centres of the journals in ins.; R, the radius of the crank in inches; V, the velocity of the crank pin in feet per second. Then the greatest bending moment due to the swaying of the rod is

$$M = 0{\cdot}82 \frac{V^2}{R} \frac{w l^2}{g} \quad . \quad . \quad . \quad (2)$$

[1] See § 278.

The stress due to this moment is $f_1 = \frac{M}{Z}$, where Z is the modulus of the section of the rod. Hence,

$$\left.\begin{aligned} f_1 &= 4\cdot92\ \frac{V^2}{R}\,\frac{w\,l^2}{g\,b\,h^2} \text{ for a rectangular rod} \\ &= 8\cdot351\ \frac{V^2}{R}\,\frac{w\,l^2}{g\,d^3} \text{ for a round rod} \end{aligned}\right\} \quad . \ (3)$$

d being the diameter, b the breadth at right angles to the plane of motion, and h the depth in the plane of motion.

Let G be the weight of a cubic inch of the material of the rod ($=0\cdot261$ lb. for iron). Then, $w = G\,b\,h$ or $\frac{\pi}{4} G d^2$. Hence,

$$\left.\begin{aligned} f_1 &= 1\cdot28\ \frac{V^2}{R}\,\frac{l^2}{g\,h} \text{ for a rectangular rod} \\ &= 1\cdot712\,\frac{V^2}{R}\,\frac{l^2}{g\,d} \text{ for a round rod} \end{aligned}\right\} \quad . \ (4)$$

If P is the pressure acting along the rod determined as above, the stress due to that pressure is

$$\left.\begin{aligned} f_t &= \frac{P}{b\,h} \text{ for rectangular rods} \\ &= \frac{4\,P}{\pi\,d^2} \text{ for round rods} \end{aligned}\right\} \quad . \quad . \ (5)$$

And the total stress is $f_t + f_1$, which must not exceed the safe limit of stress for the material of the rod. So far as the bending stress is concerned, the breadth has no influence on the strength. It is for this reason that in locomotive coupling-rods the depth is made greater than the breadth.

It would render the rules for connecting rods too complicated, to introduce the bending stress in the ordinary formulæ for proportioning them. But it is desirable, when a rod has been proportioned for the load on the piston, with a

factor of safety which allows for other straining actions, to test whether that allowance is sufficient, by examining the stress when inertia is taken into account.

292. *Strength of connecting rods.*—Connecting rods are often proportioned in an empirical way, their diameter being made proportional to the piston diameter. Thus, cast-iron rods for stationary engines have a section at the centre equal to about 0·056 of the piston area; locomotive rods are $\frac{1}{6}$ to $\frac{1}{5}$ the piston diameter; marine engine rods are $\frac{1}{8}$ to $\frac{1}{6}$ of the piston diameter. Hence, a close agreement of theoretical rules with actual practice is not to be expected. For very long rods, and when the bending stress, due to inertia, is neglected, the strength is determined by Rule II. in Table VII. But most commonly the ratio of length to diameter is such that the rules in § 39 are applicable.

Let P be the greatest longitudinal thrust transmitted through the rod; l, the length of the rod between the centres of the end journals; I, the moment of inertia of the section of the rod at the centre; A, the area of that section; n, the factor of safety. Then for long rods,

$$n\mathrm{P}=\pi^2 \frac{\mathrm{E\,I}}{l^2}$$

$$\left.\begin{aligned} \mathrm{P}&=57{,}240{,}000\ \frac{\mathrm{I}}{l^2} \text{ for wrought-iron} \\ &=27{,}970{,}000\ \frac{\mathrm{I}}{l^2} \text{ for cast-iron} \end{aligned}\right\} \quad . \quad (6)$$

For a rod having a circular section of diameter d, or a rectangular section, the smaller dimension of which is h and the greater $b=c\,h$,

$$\left.\begin{aligned} d&=\alpha\sqrt{\{l\sqrt{\mathrm{P}}\}} \\ h&=\beta\sqrt{\left\{l\sqrt{\frac{\mathrm{P}}{c}}\right\}}=\gamma\sqrt{\{l\sqrt{\mathrm{P}}\}} \end{aligned}\right\} \quad . \quad (7)$$

where $\alpha=0·02443$ for wrought-iron, and 0·0292 for cast-iron;

β=0·0214 for wrought, and 0·0256 for cast iron; and γ has the following values :—

$c=$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2
$\gamma=$·0214	·0203	·0194	·0187	·0181 for wrought iron.	
$=$·0256	·0242	·0231	·0222	·0215 for cast iron.	

For ordinary rods it is better to use the equations in § 39. Then

$$\left.\begin{aligned} P &= \frac{8500\,A\,I}{\cdot 00009\,A\,l^2 + I} \text{ for wrought iron} \\ &= \frac{2840\,A\,I}{\cdot 00027\,A\,l^2 - I} \text{ for cast iron} \end{aligned}\right\} \quad . \quad (8)$$

These equations are in an inconvenient form for determining the diameter from the thrust. The following formulæ, which have been obtained by applying Poncelet's rules for approximation, are extremely simple, and give very approximately the same results.

For connecting rods of circular section and diameter d,

$$\left.\begin{aligned} d &= 0{\cdot}01363\sqrt{\{l\sqrt{P} + 0{\cdot}79\,P\}} \text{ for wrought iron} \\ &= 0{\cdot}03394\sqrt{\{l\sqrt{P} - 0{\cdot}033\,P\}} \text{ for cast iron} \end{aligned}\right\} \quad (9)$$

For connecting rods of rectangular section, the lesser dimension being h, and the greater $b=c\,h$,

$$\left.\begin{aligned} h &= 0{\cdot}01194\sqrt{\left\{l\sqrt{\frac{P}{c}} + 0{\cdot}81\,\frac{P}{c}\right\}} \text{ for wrought iron} \\ &= 0{\cdot}02974\sqrt{\left\{l\sqrt{\frac{P}{c}} - 0{\cdot}034\,\frac{P}{c}\right\}} \text{ for cast iron} \end{aligned}\right\} (10)$$

If these equations are put in the form

$$h = C\sqrt{\{l\sqrt{P} \pm k\,P\}} \quad . \quad . \quad . \quad (11)$$

$c=$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2	
$c=$	·01194	·01129	·01079	·01039	·01004	wrought iron.
$=$	·02974	·02813	·02687	·02586	·02501	cast iron.
$k=$	·81	·72	·66	·61	·57	wrought iron.
$=$	·034	·030	·028	·026	·024	cast iron.

Other forms of cross-section than the circular and rectangular are occasionally used. For cast-iron rods, a cross-shaped section with rounded internal corners has been adopted. The modulus of this section is nearly the same as that of a square, the angles of which coincide with the middle points of the arms of the cross. For locomotive coupling-rods of wrought iron, a double-T or I-shaped section has been used, formed by slotting out the sides of a solid bar. By thus lightening the rod, the bending stresses due to the vertical oscillation of the rod are diminished.

293. *Diameter of connecting rod calculated from initial steam pressure on piston.*—Let P_1 be the load on the piston, due to the initial steam pressure. Suppose the diameter of the rod calculated for the load $P=m P_1$, where m is a factor of safety, intended to allow for the neglected straining actions, due to acceleration parallel and perpendicular to the direction of the piston's motion. Then, for engines of a given class, working in similar conditions, m may be assumed constant; and for different classes of engines, its value may be determined by examining the diameters which have been used in practice. More simply still, if d or h is the diameter or depth calculated by the formulæ above, when P_1 is put for P, then $d\sqrt[4]{m}$ and $h\sqrt[4]{m}$ will be the dimensions which should actually be taken, to allow for the straining actions additional to P_1, which have been left out of the reckoning.

$m=$	1·0	1·25	1·5	2·0	3·0	4·0	5·0	7·5	10
$\sqrt[4]{m}=$	1·0	1·06	1·12	1·2	1·3	1·4	1·5	1·6	1·8

For locomotives it appears that $m=1·25$ to $1·5$. For

stationary and marine engines m is much greater, being often 5, and ranging from 4 to 10 in different cases.

Simple rules for connecting rods.—It is common in practice to treat a connecting rod as subjected to tension, only allowing for all other straining actions in the factor of safety. Thus, let D be the diameter of the cylinder, in ins.; p the absolute initial steam pressure in lbs. per sq. in. Then the total tension in the rod is

$$P_1 = \frac{\pi}{4} D^2 p.$$

Let d be the least diameter of the connecting rod, f the safe stress, m a factor of safety—

$$\frac{\pi}{4} d^2 f = \frac{\pi}{4} m D^2 p$$

$$d = \sqrt{\frac{m}{f}} D \sqrt{p} \quad . \quad . \quad . \quad (12)$$

From some examples it appears that m may be taken at 2·5. Then let f=3,600 for cast iron, 9,000 for wrought iron, and 12,000 for steel:—

$$d = 0·0264\ D\sqrt{p} \text{ for cast iron.}$$

$$= 0·0167\ D\sqrt{p} \text{ for wrought iron.}$$

$$= 0·0144\ D\sqrt{p} \text{ for steel.}$$

The diameter at the centre of the rod may be $\frac{1}{12}$th to $\frac{1}{8}$th greater.

These rules, however, are best adapted for rods of moderate length in proportion to their diameter. For very long rods the rules previously given are better.

One end of a connecting rod is often forked so as to carry two journals. Then the forging is more complicated, and the brasses, cotters, &c. are doubled in number. A more serious objection is that it is not possible to secure

perfectly uniform distribution of the thrust to the two journals, and hence the straining action on the rod is increased in consequence of the deviation of the pressure from its centre line. That unequal distribution of the thrust is very liable to occur from the unequal tightening up of the brasses of the two journals.

CONNECTING-ROD ENDS.

294. *Proportions of steps.*—The ends of connecting rods are designed to receive crank pins or neck journals, and are fitted with gun-metal steps similar to those used for pedestals. The unit for the proportional numbers relating to the steps in connecting rods is

$$t = 0{\cdot}08\,d + \tfrac{1}{8} \quad . \quad . \quad . \quad (13)$$

where d is the diameter of an ordinary crank pin supporting the thrust transmitted by the connecting rod. When the connecting rod is attached to a journal of greater size than is sufficient, for the thrust of that connecting rod only,

$$t = {\cdot}007\sqrt{P_1} + \tfrac{1}{4} \text{ to } {\cdot}012\sqrt{P_1} + \tfrac{1}{4} \quad . \qquad (13a)$$

The flanges of the steps are of very variable thickness, but very often the space between the flanges of the steps in which the connecting-rod end is placed is $\frac{7}{10}$ths of the length of the journal.

295. *Strap end.*—Fig. 270 shows a very common form of connecting-rod end, having a loose strap enclosing the steps. This strap is kept in place by gibs and cotter. It will be seen that when the cotter is tightened up to neutralise the wear of the brasses, the rod is shortened in length. The strap is of wrought iron or cast steel, and its total section (on two sides of the rod end) is $2\,\beta\,\delta$ sq. ins. Let P_1 be the initial steam pressure on the piston, and let as above

m P_1 bc the greatest load on the strap, due to all causes of straining action, m being a factor of safety. Then

$$\beta\,\hat{c} = \frac{m\,P_1}{2f} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (14)$$

$$= \cdot 000055\, m\, P_1 \text{ for wrought iron.}$$
$$= \cdot 000037\, m\, P_1 \text{ for cast steel.}$$

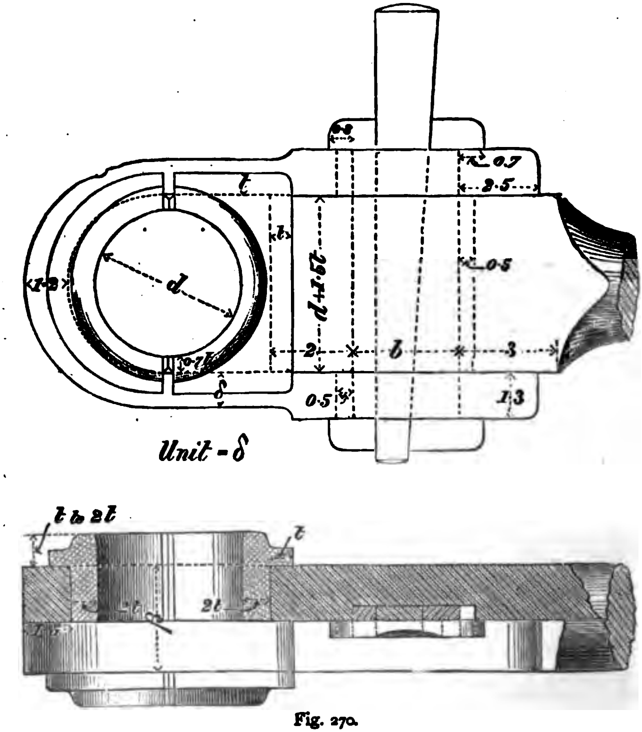


Fig. 270.

The factor of safety m appears to be large in most cases, being 3 to 4 for locomotives and 8 to 10 for other

engines. If a value is selected for β, then δ can be determined.

The total combined section of gibs and cotter is about

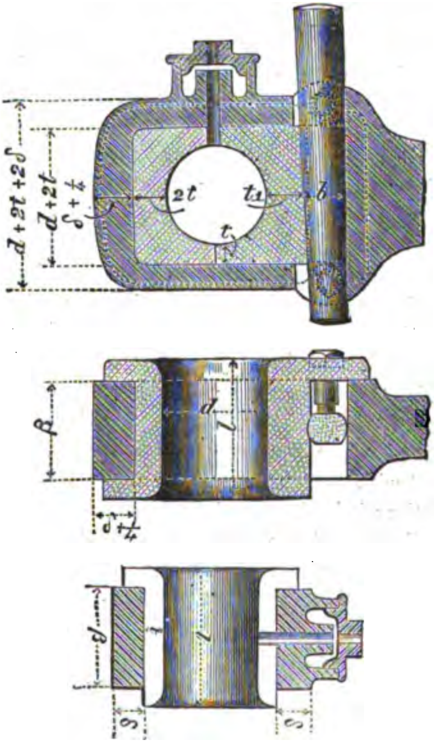


Fig. 271.

$1\frac{1}{4}\ \beta\ \delta$ when they are of the same material as the strap. If b is their total width, and $n\ b$ their thickness, then

$$b=\sqrt{\left(1{\cdot}25\,\frac{\beta\ \delta}{n}\right)}$$

where n is usually $\frac{1}{4}$ to $\frac{1}{5}$. The inclination of the sides of the cotter is $\frac{1}{2}$ inch per foot on each side, but if a set screw is used to lock the cotter, then the inclination may be 1 inch per foot. The proportions of the other parts may be obtained from the numbers given on the figure, the proportional unit being δ.

296. *Box end.*—Fig. 271 shows a connecting-rod end having no loose strap. The brass steps have a thickness $2\,t$ opposite the key, and $t_1 = 6\,t - \frac{1}{2}$ next the key. At the sides the thickness is reduced to t. The thickness and overlap of the flanges of the steps may be $\frac{1}{5}\,l - \frac{1}{8}$, so that the width of the box may be $\beta = \frac{3}{5}\,l + \frac{1}{4}$. The flanges of the steps are partially removed on one side to allow their insertion in place. The thickness δ of the sides of the box may have the same value as the thickness of strap in the last case. The mean breadth of the cotter is $0{\cdot}6\,\beta$ and its thickness $0{\cdot}3\,\beta$, and it tapers 1 in 12 on each side. It is secured by two set screws; diameter of set screws = cotter thickness. Unlike the last form, this connecting rod is lengthened when the cotter is tightened. But it may be arranged with the cotter on the other side of the brass steps, and then it is shortened by tightening the cotter. A coupling rod should have one end arranged in the former and one in the latter method. Then the length of the rod is not much altered by tightening the cotters. The proportional unit for this figure is δ.

Fig. 272 shows a locomotive connecting-rod end which, whilst it is adjusted like a box end, can be separated from the shaft like a strap-ended rod. The block forming the end is held in place by a stout pin, having a slight taper, with nuts at each end. The wedge is tapered 1 in 16, and is fixed by two peg pins driven through the adjusting bolt. The ends of the adjusting bolt are left long, and have double nuts at each end to facilitate the adjustment of the position of the wedge. The flanges of the steps are large to

cover the spaces in the wrought-iron frame of the end. In this case $\beta = 2{\cdot}3\ \delta$. For the proportions given on this rod t is

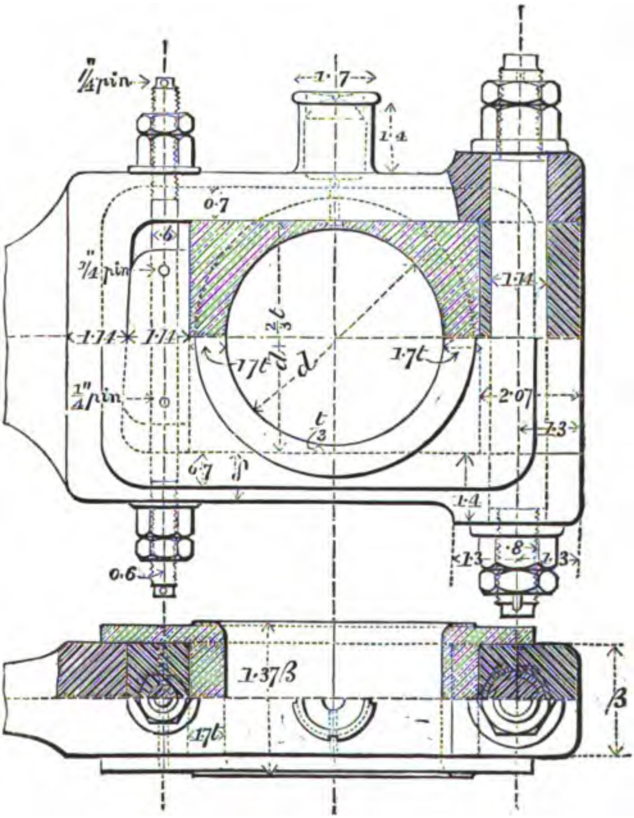


Fig. 272.

obtained by eq. 24, and the unit for the other proportional numbers is δ. The value of m is about $3{\cdot}12$.

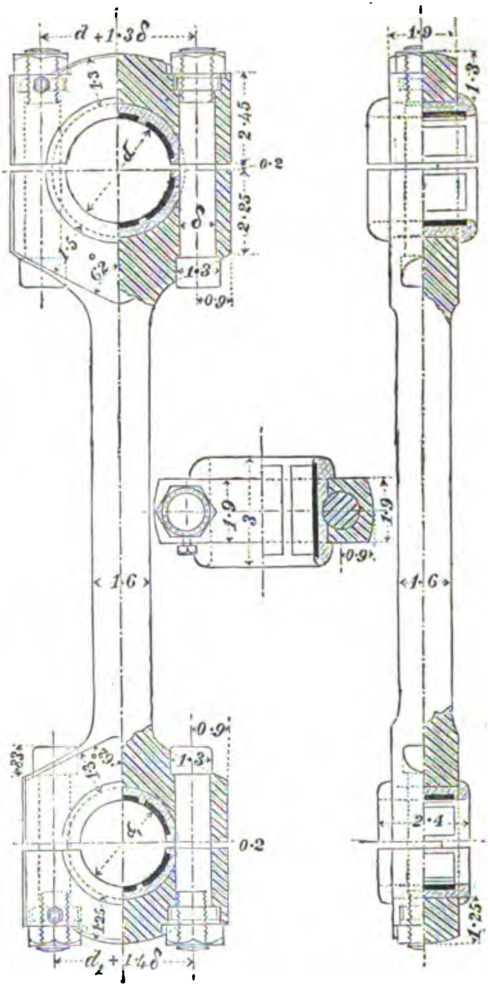


Fig. 273.

297. *Marine engine connecting-rod end.*—Fig. 273 shows another form of connecting-rod end. This is of simple and

massive form, and is often used in marine engines. The bolts here correspond to the strap in fig. 270. Hence, if δ is the diameter of these bolts,

$$\delta = \cdot 0084 \sqrt{(m\,\mathrm{P}_1)} = k\sqrt{\mathrm{P}_1} \quad . \quad . \quad . \quad (15)$$

$m =$	4	5	$7\frac{1}{2}$	10
$k =$	·0177	·0198	·0243	·0280

The numbers on the figure are proportional to δ. The brasses are lined with white metal or Babbitt's metal cast in shallow recesses.

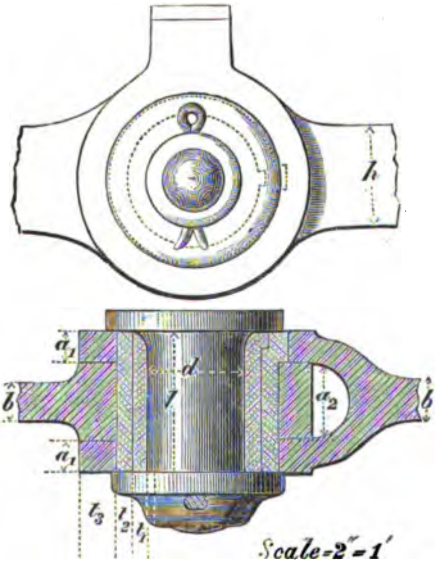


Fig. 274

298. Fig. 274 shows a coupling-rod joint which may serve as an example of a journal bearing where there is not a great amount of motion and wear. This joint is intermediate in construction between a common knuckle-

joint and a connecting-rod end. It has bushes to diminish friction and wear, but these are not divided, so that there is no adjustment after wear has taken place. The crank pin turns in a brass bush, which is protected by an outer steel bush. Both brass and steel bush are fixed in the forked-rod end by small snugs, and the solid-rod end turns on the steel bush. The pin in a joint of this kind is often larger than is necessary for strength, because, by using a large pin with a small intensity of pressure between the rubbing surfaces, there is less danger of squeezing out the lubricant. The pin is of steel. The proportions may be

$$t_1 = t_2 = 0{\cdot}1d + \tfrac{1}{8}.$$

$$a_1 = 0{\cdot}3\ d.$$

$$a_2 = 0{\cdot}8\ d.$$

$$t_3 = \frac{b\,h}{4\,a_1}.$$

For the brasses of parallel motion bars, which should be capable of being tightened without altering the position of the centres of rotation, the following ingenious plan has been suggested by Mr. Candlish ('Engineering,' xxxii. 461). The motion bars are fitted with bushes tapered internally to fit conical journals and parallel externally, with a feather to prevent the bushes from turning in the rod eyes. By regulating the position of the bushes by double nuts they can be adjusted when worn without interfering with the centres of motion.

Cross-Heads.

299. Cross-head is the name given to the part which connects together the piston rod and connecting rod of a steam engine, and with which is also connected the guiding arrangement either of slide blocks or parallel motion bars. It consists essentially of a socket to which the piston rod is

keyed, and a journal, or two journals, on which the connecting rod works. In the former case the connecting rod has a single end, in the latter it is forked. Generally there are arrangements for attaching the slide blocks to the cross-head.

The connecting rod works with less velocity of rubbing on the cross-head than on the crank-pin journal. Hence the former is of less length than the latter. Cross-head pins are usually neck journals, with a length equal to their diameter only. Putting d and l for the diameter and length of the cross-head journal for a single-end connecting rod, we have from eq. 18, p. 181,

$$d=l=\sqrt{\frac{1\cdot28}{f}}\sqrt{\text{P}},$$

where P is the maximum thrust in the connecting rod. If for P is put the piston pressure only, then the value of d so obtained must be multiplied by a factor of safety to allow for the additional straining action which has been neglected. In locomotive cross-heads that factor may be $1\frac{1}{4}$ to $1\frac{1}{2}$, and in stationary and marine engines $1\frac{1}{2}$ to 2.

300. *Forms of cross-heads.*—Only cross-heads for engines with slides will here be considered. The form of the cross-head depends primarily on the arrangement of the slides. There may be—(1) four slide bars, two on each side of the cross-head; (2) two slide bars in the plane in which the connecting rod oscillates; (3) a slipper slide on one side only of the cross-head.

Fig. 276 shows a simple cross-head for an arrangement of four slide bars. The cross-head is of wrought iron, cottered to the piston rod, and having a forked end embracing the connecting rod. A pin passing through the cross-head forms a neck journal for the connecting rod, and at the same time two end journals on which the slide blocks are fixed. The slide blocks are simple cast-iron blocks.

In large engines these blocks have brass faces on the rubbing surfaces. The pin must be fixed in the jaws of the

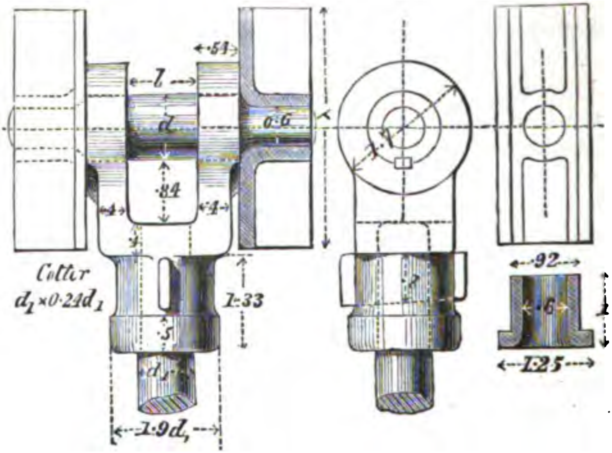


Fig. 275.

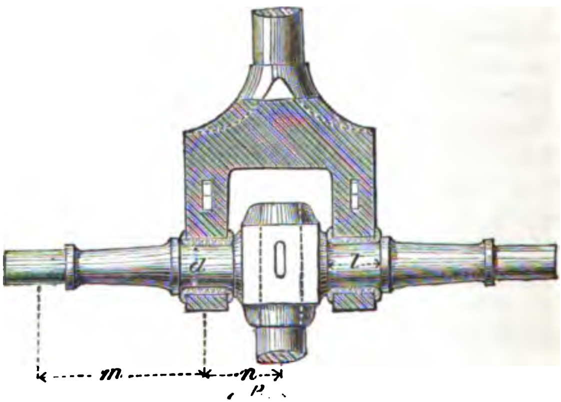


Fig. 276.

cross-head by a small key, shown in the end view, which prevents the rotation of the pin. For the connecting rod

journal $d=l$. The unit for the proportions of the other parts of the cross-head is d.

Fig. 275 shows the form of the cross-head pin when the cross-head has a single end and the connecting rod is forked. Each connecting-rod end is designed as above described, but for half the total thrust in the rod.

Fig. 277 shows a simple cross-head equivalent to that in fig. 275, but arranged with two slide bars only above and

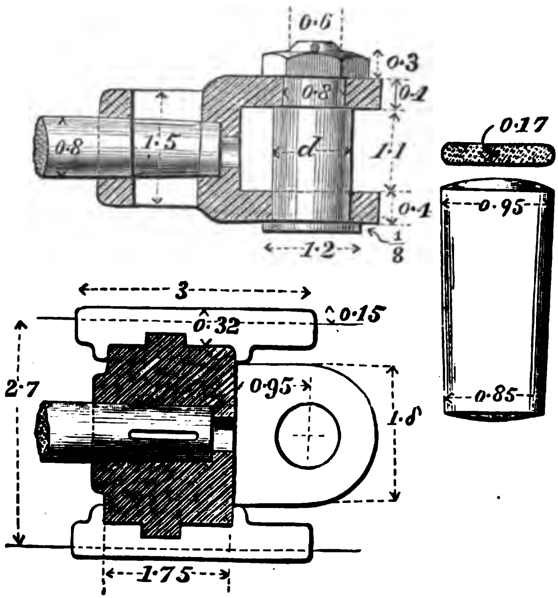


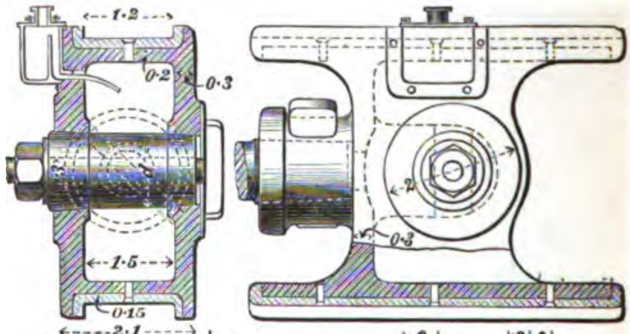
Fig. 277.

below the cross-head. The slide blocks are of cast iron, and the slide bars of steel. The design has, however, a fault not uncommon in cross-heads. In order that the vertical pressure may be uniformly distributed over the slide block, the centre of the cross-head pin should be over the centre of the slide block. If it is not so, either the pressure is very

unequal, or it is only prevented from being so by the stiffness of the piston rod.

Figs. 278, 279, show two forms of cross-head applicable

Fig. 278.



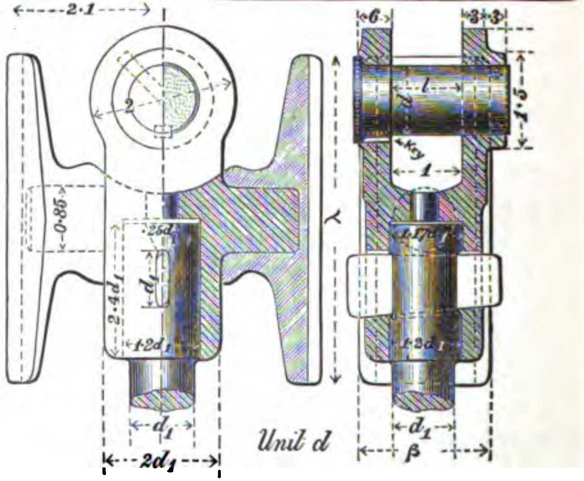


Fig. 279.

when there are two slide bars in the plane of oscillation of the connecting rods. The piston-rod socket is propor-

tioned to the piston-rod diameter, d_1. In both these examples the piston rod is enlarged at the cross-head end. This involves a split stuffing-box. The unit for the remaining parts is the cross-head pin diameter, d. In fig. 278 the cross-head is entirely of wrought iron, except the brass faces attached by set screws to the rubbing surfaces. The cross-head pin is kept in place by a T-headed bolt, which passes completely through it. The ends of the pin are tapered, and rotation of the pin is prevented by friction of the tapered parts.

In fig. 279 the cross-head of wrought iron and the slide blocks are separate, and of cast iron. The cross-head pin is kept in place by a split pin, and rotation is prevented by a small key inserted on one side.

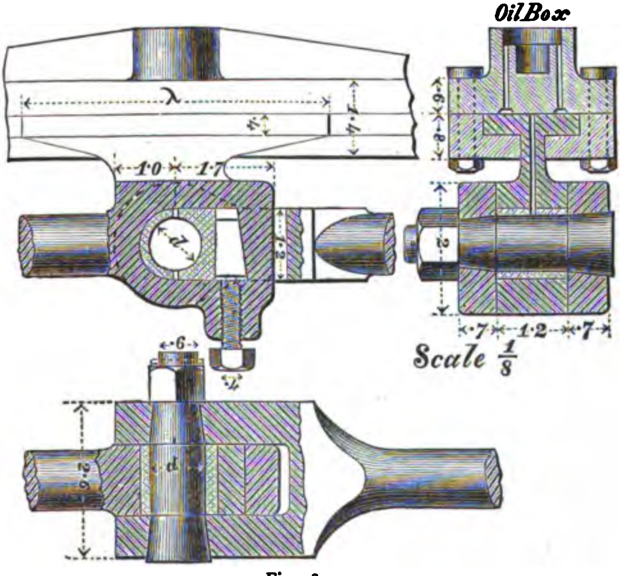


Fig. 280.

Fig. 280 shows a cross-head designed by Mr. Stroudley

for a slipper slide.[1] The cross-head is of wrought iron forged in one piece with the piston rod and slide block, and the connecting rod is forked at the end, and embraces the cross-head. The steps of the cross-head pin are of gun-metal, or of case-hardened wrought iron, and are tightened by a wedge and set screw. The cross-head pin, of case-hardened wrought iron, is fixed to the jaws of the connecting rod. In this case the slipper slide block is over the cross-head. In most cases, however, it is underneath it.

SLIDE BARS.

301. In most link-work arrangements it is necessary to guide the ends of some of the bars, so as to constrain them to move in straight lines. This can be done by an arrangement of links forming what is termed a parallel motion. Into the construction of parallel motions no elements enter which have not already been discussed. A parallel motion may be made to guide a given point with great accuracy, and with very little friction. On the other hand, it is a somewhat complicated arrangement, and if the links alter in length, in consequence of wear, it no longer properly answers its purpose. Hence parallel motions have been to a great extent superseded by a simpler arrangement of straight guiding surfaces termed slides. The only objection to these is that they involve more loss of work in friction than parallel motions. Slides are very commonly employed to guide the end of the piston rod at the point where the thrust is transmitted to the connecting rod.

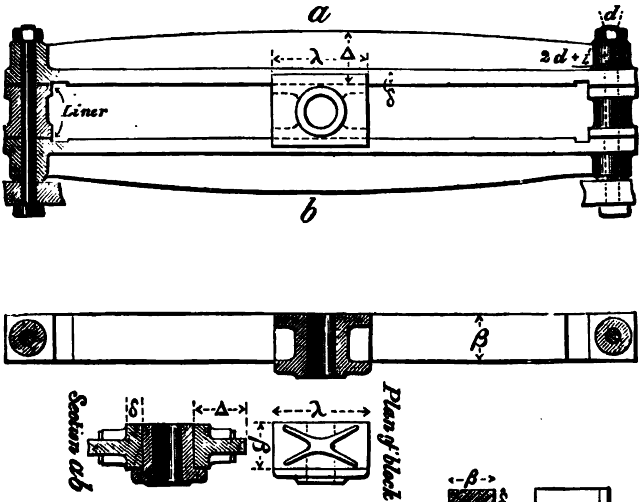
Fig. 281 shows two ordinary cast-iron slide bars with the slide block between them. The bars are of T section, and are spaced apart at the ends by distance pieces. Thin washers or liners are introduced between the distance pieces

[1] 'Engineering,' ix. p. 65.

and the bars, so that when the bars and slide block are worn, the bars can be brought closer together.

The bars are notched at the ends, and the slide block

Fig 281.



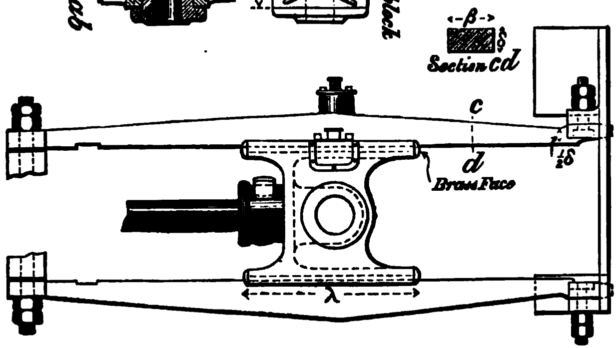


Fig. 282.

passes the edge of the notch at each stroke. This prevents the formation of a ridge at the end of the stroke, in consequence of the wear of the bar. Ample provision must be

made for lubricating the bars. The slide blocks may be of cast iron or of gun-metal. They fit on journals at the end of the cross-head pin. The arrangement will be understood, if the cross-head and slide blocks in fig. 275 are compared with the slide block and slide bar in fig. 281.

Fig. 282 shows slide bars of wrought iron (sometimes case-hardened) or steel slide bars. In the arrangement here shown, the bars are above and below the cross-head. The bars are rectangular in section, and thickest at the centre where the thrust is greatest. The cross-head here shown has brass faces. With this kind of arrangement, in large engines, provision is made to neutralise the wear of the bars by separating the surfaces of the slide blocks.

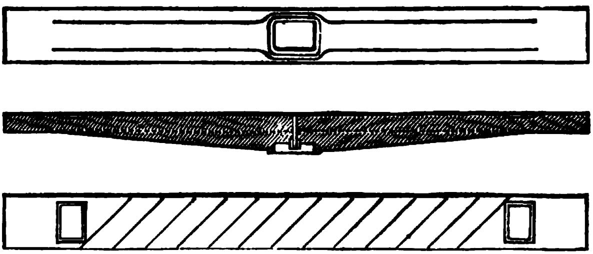


Fig. 283.

Fig. 283 shows an American slide bar of cast iron[1] partly chilled on the surface. The chilling has been effected in diagonal strips, leaving equally wide spaces of unchilled cast iron between. The arrangement seems well adapted to secure good lubrication, and at the same time a sufficient area of very hard surface.

A slipper slide is sufficiently shown in the arrangement already described, fig. 280. The slide block is a T-shaped piece, forged in one with the cross-head, and this is guided in a groove formed by a flat slide bar and two L-shaped bars. Other forms of slide bar are used, the sliding

[1] Rigg, 'Steam Engine,' p. 285.

surfaces being sometimes wedge-shaped and sometimes cylindrical.

302. *Wearing surfaces of Slide Blocks.*

Let P=total piston pressure in lbs.

Q=thrust of slide blocks on slide bars.

$a=\beta\lambda$=area of slide block surface.

r=crank radius.

l=connecting rod length.

v=mean velocity of piston.

The pressure of the slide blocks on the slide bars is greatest when the crank and connecting rods are at right angles. Then

$$Q = P\frac{r}{l} \text{ nearly} \quad . \quad . \quad . \ (16)$$

It is for this thrust that the strength of the bar must be calculated. Since, however, the obliquity of the connecting rod is constantly varying, the mean pressure between the block and slide bar is

$$Q_m = P\frac{0{\cdot}785\, r}{\sqrt{(l^2 - 0{\cdot}617 r^2)}} = k\, P \text{ very nearly} \quad . \ (17)$$

$\frac{l}{r}=$	$3\frac{1}{2}$	4	5	6	7
$k=$	·2303	·2003	·1590	·1320	·1129

Since the second term in the bracket is in most cases small compared with the first,

$$Q_m = \frac{\pi}{4}\,\frac{r}{l}\,P \text{ nearly} \ . \quad . \quad . \ (18)$$

The work wasted in friction is

$$T = Q_m v = \frac{\pi}{4}\,\frac{r}{l}\,P\,v \text{ foot lbs. per second.}$$

The heat produced is

$$H = \frac{T}{J} = \frac{\pi}{3088}\,\frac{r}{l}\,P\,v \text{ units per second;}$$

and supposing that h units are dissipated per second by conduction through the metal pieces from each unit of area of the slide block,

$$a = \frac{\pi}{3088 h} \frac{r}{l} \text{P} v \quad . \quad . \quad . \quad (19)$$

Hence it would appear that the slide block surface should be proportional to the ratio $\frac{r}{l}$, to the mean velocity of the piston, and to the mean pressure on the piston, or for engines working in similar conditions, to the area of the piston.

Let Q be the maximum pressure on the slide block, a the area of the side block, or blocks. Then

$$a = \frac{\text{Q}}{\gamma},$$

where γ is the greatest pressure permitted per sq. in. of slide block surface. From data collected by Mr. Rigg,[1] it appears that γ ranges in different examples from 22 to 120 lbs. per sq. in. With steel blocks or bars the pressure may probably reach 60 lbs. per sq. in. without excessive wear. With cast-iron blocks and slides it is better to not exceed 40 lbs. per sq. in. If β is the width, λ the length, of a slide block, then $a = \beta \lambda$, if the pressure comes on one slide block; and $a = 2 \beta \lambda$, if the pressure is distributed to two slide blocks.

303. *Strength of the Slide Bar.*—Let m and n be the distances from the centre of the connecting rod eye to the points of support of the slide bar, when the crank and connecting rod are at right angles. Then the greatest bending moment on the slide bar, immediately under the connecting rod, is

$$\text{P} \frac{r}{l} \cdot \frac{m n}{m + n} \quad . \quad . \quad . \quad (20)$$

Hence, if the section of the bar is rectangular, of breadth β and thickness δ,

[1] 'Treatise on the Steam Engine,' p. 124.

$$\tfrac{1}{6}\,\beta\,\delta^2 f = \mathrm{P}\,\frac{r}{l}\cdot\frac{m\,n}{m+n},$$

$$\delta = \sqrt{\left\{\frac{6}{f}\cdot\frac{r}{l}\cdot\mathrm{P}\frac{m\,n}{(m+n)\,\beta}\right\}}$$

$$= k\sqrt{\left\{\mathrm{P}\,\frac{m\,n}{(m+n)\,\beta}\right\}}\quad . \quad . \quad . \; (21)$$

The limiting stress should be taken at 6,000 lbs. for wrought iron or steel to allow for the straining actions due to reaction, and to secure stiffness; and at about 3,000 lbs. for cast iron. Hence,

$\frac{l}{r}=$	$3\frac{1}{2}$	4	5	6	
$k=$	·0169	·0158	·0141	·0129	for wrought iron,
$=$	·0239	·0224	·0200	·0183	for cast iron.

The T-shaped section for cast iron is more rigid, but not much stronger than if the feather were omitted.

When the slide bars are horizontal, the weight of the connecting rod, cross head, &c., rests on the lower bar. If, then, the engine runs only in one direction, it may be arranged so that the thrust, due to the pressure transmitted, acts on the upper bar, provided at least that the crank is driven by the piston, and that the crank does not for part of the stroke drag the piston. Then the weight and thrust partially neutralise each other, and friction and wear is diminished. If the engine runs in both directions, but more constantly forwards than backwards, the surface of the slide block, which receives the thrust when running forwards, is often greater than that which receives the thrust when running backwards. This is the case in fig. 280. When the engine runs forward, the thrust is upward. When running backward, the thrust is downward.

The surfaces of slides are usually plane. Sometimes the slide bars are cast on the cylinder, and then it is most

convenient to bore out the space for the slide block concentric with the cylinder. Necessarily then the surface of the slide bars is cylindrical.

In slide bars having large surfaces and well lubricated the wear is small. Hence sometimes adjustments to neutralise wear are omitted.

CHAPTER XV.

PISTONS AND STUFFING-BOXES.

304. A piston, or plunger, is a sliding piece which is either driven by fluid pressure or acts against fluid pressure as a resistance. Pistons and plungers are commonly circular in section, and are guided by cylindrical bearing surfaces, so as to reciprocate in a straight path. But other forms of piston are occasionally used.

A plunger is a single-acting piston—that is, a piston receiving the action of the fluid on one face only—and it is guided, not by the cylinder itself, but by a stuffing-box in the cylinder cover. The bearing surface of the plunger therefore requires to be longer than the stroke. The stuffing-box forms the only joint requiring attention to keep it staunch, and it is accessible without removing the plunger. A piston is equivalent to a short plunger entirely contained within the cylinder and guided by it. The force is transmitted through a piston rod of relatively small area. Hence the piston has two faces on which the fluid pressure can act, and it is usually double-acting. With a piston there are two joints requiring to be kept staunch, one within the cylinder, and one where the rod passes through the cylinder cover. A large hollow piston rod is termed a trunk. The pistons of pumps are often termed buckets.

305. *The volume swept through* by an ordinary piston is the product of the transverse section of the piston normal to the direction of motion, and the length of its path. With an incompressible fluid, such as water, the volume swept through

is the volume of water lifted, in the case of a pump, or acting on the machine, in the case of a pressure engine or ram.

Work done on a Piston.—The work done on a piston by fluid pressure is the product of the volume swept through by the piston and the intensity of the fluid pressure. If the fluid pressure is variable, the mean intensity of the fluid pressure is to be taken. If the work is to be in foot lbs., the volume swept through may be in cub. ft. and the pressure in lbs. per sq. ft., or the volume swept through in units of 12 cub. inches and the pressure in lbs. per sq. inch.

Velocity of Piston.—Ordinarily a piston drives or is driven by a crank, rotating with nearly uniform velocity Then the motion of the piston is approximate harmonic motion, varying from rest at each end of the stroke to a maximum near mid stroke. The acceleration is greatest at the beginning of the stroke, vanishes near mid stroke, and changes sign and increases to another maximum at the end of the stroke.

306. *Influence of the weight of the Piston on the Crank-Pin pressure.*—When a piston is driven by a constant pressure, it is generally desirable to make the piston as light as possible, because the inertia of the piston causes the crank-pin effort to be more irregular than it otherwise would be. When, however, the pressure on the piston varies, the inertia of the piston may be used to diminish the variation of the crank-pin effort, and to make the total pressure on the crank pin nearly uniform.

Usually when the inertia of the reciprocating parts is intended to equalise the effort on the crank pin in expansive engines, the weight, w, of the piston, piston rod, and cross-head is so adjusted that at the intended speed of the engine

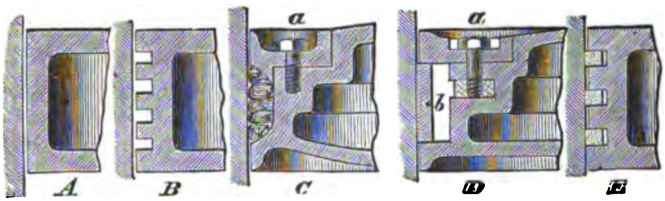
$$\frac{w\,v^2}{g\,R\,\omega} = \text{about } \tfrac{1}{2}\,p,$$

where v is the velocity of the crank pin, R the radius of the

crank, ω the area of the piston, and p the initial steam pressure. (Units, feet and lbs.)

307. *Construction of Pistons.*—Various arrangements have been adopted to diminish the leakage between the piston and the sides of the cylinder in which it slides. The piston may be simply turned to fit the cylinder accurately (A, fig. 284); but, however good the fit at first, the wear of the cylinder and piston will gradually enlarge the clearance between them, and the leakage will steadily increase. If a series of recesses are cut round the piston circumference (B, fig. 284), the leakage for any given width of clearance

Fig. 284.



space is less, because the fluid loses its energy of motion, at each sudden enlargement of the section of the annular space between the piston and cylinder, through which it is escaping. Pistons of this kind are used for quick-running pumps, where a small leakage is not very prejudicial. Leakage may be prevented by placing, in a recess in the piston, a packing of gasket or tallowed rope (c, fig. 284). This soft and elastic packing is compressed against the cylinder by a junk ring, shown at *a*, which is fixed by studs or set screws. As the gasket wears away, it can be replaced, and thus the permanent staunchness of the piston is secured. Pistons of this kind are not now much used for steam cylinders, though they are still employed for air pumps and cold water pumps. The objection to them is that the repacking of the piston is troublesome.

and the friction of the piston is considerable; also with high-pressure steam hemp packing is charred. To diminish the wearing away of the gasket, a face ring or spring ring, shown at *b*, was introduced (D, fig. 284), made of cast iron and divided on one side, to allow it to expand to the cylinder diameter as it wore away. The space behind the spring ring was at first filled by gasket packing, but it was found better to substitute steel springs for gasket, which retain their elasticity much longer, and press the spring ring outwards quite as effectively. In small pistons the elasticity of the spring ring itself is sufficient to maintain contact with the cylinder. The spring ring is then free from the piston. Various arrangements of this kind have been used. Sometimes the spring ring is a cast-iron ring, of uniform or varying thickness. Ramsbottom's rings are shown at E, fig. 284. These consist of a continuous spiral steel ring of about 3 coils, or of 3 separate steel rings, each split on one side. The rings are initially of one-tenth larger diameter than the cylinder, and, when compressed within it, press outwards with sufficient force to prevent leakage. The width of the rings (parallel to the axis of the cylinder) is about $0{\cdot}175 + \frac{d}{120}$ and the thickness about $0{\cdot}06 + \frac{d}{48}$, where d is the diameter of the cylinder. Ramsbottom's rings are usually of steel. Cast iron answers very well for small spring rings. It retains its elasticity till the ring is half worn through. Cast-iron rings are sometimes of uniform thickness, but very often they are one-half thicker at the middle of the ring than at the ends where the ring is split. At F, fig. 284, is shown an arrangement for admitting the steam pressure in the cylinder to the back of the rings. In principle this is a good arrangement, but, in this form, it does not succeed very well, and is not very often adopted. In Stroudley's piston the steam is admitted to the back of the spring ring on the opposite side to that on which the steam acts. This ring, passing a little beyond the bored part of the cylinder, prevents the formation of a shoulder at the end of the cylinder.

Bramah's cup leather is a perfectly successful application of the same principle. For pumps and blowing cylinders, wood blocks have been used to replace the spring ring. The packing of a piston may or may not share with the piston rim the pressure of the piston against the cylinder sides, whether due to the weight of the piston, as in horizontal engines, or to other causes.

The spring rings or metallic packing of pistons may be of cast iron, of wrought iron, of steel, or of gun metal. For steam cylinders cast iron wears better than wrought iron, and about as well as gun-metal. Gun-metal is chiefly employed in pumps and in pistons of complicated types, the action of which would be impaired by corrosion. Steel is a good material for packing, especially where considerable elasticity is necessary. Cast-iron and wrought-iron rings may be made more elastic by hammering.

308. *Strength of Pistons.*—Pistons are of a complicated form, and it is not easy to determine their strength theoretically. If the piston were a simple metal disc, supported at the centre and uniformly loaded, the greatest stress would be

$$f = k\frac{d^2}{t^2}p \quad . \quad . \quad . \quad (1)$$

where d is the diameter of the piston, t its thickness, p the greatest difference of the pressures on the two sides,[1] estimated per unit of area, and k is a constant. Putting $f =$ 8,000 for wrought iron and 3,000 for cast iron, we get

$$\left.\begin{aligned} t &= \cdot 0051\, d\sqrt{p} \text{ for wrought iron} \\ &= \cdot 0083\, d\sqrt{p} \text{ for cast iron} \end{aligned}\right\} \quad . \quad (2)$$

These values of t will be taken as empirical units for the proportions of pistons. Since, however, the form of pistons varies greatly, and also the conditions under which they work, the draughtsman should not depend solely on the following proportional figures, but should deduce the

[1] That is, p is very nearly the initial absolute steam pressure in the case of a condensing engine, and the initial steam gauge pressure in a non-condensing engine.

proportional figures for himself, from good examples of pistons of a similar kind to the one he is designing.

Piston rods.—From reasoning similar to that in § 293, putting d for the cylinder diameter and p for the steam pressure, the piston-rod diameter may be—

$$\text{If of wrought iron, } \delta = 0{\cdot}0167\, d\sqrt{p}.$$
$$\text{If of steel } \qquad \delta = 0{\cdot}0144\, d\sqrt{p}.$$

Steel has the advantage that it is less likely to become fluted

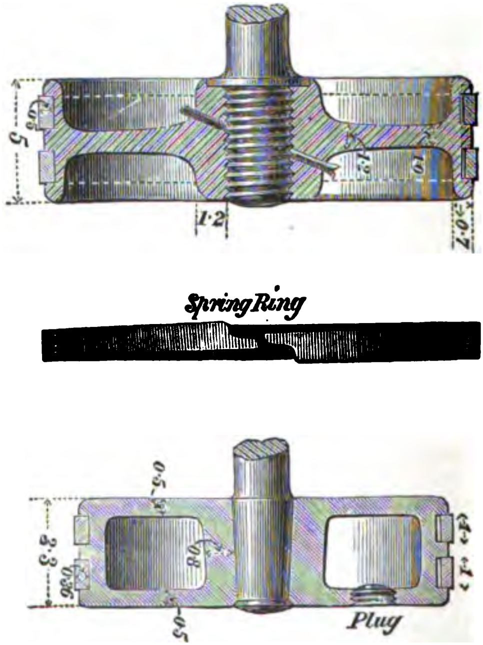


Fig. 285.

in wear than wrought iron. As these rules allow a large margin of strength, the screwed or cottered part of the rod is sometimes rather less than δ.

309. *Locomotive Pistons.*—Fig. 285 shows two forms of piston used in locomotives. One is constructed chiefly of

cast, the other chiefly of wrought iron. Wrought iron is preferred by some engineers on account of its toughness and strength. But cast iron is much cheaper and answers well. The spring rings in both cases are of cast iron and require no springs or packing. These rings are of uniform section, about $1\frac{1}{2}$ inches wide by $\frac{1}{2}$ inch thick, in pistons of average size. The split is made with a half lap, to prevent leakage at that point. The rings are sprung into the recesses in the piston, and should be so placed that the splits in the

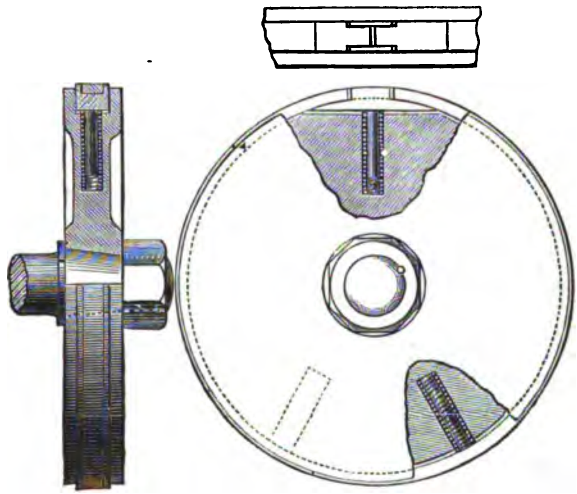


Fig. 286.

two rings are on opposite sides of the piston. This equalises the wear of the cylinder. A small screw is sometimes used to prevent the rings turning round in the grooves. The piston rod is screwed into the wrought-iron piston and fixed by a split pin. In the case of the cast-iron piston, the rod is slightly coned at the end, and, when in place, is riveted over. The holes filled by screw plugs are intended for the removal of the sand core after casting.

Fig. 286 shows another locomotive piston. In this three spiral springs are placed behind the spring ring, and

assist the elasticity of the latter in keeping the piston tight. A brass tongue-piece prevents leakage at the joint in the spring ring. The piston rod has a strong taper to enable it to be easily removed, and it is secured by a screwed end and nut. The spiral springs are so placed as to prevent the body of the piston bearing on the bottom side of a horizontal cylinder.

310. *Stationary Engine Pistons.*—Fig. 287 shows one form of stationary engine piston. It is made of cast iron, with a junk ring to confine the metallic packing. The packing consists of three cast-iron rings of the sectional

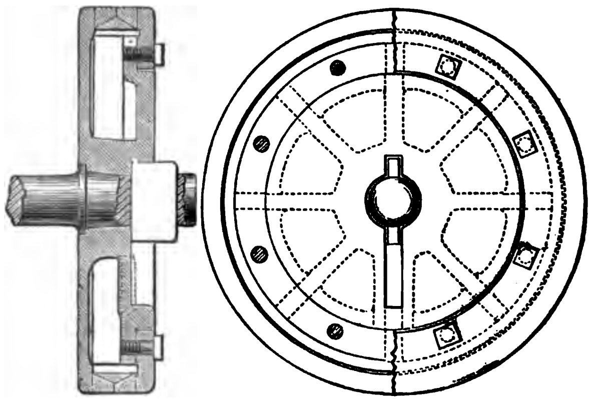


Fig. 287.

form shown. The outer rings are turned $\frac{1}{16}$ inch larger than the cylinder diameter, and are split. The inner ring may or may not be split. By screwing down the junk ring the two outer rings are forced outwards, as they slide down the conical surfaces of the inner ring, and thus any desired amount of pressure can be obtained between the piston and cylinder. The inner ring has sometimes been made in the form of a spiral spring. It then presses the outer rings both apart and outwards. In this piston the rod is tapered at the end and fixed by a cotter.

Joy's Piston (fig. 288).—The piston is a simple block,

into which the piston rod is screwed and pinned. The diameter of this block is $\frac{1}{16}$ inch less than the bore of the cylinder. A recess is cut with a tool set to $\frac{1}{2}''$ pitch, and making 3 inches more than two revolutions. The cast-iron ring from which the packing is made is turned and bored, $\frac{5}{8}$ inch thick, and $\frac{3}{4}$ inch larger in diameter than the cylinder. It is then placed on a mandrel, and a spiral groove cut with a $\frac{1}{4}$-inch tool set at $\frac{5}{8}$-inch pitch, so as to form a spiral spring of $\frac{5}{8}$ inch by $\frac{1}{2}$ inch in section. The spiral spring gives probably a more uniform pressure on the cylinder than a simple ring, and its axial elasticity prevents its knocking in the groove.

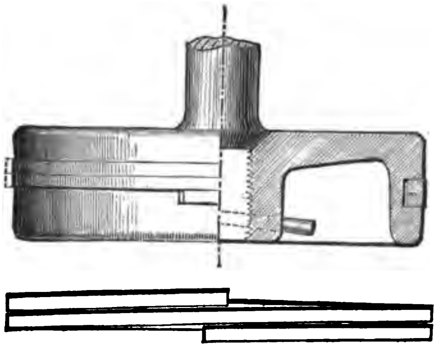


Fig. 288.

311. *Marine Engine Pistons.*—Marine engine pistons are often of very large size, and are usually of cast iron, of a box-shape and stiffened by numerous ribs. Fig. 289 shows a piston of this kind. The spring ring of cast iron is of uniform thickness. Leakage at the split is prevented by a brass tongue-piece, fixed to one end of the spring ring by screws. The split in the spring ring is shown square across the spring ring. But it is better to cut it obliquely so that the edges may not score the cylinder. The spring ring is pressed outwards by numerous plate-springs, placed in recesses cast in the rim of the piston. The spring ring and springs are kept in place by a junk ring. This last is attached

to the piston by bolts, which have brass nuts placed in recesses behind the plate springs. To prevent these bolts slacking back, in consequence of the vibration of the piston, various locking arrangements are used. In the piston shown

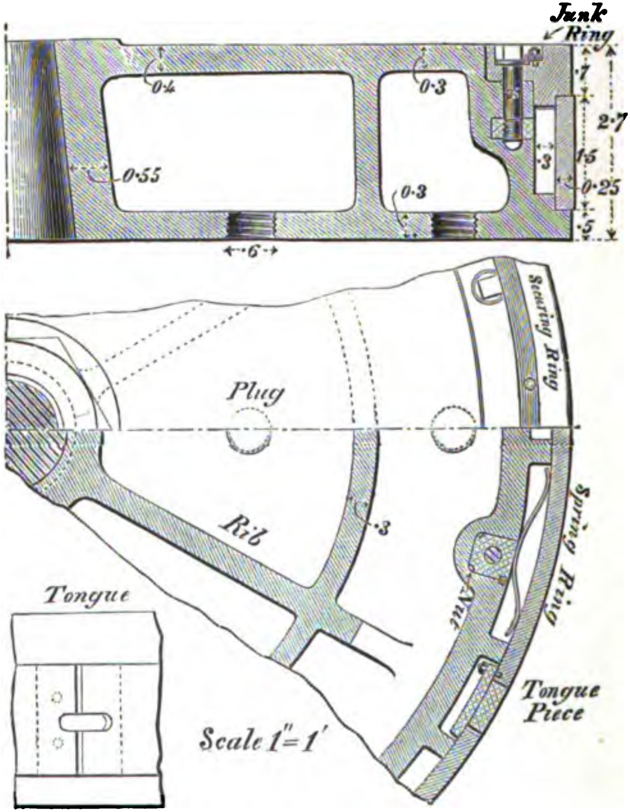


Fig. 289.

(a type used by Messrs. Humphreys and Tennant), a securing ring bears against the heads of all the junk ring bolts. This ring is attached to the piston by studs, the nuts being fixed by split pins. Holes are cast in the piston for clearing out

The amount of cotton wick necessary to supply sufficient oil is ascertained by trial.

343. *Needle lubricator.*—Fig. 323 shows Lieuvain's needle lubricator, oftenest used for the bearings of shafting. It consists of a glass reservoir having a wooden plug. This is filled and inverted over the bearing. A wire needle or pin passes through the plug loosely and rests on the journal. When the shaft is running, the vibration of the needle causes a slow descent of the oil. When the shaft is at rest, the capillary attraction stops the flow of the oil. The rate of supply of oil may be regulated by making the needle thicker or thinner.

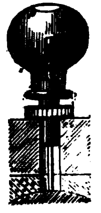


Fig. 323.

APPENDIX.

AFTER the chapter on Toothed Gearing was in type, the author received from Mr. Briggs, of Philadelphia, the following paper on the use of worm gearing. Mr. Briggs was the first to notice that the ordinary method of drawing the teeth of worm wheels was erroneous, and his practical experience in the use of worm gearing gives value to his observations.

Worm Gearing.

'There is a current opinion among machinists in general that worm gearing offers so disastrous a frictional resistance in wear, that its use, except for purposes where little power is to be transmitted and where certain slow movements are to be effected, is not permissible in good mechanism. This view is supported by most of the text-books; and the English writers, beginning with Professor Willis, cannot be said to advise or recommend worm gearing, while they invariably represent the laying out of the teeth by considering the worm as a rack with inclined teeth, where the pitch lines of the worm and wheel are taken on a plane passing through the axis of the worm, at the same relative distances on the respective teeth of wheel and worm as is usual for any rack and wheel gearing.

'Now, the fact is, that the use of worm gearing for hoists, cranes, boring-bars, lathes, &c., has been growing in favour, and it is found that neither excessive loss of power nor excessive wear of gearing ensues. In regard to friction, it is established that for ordinary ratio of wheel to worm, say not to exceed 60 or 80 to 1, well-fitted worm gear will transmit motion backward through the worm, exhibiting a lower coefficient of friction than is found in any other description of running machinery. For

the text-books, also, it should be remarked that Weisbach (1850 about), in his illustration of a worm gear, shows (without description) the pitch-lines located to give an equality of bearing face to both the teeth of wheel partly encircling the worm and to those of the worm thus segmentally surrounded.

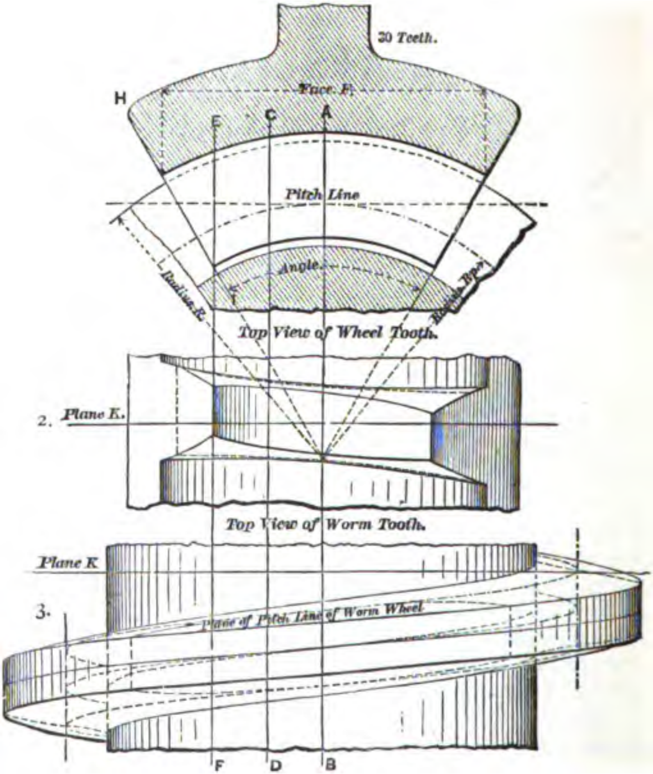


Fig. 324.

'It remains to be shown how to lay out a worm gear and worm so that this result will be reached; and to exhibit this, the accompanying figures of a worm in position have been prepared.

'Accept the teeth on the worm to be 0·65 of the pitch, radially, of which 0·60 p is to be the line of contact with the teeth of the wheel (on the radius and also on the plane through

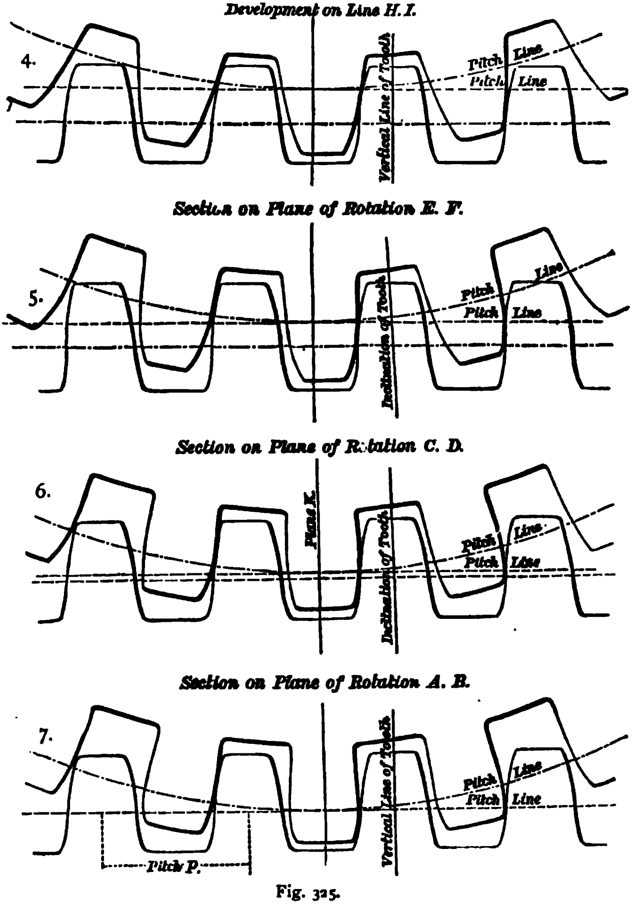


Fig. 325.

the middle of wheel), with 0·05 p for clearance between the roots and points of worm and wheel teeth.

'Let the teeth of the wheel follow the circle of the worm through the arc $2a$, which ought not to exceed 60°, and is shown as 60° in the figures. Let R = outside radius of worm; R_p = radius of pitch line of worm; p = pitch; F = width of face of wheel at the root of the teeth. Then

$$R_p = \tfrac{1}{2}\{R + (R - 0{\cdot}6p)\,\text{Cos}\,a\}$$
$$F = 2\,(R + 0{\cdot}05p)\,\text{Sin}\,a.$$

'To simplify the process of drawing worm wheels, it has been usual to make $R = 2p$ and $2a = 60°$. Then

$$R_p = 1{\cdot}606p.$$
$$F = 2{\cdot}05p.$$

'It will be found better to limit the number of teeth in the worm wheel to not less than 30; and if any less ratio of speed than 1 in 30 be demanded, to employ double or treble threaded screws.

'The figures consist of (1) a cross-section of worm wheel and worm; (2) a top view of wheel teeth; (3) a top view of worm teeth; (4) a side view development on the line or plane H I, or on the inclined face of the wheel teeth, and forming a radial section through the worm; (5, 6, and 7) horizontal sections on the planes or lines A B, C D, and E F, where the plane A B passes through the middle of the wheel and on the axis of the worm, and the planes C D and E F are parallel to A B, as shown on cross-section of worm wheel and worm.

'The only effect of this method of setting out pitch lines for the teeth of screw gearing is to bring the bearing or working lines of contact for both orders of teeth more nearly to the true pitch line, and not to throw much effort or work on the points of the teeth of the worm wheel outside of the true pitch line. It cannot be denied that it is possible to run two gears together, the pitch lines of which shall be above the point of the teeth of the one and above the roots of the teeth of the other, but it is insisted that such possible construction is not good practice.'

It will be seen that Mr. Briggs does not explicitly state how the curves of his teeth are drawn. Fig. 1 can be drawn from the proportions he gives and the known pitch and number of teeth of the worm wheel. The section on the plane A B can then be drawn, the pitch and root lines being taken from fig. 1,

and the teeth drawn as for a cycloidal wheel and rack. Fig. 3 can next be drawn from the data in the section on the plane A B, and the ordinary rules for drawing helixes. So far there is no difficulty. To draw the sections on the planes C D and E F, it is perhaps accurate enough for practical purposes to proceed thus. Draw first the worm teeth by setting off the distances from the central helix to the other helixes in fig. 3. This gives three points on each tooth curve if three helixes have been drawn, and a line sketched through them will approximate to the true curve. The worm wheel teeth curves may then be drawn by the method in § 206, Chapter IX.

INDEX.

PRINTED BY
SPOTTISWOODE AND CO., NEW-STREET SQUARE
LONDON

Date

1.

2.

3.

4.

5.

6.

UNIV. OF MICH.

Mor............ Sample........ Vols. of this title..

Duck........... S. B.

Cloth........... $............

Collated.................. Finished...................

Sewed..... .... Charged...................

ELECTRICITY and MAGNETISM. By FLEEMING JENKIN, F.R.SS.L. & E. late Professor of Engineering in the University of Edinburgh. With 177 Woodcuts. Price 3*s.* 6*d.*

THEORY of HEAT. By J. CLERK MAXWELL, M.A. LL.D. Edin. F.R.SS. L. & E. With 41 Woodcuts. Price 3*s.* 6*d.*

TECHNICAL ARITHMETIC and MENSURATION. By CHARLES W. MERRIFIELD, F.R.S. Price 3*s.* 6*d.* KEY, by the Rev. JOHN HUNTER, M.A. Price 3*s.* 6*d.*

INTRODUCTION to the STUDY of INORGANIC CHEMISTRY. By WILLIAM ALLEN MILLER, M.D. LL.D. F.R.S. With 72 Woodcuts. Price 3*s.* 6*d.*

TELEGRAPHY. By W. H. PREECE, F.R.S. M.I.C.E. and J. SIVEWRIGHT, M.A. C.M.G. With 160 Woodcuts. Price 5*s.*

The STUDY of ROCKS, an ELEMENTARY Text-Book of PETROLOGY. By FRANK RUTLEY, F.G.S. of Her Majesty's Geological Survey. With 6 Plates and 88 Woodcuts. Price 4*s.* 6*d.*

WORKSHOP APPLIANCES, including Descriptions of some of the Gauging and Measuring Instruments—Hand-Cutting Tools, Lathes, Drilling, Planing, and other Machine Tools used by Engineers. By C. P. B. SHELLEY, M.I.C.E. With 291 Woodcuts. Price 4*s.* 6*d.*

STRUCTURAL and PHYSIOLOGICAL BOTANY. By Dr. OTTO WILHELM THOMÉ, Rector of the High School, Cologne, and by A. W. BENNETT, M.A. B.Sc. F.L.S. With 600 Woodcuts and a Coloured Map. Price 6*s.*

QUANTITATIVE CHEMICAL ANALYSIS. By T. E. THORPE, F.R.S. Ph.D. Professor of Chemistry in the Andersonian University, Glasgow. With 88 Woodcuts. Price 4*s.* 6*d.*

QUALITATIVE ANALYSIS and LABORATORY PRACTICE. By T. E. THORPE, Ph.D. F.R.S. Professor of Chemistry in the Andersonian University, Glasgow; and M. M. PATTISON MUIR, M.A. and F.R.S.E. With Plate of Spectra and 57 Woodcuts. Price 3*s.* 6*d.*

INTRODUCTION to the STUDY of CHEMICAL PHILOSOPHY; the PRINCIPLES of THEORETICAL and SYSTEMATICAL CHEMISTRY. By WILLIAM A. TILDEN, D.Sc. London F.R.S. With 5 Woodcuts. Price 3*s.* 6*d.* With Answers to Problems, 4*s.* 6*d.*

ELEMENTS of MACHINE DESIGN; an Introduction to the Principles which determine the Arrangement and Proportion of the Parts of Machines, and a Collection of Rules for Machine Design. By W. CAWTHORNE UNWIN, B.Sc. M.I.C.E. With 325 Woodcuts. Price 6*s.*

PLANE and SOLID GEOMETRY. By H. W. WATSON, M.A. formerly Fellow of Trinity College, Cambridge. Price 3*s.* 6*d.*

21

London: LONGMANS, GREEN, & CO.

the loam core, and these are afterwards fitted with screw plugs or plugs secured by screwed pins.

In this piston the rod is tapered and the piston is secured to it by a large nut on the upper side of the piston. When such a piston works in a horizontal cylinder, a block placed between the spring ring and piston body on the bottom side of the piston keeps the latter from bearing on the cylinder. This block may extend round about one-fourth of the circumference.

In the case of large horizontal engines the piston rod is carried through both cylinder ends so as to support the weight of the piston and prevent wear of the cylinder.

Some large marine engine pistons have been made of cast steel, and these are one-third lighter than cast-iron pistons.

312. *Flexure by bending of a bar initially curved.*—In the case of a straight bar subjected to a bending moment, it is known (see § 27) that the bar takes a curvature given by the equation

$$\mathrm{M}=\frac{\mathrm{E\,I}}{\rho},$$

where M is the moment of the bending forces on one side of a section taken normal to the axis of the bar, I is the moment of inertia of the section, and ρ is the radius of curvature of the bent bar at the section.

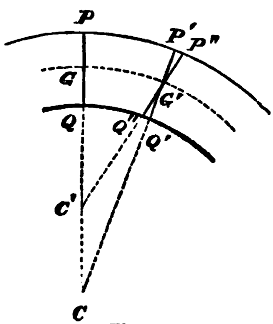


Fig. 290.

Let fig. 290 represent a bar initially curved, and let G G′ be a line passing through the centres of gravity of sections normal to G G′. Then if P Q, P′ Q′ are two normal sections at a small distance apart, and if P Q, P′ Q′ meet at C, C G$=\rho$ is the radius of curvature at P Q of the unstrained bar. Now let a bending moment M be applied to the right of P Q. The particles initially in P′ Q′ will be found after bending in a new plane P″ Q″ still normal to the axis of the bar. The

radius of curvature after bending will be $C' G = \rho'$. Then on the same assumptions made in treating straight bars, it will be found that

$$M = E I \left(\frac{1}{\rho'} - \frac{1}{\rho}\right)$$

that is, to pass from straight bars to curved bars, we have only to substitute in the equation for the curvature due to bending $\frac{1}{\rho}$, the augmentation of curvature $\frac{1}{\rho'} - \frac{1}{\rho}$.

313. *Theory of a cast-iron spring ring of unequal thickness.*—Let A B be a portion of a spring ring, which, initially circular on the outside, has been sprung into a cylinder of smaller diameter than itself. Let I be the moment of inertia of a cross section at B, which without sensible error may be taken normal to the outside curve of the ring. Let ρ be the radius of the outside of the ring initially, and r its radius when sprung into the cylinder. Let b be the breadth, t the thickness of the ring at B. From the conditions in which the ring is placed, it is necessary to make the breadth b of the ring constant, and it is required to determine the variable thickness t of a ring which will produce a uniform pressure of p lbs. per sq. in. on the surface of the cylinder into which it is sprung.

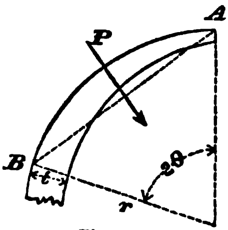


Fig. 291.

The pressure of the ring on the cylinder per unit of length is $p\,b$, and the resultant of the uniform pressure from A to B is $p\,b \times$ chord A B, or

$$P = 2\,p\,b\,r \sin\theta.$$

The moment of this about B is

$$2\,p\,b\,r^2 \sin^2\theta.$$

Putting this value of the bending moment in the equation above,

$$2\,p\,b\,r^2 \sin^2\theta = E I \left(\frac{1}{r} - \frac{1}{\rho}\right)$$

But for a rectangular section, § 38,

$$I=\frac{b\,t^3}{12}.$$

$$\frac{1}{r}-\frac{1}{\rho}=\frac{24\,p\,r^2\sin^2\theta}{E\,t^3} \quad . \quad . \quad (3)$$

For $2\,\theta=180$, let t_1 be the thickness of the ring,

$$\frac{1}{r}-\frac{1}{\rho}=\frac{24\,p\,r^2}{E\,t_1{}^3} \quad . \quad . \quad . \quad (4)$$

Hence

$$\frac{t}{t_1}=\sqrt[3]{(\sin^2\theta)} \quad . \quad . \quad . \quad (5)$$

$2\,\theta=$	$t=$
10°	0·197 t_1
20°	·311
40°	·489
60°	·630
80°	·745
100°	·837
120°	·908
140°	·960
160°	·990
180°	1·000

From which table the thickness at different points in the ring can be calculated when t_1 is known.

Fig. 292 shows a ring drawn from these values. It is found that about $\frac{2}{3}$ of the inside curve of the ring agrees very closely with a circle struck from a point C at a distance O C$=a$ from the centre of the outside of the ring. It is easy to show that $a=0{\cdot}206\,t_1$ nearly. The ring can therefore be bored out to a radius $r-0{\cdot}794\,t_1$, and the points near A then thinned to agree with the proportions above.

From eq. 2,

$$p=\frac{E\,t_1{}^3}{24\,r^2}\left(\frac{1}{r}-\frac{1}{\rho}\right).$$

Let ρ, the initial radius of the ring, be taken $1 \cdot 1\, r$, and put $E = 17{,}000{,}000$ for cast iron. Then

$$p = 64400 \frac{t_1^3}{r^3}.$$

Often in practice $t = \cdot 06\, r$. Then

$$p = 14 \text{ lbs. per sq. in.};$$

a value which does not appear excessive as regards the wear of the cylinder.

It has been stated, however, that in experiments with Ramsbottom's rings, they have been found to be practically

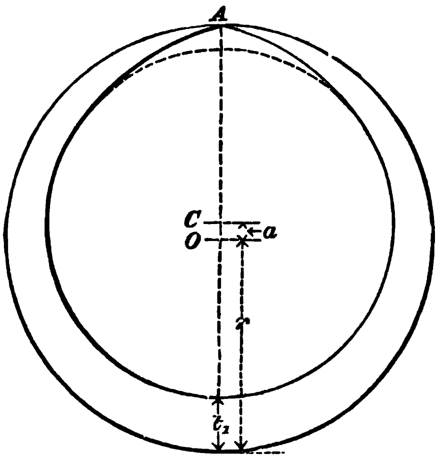


Fig. 292.

steam tight when giving a uniform pressure of $3\frac{1}{2}$ lbs. per sq. in., even when the difference of pressure on the two sides of the piston amounted to 100 lbs. per sq. in. If we insert $p = 3\frac{1}{2}$ in the equation above, we get when $\rho = 1 \cdot 1\, r$

$$t_1 = 0 \cdot 04\, r,$$

a proportion which does not seem very exceptional.

Sometimes two eccentric rings are used placed one inside the other, as shown in fig. 293. The proportions of such rings given by Von Reiche are:

Total thickness of two rings $\delta = 0{\cdot}8 + 0{\cdot}12\,r$
Greatest thickness of outside ring $\frac{2}{3}\,\delta$
Least „ „ $\frac{1}{3}\,\delta + 1{\cdot}2$
Greatest thickness of inside ring $\frac{2}{3}\,\delta - 1{\cdot}2$
Least „ „ $\frac{1}{3}\,\delta$
Breadth of rings . . . $0{\cdot}12\,r + 0{\cdot}9$

It is easy to see that such rings will produce approximately the same pressure on the cylinder as a single ring designed as described above, and of a maximum thickness equal to the total thickness of the two rings.

The breadth of the ring does not affect the intensity of pressure against the cylinder, nor for a given pressure per unit of area will a narrow ring wear longer than a wide one. On the other hand, a wide ring will wear the cylinder more than a narrow one.

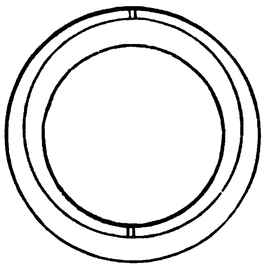


Fig. 293.

The theory above is simplified from an investigation by Prof. Robinson,[1] which contains also an attempt to determine theoretically the proper pressure of the ring on the cylinder to prevent leakage. This latter part of the investigation does not appear to the author to be well based.

314. *Theory of a spring ring of uniform thickness.*—A spring ring of uniform thickness may be made to give a uniform pressure on the cylinder, if it is initially bent to a form of varying curvature. Let A B C, fig. 294, be the spring ring when sprung into the cylinder, A′ B C the ring when unstrained. Let b be the breadth and t the thickness of the ring; I the moment of inertia of a cross section; ρ the radius of curvature of a point B when the ring is unstrained; r the radius of the cylinder into which the ring is sprung. The resultant pressure between A and B is as before,

$$2\,p\,b\,r \sin\theta.$$

[1] Van Nostrand's Magazine, June 1881.

Its moment about B is

$$M = 2\,p\,b\,r^2 \sin^2 \theta.$$

Inserting this in the equation above,

$$2\,p\,b\,r^2 \sin^2 \theta = \mathrm{E\,I}\left(\frac{1}{r} - \frac{1}{\rho}\right)$$

$$\frac{1}{r} - \frac{1}{\rho} = \frac{24\,p\,r^2 \sin^2 \theta}{\mathrm{E}\,t^3}.$$

Hence

$$\rho = \frac{\mathrm{E}\,t^3\,r}{\mathrm{E}\,t^3 - 24\,p\,r^3 \sin^2 \theta} \quad . \quad . \quad (6)$$

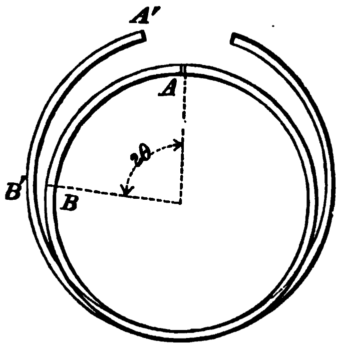


Fig. 294.

Let $\mathrm{E} = 30{,}000{,}000$ for steel, and let $t = 0{\cdot}04\,r$, which is about the usual proportion in practice. Then

$$\rho = \frac{80\,r}{80 - p \sin^2 \theta}.$$

The following table gives values of ρ for $\theta = 0°$ to $90°$, or $2\,\theta = 0°$ to $180°$, and for $p = 3\frac{1}{2}$ and $p = 14$, the values assumed in the previous case.

$2\,\theta =$	$\rho =$ for $p = 3\frac{1}{2}$	$= \rho$ for $p = 14$
0°	1·000 r	1·000 r
30	1·003	1·011
60	1·012	1·047

90	1·021	1·096
120	1·033	1·152
150	1·042	1·195
180	1·045	1·214

The form to which the spring ring ought to be bent in order that when sprung into the cylinder the pressure may be uniform, may be obtained thus :

Let o be the centre of the cylinder. Divide its circumference at $a\,a'\,a''$ . . . . into 12 equal parts. Take $a\,c$=the

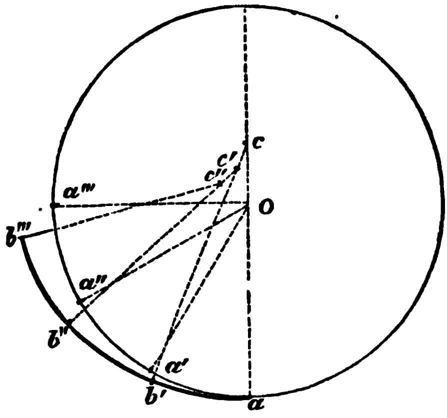


Fig. 295.

value of ρ for $2\,\theta=180$, and with centre c draw the arc $a\,b'$. Make $a\,b'=a\,a'$ and join $b'\,c$. Take $b'\,c'=\rho$ for 150°, and with centre c' draw the arc $b'\,b''$. Proceeding thus, the whole curve can be drawn.

Fig. 296 shows the curves drawn for the values of ρ for $p=3\frac{1}{2}$ and $p=14$ given above. The former curve agrees roughly with a circle of radius $\frac{1}{25}$th greater than the cylinder radius; the latter curve with a circle of radius $\frac{1}{7}$th greater than the cylinder radius. But it will be seen that both curves deviate considerably from circular curves, and hence

rings made by turning them to a circular form and cutting out a portion cannot give a nearly uniform distribution of pressure.

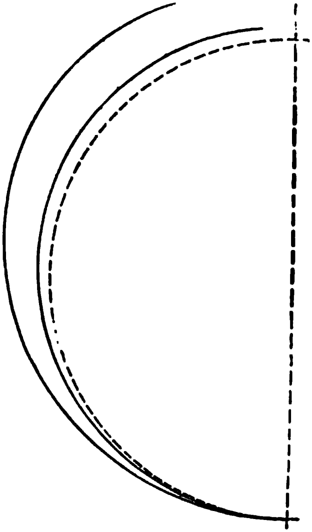


Fig. 296.

315. *Hydraulic Pistons.*—Fig. 297 shows a combined piston and plunger with a cup-leather or hat-leather arrangement for preventing leakage. The fluid pressure acting on the flexible leather cups, aided by their own elasticity, makes an exceedingly staunch joint, whatever the pressure may be. The hat-leathers are so arranged that one acts when the piston moves in one direction, the other when the piston moves in the reverse direction. The leathers used in this case are often termed hat-leathers, and are moulded in the same way as the cup-leathers described below.

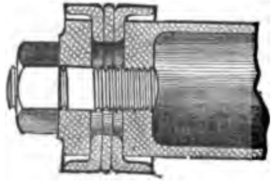


Fig. 297.

Stuffing-boxes.

316. Stuffing-boxes are used to prevent leakage of steam or water, at the points where moving parts pass through the sides of vessels containing fluids. Thus a stuffing box is used where the piston rod of an engine passes through the cylinder cover, or where a rotating shaft passes through a centrifugal pump case. In ordinary stuffing-boxes, soft packing is used to prevent leakage, but various forms of metallic packing have also been employed.

In fig. 298 is shown a stuffing-box for a vertical rod, and in fig. 299 a stuffing-box for a horizontal rod. In both cases the stuffing box is cast on the cylinder cover. The stuffing-box is larger than the rod which traverses it, by the space necessary for the soft packing. At the bottom of the box there is a brass bush, which, being softer than the rod, preserves the latter from injury. When the bush has worn oval, it is easily replaced by a new one. To keep the packing in place and to compress it sufficiently to prevent leakage, a loose piece termed a gland is used. This is entirely of brass (fig. 299) or bushed with brass (fig. 298),

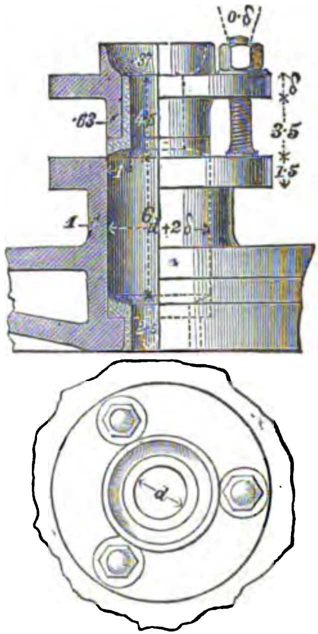
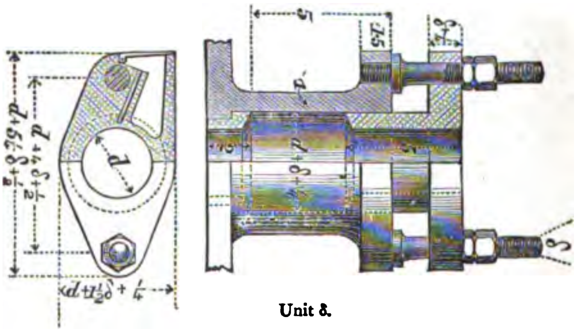


Fig. 298.

and often has an oil-box formed in it (fig. 299). The gland is forced down on the soft packing by two or more bolts or studs.

317. *Proportions of Stuffing-Box and Gland.*—Let d be the diameter of the rod or shaft traversing the stuffing-box. The diameter δ of the gland bolts may be $= \frac{1}{4} d + \frac{1}{4}$, if there are two; and $= \frac{1}{5} d + \frac{1}{4}$, if there are three. Then δ will be taken as the unit for the proportions of the box. The



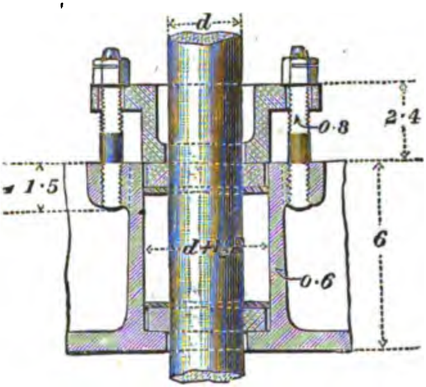
Unit δ.

Fig. 299.

thickness of packing in the box is very variable. In ordinary stuffing-boxes, not of very great diameter, the packing thickness varies from $\frac{1}{2}\,\delta$ to δ. But in large stuffing-boxes for trunks or hollow piston rods a less thickness is employed. The length of the box is also variable. The greater the length of box, the less frequently will it be necessary to renew the packing. On the other hand, the space available for the stuffing-box is sometimes restricted. From $5\,\delta$ to $8\,\delta$ is an average length. The thickness of the stuffing-box flange may be $1\frac{1}{4}\,\delta$ to $1\frac{1}{2}\,\delta$, and the thickness of the gland flange may be δ for cast iron, or $1\frac{1}{4}\,\delta$ for brass. If an oil-box is cast in the flange, the thickness is somewhat greater. The length of gland may be $\frac{3}{4}$ to $\frac{4}{5}$ the stuffing-box length. The thickness of the stuffing-box should not be less than $\frac{1}{6}\sqrt{d}$

or less than $\frac{3}{8}\,\delta$. The length of the brass bush may be about $2\frac{1}{2}\,\delta$, but when the stuffing-box serves to guide the rod, as in oscillating engines, a much greater length of bush is used.

318. *Yarrow's Stuffing-Box.*—As ordinarily constructed the gland of a stuffing-box fits the piston rod, and at the same time it is fixed in position by the stuffing-box. Hence it must act as a guide to the piston rod. The gland ought, therefore, to be exactly coaxial with the cylinder and parallel to the line of stroke of the slide block. If initially or in



Unit $\frac{1}{4}d + \frac{1}{4}$

Fig. 300.

consequence of wear these conditions are not satisfied, the piston rod must bear heavily on one side of the gland, and will wear it oval. In quick-running engines the piston rod will be heated on the side bearing on the gland, and this will cause it to bend in a direction which increases the injurious pressure. Messrs. Yarrow have found it advantageous to guide the parts connected by the piston rod by the piston and slide block only, and to construct the stuffing-box so as to permit a small lateral adjustment of position of the piston rod without causing pressure and heating. Fig. 300 shows

this stuffing-box. The gland is bored out a little larger than the piston rod (about $\frac{1}{8}''$ larger) and the ordinary fixed bush at the bottom of the stuffing-box is dispensed with. Two rings fit closely to the piston rod, but are turned a little smaller than the stuffing-box, and between these and the soft packing are two thin washers filling the stuffing-box, but $\frac{1}{8}''$ smaller than the rod.

Small Stuffing-Box.—For small rods, such as those round the spindles of valves and cocks, the form of stuffing-box shown in fig. 301 is used. The stuffing-box has a screw thread on the outside, and a six-sided cap fits over the gland and is screwed to fit the stuffing-box. Taking the unit as $\delta = \frac{1}{4}\ d + \frac{1}{4}$, the internal diameter may be $2\frac{1}{2}$ δ, the external diameter $5\frac{1}{2}$ δ, and the other proportions as given in the figure.

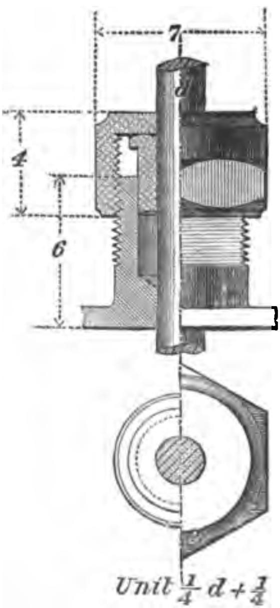


Fig. 301.

319. *Packing for Stuffing-Boxes.*—A loose kind of hemp rope termed spun yarn, steeped in melted tallow, is commonly used for stuffing-box packings. Occasionally brass turnings are sprinkled over the spun yarn, and cause the packing to wear longer. Packings of india-rubber wrapped in canvas, termed elastic core packing, and metallic packings of woven wire, are also used. These are formed into ropes of such a form that when cut of a length equal to the circumference of the rod, they form a ring exactly fitting the stuffing-box. Lately asbestos prepared in discs or ropes has been used. Elastic core or asbestos packing is better than spun yarn for engines working with high-pressure steam.

Mr. Tweddell has found hemp packing in ordinary stuffing-boxes efficient under pressures of 1,500 to 2,000 lbs. per sq. in.; and where the rod passing through the stuffing-box is continuously at work, the hemp packing gives less trouble in such cases than a cup leather.

320. *Cup-leather Packing.*—When great hydraulic pressure is to be resisted, a peculiar packing is used, invented by Bramah, and already alluded to. The leakage is prevented by a flexible leather ring, kept in contact with the piston rod or ram on one side and the cylinder on the other by the fluid pressure. The leather is moulded into an annular shape in plan and to a U-shape in section (fig. 302). This ring is placed in a recess in the cylinder or in a stuffing-box, in such a way that the fluid has free access to its interior; the fluid pressure acting within the ring presses it against the plunger and the sides of the recess, and this, aided by the elasticity of the ring, makes a perfectly tight joint. When the cup leather is large, it is provided with an internal brass ring, and a thin guard ring of brass on the edge most liable to wear. A packing of hemp or cotton is used as a bed for the leather.[1]

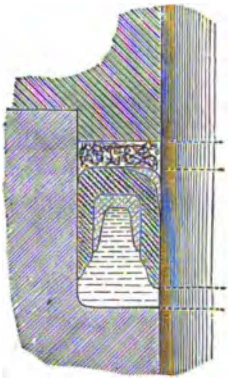


Fig. 302.

Mr. Welch gives the following rules for the proportions of cup leathers for great pressures. Let D be the diameter of the ram or plunger. Then the thickness of the leather should be

$$t = 0{\cdot}156\,D^{0{\cdot}28} \quad . \quad . \quad . \quad (7)$$

$$\log. t = \frac{7}{25} \log. D - 0{\cdot}8069,$$

[1] Mr. Tweddell, who introduced this arrangement of the cup leather, informs the Author that he no longer uses a guard ring.

and the width and depth of the ring measured outside should be each $2\frac{1}{2}t$. ('Proc. Inst. Mech. Eng.' 1876.)

D=	3	6	9	12	15	18	21	24
t=	0·212	0·258	0·288	0·313	0·333	0·351	0·366	0·380

Fig. 303 shows a press for moulding cup leathers.[1]

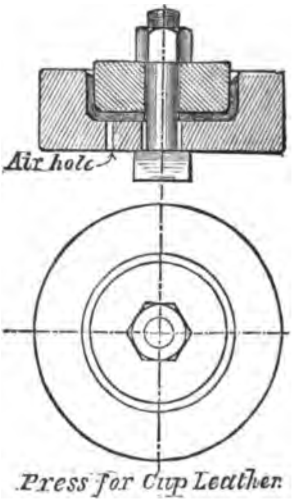


Fig. 303.

The best oil-dressed leather is steeped in warm water, and then forced into the mould and left till it is again hard.

[1] Anderson, 'Chatham Lectures on Hydraulic Machinery.'

CHAPTER XVI.

VALVES, COCKS, AND SLIDE VALVES.

VALVES.

321. In all machinery put in motion by the action of a fluid (water or steam) or employed in pumping fluids, valves are required to regulate the admission and discharge of the fluid. With reference to the mode in which the motion of valves is obtained, they may be divided into four classes: (1) Valves opened and closed by hand; (2) Valves opened and closed by independent mechanism; (3) Valves opened and closed by mechanism so connected with the machine as to render the times of opening and closing synchronous with the motions of the machine; (4) Valves opened and closed by the action of the fluid.

The mode in which a valve is actuated does not affect its construction, and a more convenient division, for the present purpose, depends on the way in which the valve moves relatively to its seat. Thus we have: (1) flap or butterfly valves, which rotate in opening; (2) lift valves, or puppet valves, which rise perpendicularly to the seat; (3) sliding valves, which open by moving parallel to the seat.

Fig. 304 gives a general view of the types of automatic valves. At B is the simplest form of leather flap valve, the leather being stiffened with metal plates. Such valves work very well under low pressures, and when the number of beats per minute is not too great. A similar valve of brass

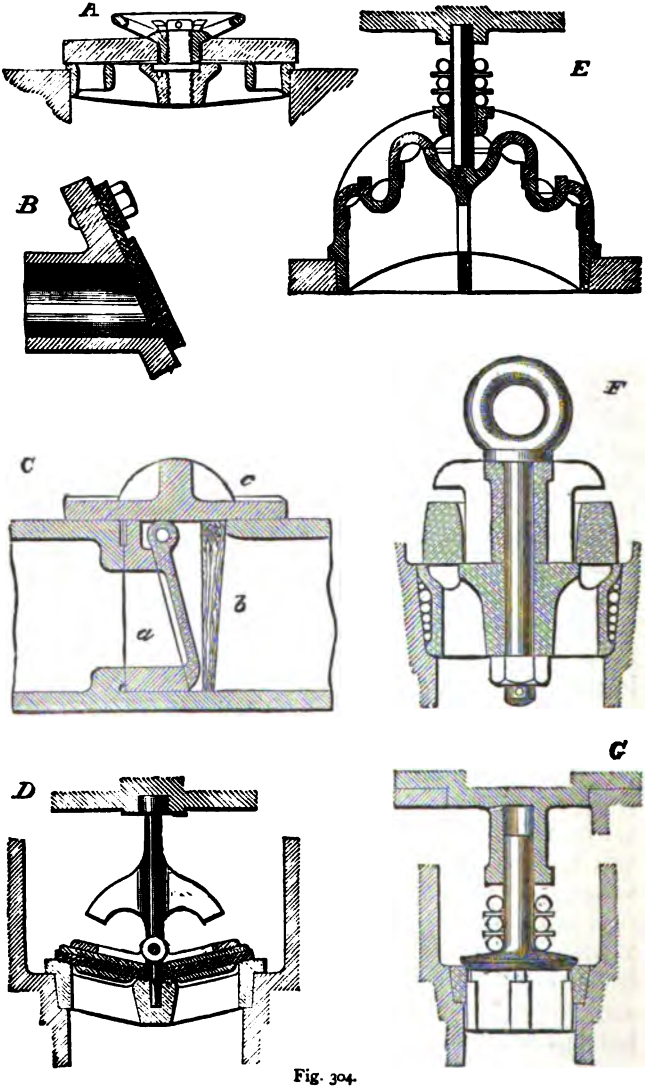


Fig. 304.

is shown at C hinged to a metal seating *a*, which is fixed by wood wedges *b*, and can be lifted out through the aperture covered by the plate *c*. This is a common form of condenser foot valve. At A is an india-rubber valve which acts in a similar way. The india-rubber valve is in the form of a circular disc covering a gridiron seating. The bars of the gridiron support the flexible india-rubber, and should be placed closer the greater the pressure on the valve. The valve is held by a bolt at the centre, and takes a dished form in rising. A guard plate prevents the valve from rising too high. At D are two flap valves placed back to back, and then termed butterfly valves. They beat on a brass seating, and have a guard to limit the lift. G is a mushroom lift or puppet valve; it consists of a metal disc fitting accurately a flat, or, more commonly, a conical seat. It is guided, in rising, either by a spindle or by three feathers underneath the valve, sliding in the valve seat. The bearing surface of the seat must be made narrow (about $\frac{1}{16}$ to $\frac{3}{16}$ inch), or it will not be tight. Consequently as the valve is made larger the intensity of the pressure on the seat increases. Hence there is a difficulty in using valves of this kind of large size. Further, the lift of such a valve should be proportional to its diameter, to give adequate water way when open. But when the valve is large, and the lift great, the shock of the valve in closing becomes severe and damages the valve and seating faces. At F is a ring valve, in which these objections are to some extent obviated. When open the water escapes at both the inside and outside edges of the valve, and hence for a given waterway only half as much lift is necessary as would be required by a mushroom valve of the same diameter. At E is a valve used for large pumping engines, which consists of two ring valves, and gives four edges for the escape of the water. At G and E the valves are provided with an arrangement to accelerate the closing of the valve, introduced by Mr. H. Davey. This consists of india-rubber

washers, forming a spring. The valve has a certain amount of free lift, and the rest is obtained by compression of the india-rubber.

322. *Flap or Butterfly Valves.*—Fig. 305 shows the simplest form of flap valve, formed of a leather disc, strengthened and stiffened by two plates of iron, of brass or of lead, which at the same time give weight enough to the valve to close rapidly, when the pressure beneath it ceases. A butterfly valve consists of two flap valves placed hinge to hinge, or sometimes edge to edge. In the latter position the direction of motion of the fluid is less interfered with. The lifting of the valve is usually restricted to an angle of

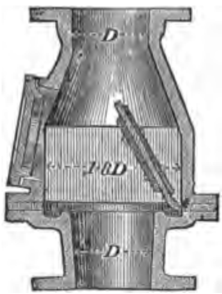
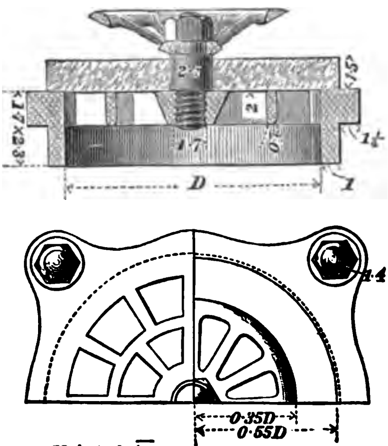


Fig. 305.

30° or 40°; width of seat $\frac{1}{8}\sqrt{D}+\frac{1}{8}$. Valves of this kind are most commonly lifted by the fluid.

323. *India-rubber Disc Valve.*—A form of valve very extensively used for condensers and pumps, consists of a circular disc of india-rubber, secured by a bolt at the centre, and resting on a brass gird which forms the seating. The india-rubber being flexible lifts easily from the grating, when any fluid pressure is applied beneath it, and closes again readily, and without violent shock, when the reflux begins. To prevent the india-rubber rising too high, a perforated guard plate is placed over the valve. Figs. 306

and 307 show two of these valves. In one the valve seat is attached to the cast-iron casing of the condenser by bolts, and the india-rubber and guard plate are attached to it by a stud. In the other the seating, india-rubber, and guard plate are all secured by the same central bolt, which bears against a cross-bar on the other side of the casing to that on which the valve is placed. In each of these figures the valve and guard plate are removed from one half of the plan, in order to show the

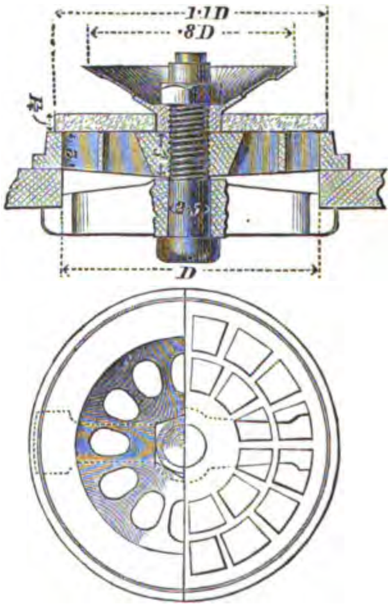


Unit $t = \frac{1}{4}\sqrt{D}$.

Fig. 306.

grating on which the valve rests. The india-rubber should not be too thin; $\frac{3}{4}$ inch to $\frac{7}{8}$ inch thickness is sufficient, and the apertures of the grating should be so small that there is no great flexure of the india-rubber when resting on the grating. Moderate-sized valves of this description answer better than large ones. It is more satisfactory to use several valves of 7 inches to 9 inches diameter than to use a single large one.

The throttle valve used on many engines, which consists of a circular or square metal disc, capable of turning about a shaft passing through it in the direction of a diameter, is a kind of double-flap valve. The disc is placed in a pipe, and closes the passage-way when placed across the pipe, whilst it offers little resistance when parallel to the axis of



Unit $t = \frac{1}{4}\sqrt{D}$.

Fig. 307.

the pipe. This valve is an imperfect equilibrium valve, the pressure on one half partly balancing the pressure on the other, so that the force required to move the valve is only equal to the difference of these two pressures. The equilibrium is exact, however, only while the valve is shut or so long as there is no sensible current passing it. If a rapid

current is established, the pressure on that half of the valve which first deflects the current is greater than on the other half, thus tending to close the valve.

324. *Lift or Puppet-valves.*—These are vary various in form, the simplest being a circular disc, usually of metal, with a flat or bevelled edge, which fits a circular metal seating (fig. 308). These valves are generally placed with the axis of the valve vertical, so that their weight tends to keep them closed, but they may be otherwise placed if springs or rods are used to close them.

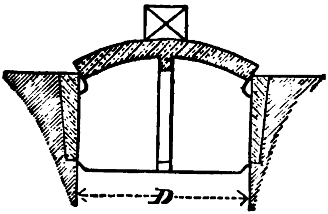


Fig. 308.

Let D be the diameter of the valve seating, then the water way through the valve seating is $\frac{\pi}{4}D^2$. If h is the height of lift, the waterway round the edge of the valve when open is $\pi D h$. In order that these two may be equal, the lift h must be one-fourth of the diameter D. It is only possible to give so great a lift in small valves, because with a great lift the valve acquires too much velocity in closing, and there is a violent shock, causing vibration and damage to the valve seat. It is necessary often to restrict the lift to a less amount than would otherwise be desirable, and then the resistance to the passage of the fluid is increased. Hence it is better to use two valves of diameter 0·707 D than a simple valve of diameter D. Let s be the width of the bearing surface of the valve and seat, measured on a plane at right angles to the direction of lift of the valve. Then $\pi D s$ is nearly the area of the bearing surface. If p is

the pressure in lbs. per sq. in. on the valve when closed,

$$f=\frac{\frac{\pi}{4}D^2 p}{\pi D s}=\frac{pD}{s} \quad . \quad . \quad . \quad (1)$$

is the pressure per unit of area of the bearing surface of the seat. Now s cannot be made more than about $\frac{1}{6}\sqrt{D}$ without danger of rendering the valve leaky.[1] Hence the intensity of pressure on the bearing surface increases with the diameter of a mushroom valve, and may reach a value greater than the metal of the valve or seat can permanently sustain.

The diameter of the under side of the valve exposed to pressure when the valve is closed is $D-2s$. Let w be the weight of the valve per unit of area. Then to lift the valve there will be required a pressure, p^1, given by the equation

$$\frac{\pi}{4}(D-2s)^2 p_1=\frac{\pi}{4}D^2(p+w)$$

or when s is small compared with D

$$p_1=\frac{(p+w)D}{D-4s} \quad . \quad . \quad . \quad (2)$$

which is evidently greater the wider the bearing surface s is made. The calculation assumes that the valve fits its seating so well that there is no pressure between the faces. If we neglect w, and take

$$s=\tfrac{1}{8}'', \ D=3 \text{ in.},$$
$$p_1=1{\cdot}2\,p,$$

or an excess of 20 per cent. of pressure to lift the valve. This increase of pressure required to open the valve is a cause of shock, because immediately the valve lifts, the pressure acts on a larger area. It is a defect of multiple-beat valves that this excess of pressure to open the valve is often very large. In multiple-beat valves the area of the valve on which the water acts in lifting them is reduced,

[1] For safety valves $s = \frac{1}{10}$ to $\frac{1}{8}$ inch.

and at the same time the bearing surface of the seating is increased. Thus in a ring valve (fig. 304, *F*) of diameters D_1 and D_2, and with a width of bearing surface s

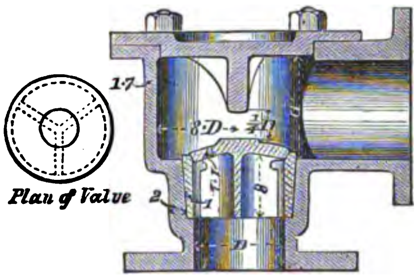
$$p_1 = \frac{(p+w)(D_2^2 - D_1^2)}{D_2^2 - D_1^2 - 4s(D_2 + D_1)}$$

$$= \frac{(p+w)(D_2 - D_1)}{D_2 - D_1 - 4s}.$$

Neglecting w, and taking $D_2 = 12$, $D_1 = 10$ ins., $s = \frac{1}{8}$ inch,

$$p_1 = 1{\cdot}33\, p.$$

The requirements of a perfect automatic valve are, large lifting area, great waterway, and little lift.[1]



Unit $t = \frac{1}{8}\sqrt{D}$

Fig. 309.

Fig. 309 shows a conical disc valve and casing. The valve is guided in rising and falling by three feathers which fit the cylindrical part of the seating, and are shown in the plan of the valve. The lift of the valve is limited by a projection on the cover of the casing. The fitting part, or face of the valve, should be narrow, as it is then more easy to make it tight. It must, however, present area enough to resist deformation by the hammering action of the valve. The inclination of the face of the valve is usually 45° with

[1] Davey, 'Proc. Inst. Civil Eng.' vol. liii.

the axis of the valve. Conical disc valves may either be actuated by the fluid pressure or by hand. In the latter case they are opened and closed by a screwed rod.

Fig. 310 shows a ball valve, which acts in precisely the same way as a disc valve, except that as the surface of the ball is accurately spherical, it fits the seating in every position. The only guide required is, therefore, an open cage, which limits the play of the valve. Such valves are often used for small fast-running pumps. To lighten the ball it is often made hollow.

The proportions of valves depend partly on the diameter. Thus the area of the waterway must be constant, and

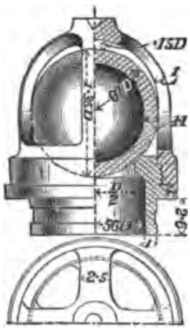


Fig. 310.

the linear dimensions of the casing are proportional to the valve's diameter. But the thicknesses are in most cases excessive as regards strength, especially in small valves, and do not increase in the same proportion as the diameter. For these the empirical proportional unit

$$t=\tfrac{1}{6}\sqrt{D}$$

will be adopted, where D is the diameter of the valve.

325. *Screw-down valve.*—Fig. 311 shows a lift valve arranged to be worked by hand instead of automatically. Such

screw-down valves are now largely used in place of cocks, being tighter and free from liability to stick. The valve rod passes through a stuffing-box, and is made so long that the screwed part does not come in contact with the stuffing box packing. The valve is fixed on the end of the rod by a pin in such a way that it can turn round. It does not then grind on the seat when screwed down. For steam the valve faces are of gun metal. For water one of the valve faces may be of

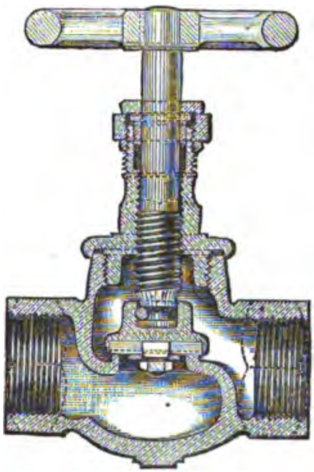


Fig. 311.

leather, of india-rubber, of asbestos, or in large valves of hippopotamus hide. When used for water fittings it is convenient that the pressure should be on the under side of the valve when closed. Then there is no danger of leakage at the stuffing-box.

326. *Double-Beat or Cornish Valve.*—The objection to a great lift in metal valves has already been mentioned. In the double-beat valve, two valve faces are obtained in the same valve, and two annular spaces are opened when the

valve lifts. For a given area of opening, the lift is only about one-half that of a simple lift valve of the same diameter. Fig. 312 shows a Cornish valve for a pumping engine. This valve is raised and lowered by a cam acting on an arrangement of levers. The lower seating is carried directly by the steam-chest. The upper seating is carried by four feathers or radiating plates cast with the lower seating. The valve itself is ring-shaped. Since the

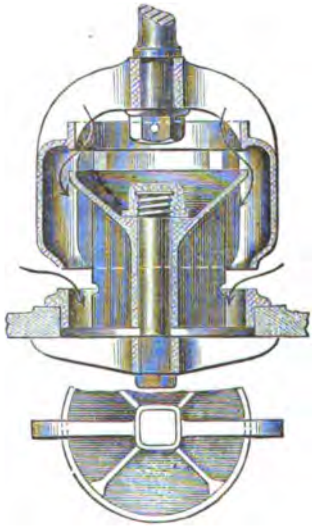


Fig. 312.

two valve faces are nearly of the same diameter, another subsidiary advantage is gained in this form of valve. The valve is pressed down on its seat, partly by its weight, partly by the steam pressure acting on one side of it. If the valve were a simple disc-valve, the steam pressure would act on an area $\frac{\pi}{4}$ D^2, where D is the diameter of the valve. As the valve is annular, however, the steam presses

only on the area $\frac{1}{4}\pi(D_1{}^2 - D_2{}^2)$ where D_1 and D_2 are the diameters of the two faces. Hence the valve is easily lifted.

327. *Sliding Valves.*—Sliding valves are more commonly used than any others for stop-valves, which are opened and closed by hand; they may be divided into two classes: (1) those with plane faces and seats; (2) those with cylindrical or slightly conical faces and seats. The former class includes engine slide-valves and the sluices, often of very large size, which are used as stop valves on water mains. The latter class includes the hand-worked valves commonly known as cocks.

SLIDE-VALVES.

328. *Engine Slide-Valves.*—Of the various valves used to effect the distribution of steam to steam-engine cylinders, the slide-valve is by far the most frequently adopted. The full treatment of the action of the slide-valve is beyond the scope of this treatise, and a short description of the most simple form of slide-valve is all that can be attempted. In its simplest form the slide-valve consists of a dish-shaped rectangular piece, the face of which is very accurately planed and scraped to a true surface. It slides upon a seating formed in the steam-chest, which is also an accurate plane, and in which are formed the passages through which the steam passes, termed the *ports*. The slide-valve is pressed down on the seating by the steam pressure on its back, which is greater than the pressure on its face, because part of the lower surface of the valve communicates with the atmosphere or the condenser. The section of the valve is D-shaped, as shown in figure 313, and it has two flat faces, in section, which, when the valve is in its middle position, cover the two passages leading to the two ends of the cylinder, or steam-ports; at the same time the hollow part of the valve covers the middle passage through which the steam is discharged, and which is termed the exhaust-port. When the valve moves in either direction from its middle position, it uncovers one steam-port and allows

steam to pass to one end of the cylinder, whilst, at the same time, the hollow part of the valve passes over the other steam-port and puts it in communication with the exhaust-port. The reciprocating motion of the slide-valve, which opens the ports alternately, is effected by an eccentric, which may be regarded as a very short crank, keyed on the same shaft as the engine crank. It is obvious that the travel of the valve each way from its mid-position is equal to the radius of the eccentric, unless modifying arrangements are interposed.

In the earliest slide-valves the width of the faces of the valve was sensibly equal to the width of the steam-ports. Then, the moment the valve passed its mid-position, it began

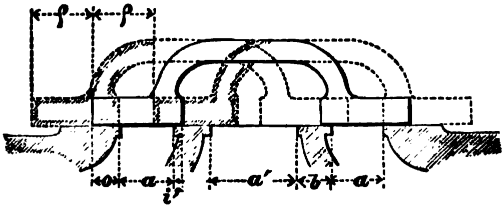


Fig. 313.

to open one port to steam and the other to exhaust. Apart from a circumstance to be mentioned presently, the steam-piston must be at the end of its stroke at the moment steam begins to be admitted on one side and exhausted from the other. It follows that with this form of valve, the valve is at mid-stroke when the steam piston is at the end of its stroke. Consequently the eccentric must be at right angles to the crank.

It was discovered, however, that with this arrangement the steam entered and left the cylinder with difficulty at the beginning of each stroke, in consequence of the very gradual opening of the slide-valve. To afford a wider opening to the steam, it was found necessary that the valve should be already a little open at the beginning of a stroke. To secure this it is only necessary to fix the eccentric a

little more than 90° in advance of the crank. The width of the port open at the beginning of a stroke is termed the *lead* of the valve.

Next it was found desirable to make the faces of the valve wider than the steam-ports, so that when the valve is placed in its middle position, the valve faces overlap the edges of the ports. The width of the overlap on the steam edge of the valve is termed the outside lap, and the width of the overlap on the exhaust edge of the valve is termed the inside lap. The former is generally greater than the latter. The valve must be open to the extent fixed for the lead, when the piston is at the end of the stroke, and the crank connected with it on one of its dead centres. To attain that position, the valve must have moved from its middle position a distance equal to outside lap + lead. The eccentric must therefore have passed its middle position by the angle necessary to move the valve through that distance. It will therefore be in advance of a position at right angles to the crank, by an angle termed the angular advance of the eccentric. Two important advantages are obtained in a valve thus arranged: (1) The steam is cut off from the cylinder before the end of the stroke, and the slide-valve acts as an expansion valve. (2) By making the outside lap greater than the inside lap, the lead to exhaust is made greater than the lead to admission, by the amount of the difference of the laps. Now, the greater freedom of exhaust thus secured reduces the back pressure on the piston in a very important degree. There is a limit to the amount of lap which can be used with an ordinary slide-valve. To whatever extent the opening of the exhaust is made earlier, to the same extent the closing of the exhaust is made earlier also. If the exhaust is closed too soon, steam is retained in the cylinder and compressed, as the piston returns, into the small clearance space at the end of the cylinder. This action is termed cushioning. A moderate amount of cushioning is useful, but excessive cushioning would be prejudicial. To prevent this the

outside lap is usually not greater than is sufficient to close the steam-port at $\frac{5}{8}$ths of the stroke. When more expansion is wanted, a double slide-valve or some other arrangement is used.

Fig. 313 shows a section of a slide valve and of the steam-ports, taken parallel to the direction in which the valve moves. The dotted lines show the positions of the valve at the ends of its stroke, in either position completely uncovering one port to exhaust, and partially uncovering the other to steam.

329. *Area of Steam-ports.*—The area of the steam-ports must be so arranged that the mean velocity of the steam does not exceed 80 to 100 feet per second. Let Ω be the piston area and ω the area of each steam-port, then the ratio of port area to piston area is as follows :—

Piston velocity feet per minute	$\frac{\omega}{\Omega}$
200	·04
300	·055
400	·07
600	·10

For locomotives which run at a high, but variable speed, $\omega = \cdot 07\Omega$. Let l be the length and a the width of each steam-port. Then $al = \omega = \left(\frac{\omega}{\Omega}\right)\Omega$. The proportions of the port are variable ; the length l may be from 0·5 to 0·8 of the cylinder diameter, and the ratio $\frac{l}{a}$ is 6 in small engines, 7 in medium engines, and 9 in large engines. Let D be the cylinder diameter, and let $l = x$ D, $a = y$ D. Then suitable values of x and y are given in the following table :—

Piston speed	x	y	x	y	x	y
200	·4	·078	·5	·062	·6	·052
300	·5	·086	·6	·072	·7	·062
400	·6	·091	·7	·078	·8	·068
600	·7	·112	·8	·098	·9	·087

The whole width of the steam-port is opened to exhaust, but often only 0·6 to 0·9 of the width to steam.

330. *Proportions of the Slide-valve*:—

Let a=width of steam port.

na=greatest width opened to steam.

o=outside lap.

i=inside lap.

e=lead; e'=inside lead.

b=width of bar between steam and exhaust ports.

a'=width of exhaust port.

ρ=half travel of valve, or radius of eccentric.

r=radius of crank or half stroke of engine.

ϵ=ratio of eccentric radius to length of eccentric rod.

ξ=distance valve has travelled from its mid-position when the crank has moved through an angle ϕ from the dead point.

l=distance piston has travelled from beginning of stroke, at the same moment.

θ=angle of advance of eccentric, so that the eccentric is $90°+\theta$ in advance of the crank.

The width b of the bars is fixed empirically. In small engines it may be $=\frac{a}{2}+\frac{1}{4}$, and in large engines it is determined almost entirely with reference to convenience of casting, and should be at least equal to the cylinder thickness. The inside lap, i, is generally small, and may be from ·075a to ·1a. The outside lap, o, is determined by the point at which steam is to be cut off. Very commonly o is from 0·25a in slow to 0·66a in fast engines. Then, if the valve and eccentric are directly connected,

$$\rho=n\,a+o=a+i \quad . \quad . \quad . \quad (3)$$

If these equations are satisfied o and i are not independent when a and $n\,a$ are fixed.

The lead e may be from $\frac{1}{16}\rho$ in slow to $\frac{1}{8}\rho$ in fast engines.

The angular advance of the eccentric, θ, is determined by the equation—

$$\text{Sin } \theta = \frac{o+e}{\rho} \text{ nearly} \quad . \quad . \quad . \quad (4)$$

This determines θ, and then the angle between the crank and eccentric radius is $90°+\theta$. The following table gives some values of θ :—

$\frac{e}{\rho}=$	When $\frac{o}{\rho}=$ ·1	·15	·2	·25	·3	·35
$\frac{1}{16}$	9° 21′	12° 16′	15° 13′	18° 12′	21° 15′	24° 22′
$\frac{3}{32}$	11 10	14 6	17 5	20 6	23 11	26 21
$\frac{1}{8}$	13 1	15 58	18 58	22 2	25 10	28 22

The equation is not quite exact, because of the obliquity of the eccentric rod, but the error is not great.

The inside lead$=e'=\rho \sin \theta - i$. The inside and outside lead and lap are connected by the equation, $o+e=i+e'$.

The width a' of the exhaust port must be equal to or greater than $2\rho - b$.

The width of the hollow under the valve (measured parallel to the direction of the valves' motion)$=2(a+b-\rho)+a'$.

331. *Travel of valve and corresponding crank angle when the influence of the obliquity of the eccentric rod is neglected.*—Let a line through the two dead centres of the crankpin circle be termed the line of stroke. Generally this line is also parallel to the axis of the cylinder. If the obliquity of the connecting rod is neglected, the valve is in its mid position when the eccentric radius is at right angles to the line of stroke. Let that position be termed, for shortness, the mid position of the eccentric. As the eccentric moves through an angle α from its mid position, the valve travels a distance

$$\xi = \rho \sin \alpha \quad . \quad . \quad . \quad . \quad (5)$$

which will be + or − according as α lies between 0° and 180°, or between 180° and 360°, α being measured in the direction of motion of the crank. Since the eccentric is $90°+\theta$ in advance of the crank,

$$\alpha=\phi+\theta,$$

where ϕ is the angle through which the crank has moved from its position at the beginning of the stroke. Hence

$$\xi=\rho \sin(\phi+\theta) \quad . \quad . \quad . \quad (6)$$

The opening of the port to steam is

$$w=\xi-o=\rho \sin(\phi+\theta)-o,$$

and the opening of the port to exhaust is

$$w'=-\xi-i=-\rho \sin(\phi+\theta)-i.$$

When admission begins and when steam is cut off, $w=O$; and when exhaust or compression begins, $w'=O$. Inserting these values, we obtain four values of the crank angle for each edge of the valve and for one revolution of the engine.

For			$(\phi+\theta)$ lies between
Admission	$w=o$	$\sin(\phi_1+\theta)=\frac{o}{\rho}$	0° and 90°
Cut off	$w=o$	$\sin(\phi_2+\theta)=\frac{o}{\rho}$	90° and 180°
Release	$w'=o$	$\sin(\phi_3+\theta)=-\frac{i}{\rho}$	180° and 270°
Compression	$w'=o$	$\sin(\phi_4+\theta)=-\frac{i}{\rho}$	270° and 360°

From these equations the values of $\phi+\theta$, and therefore of ϕ, can be obtained. The angles are connected by the relations $\phi_2=180°-\phi_1-2\theta$; $\phi_4=180°-\phi_3-2\theta$.

The following form of the same equations is sometimes more convenient.[1]

Since $$\text{Sin } \theta=\frac{o+e}{\rho},\ o=\rho \sin\theta-e.$$

[1] Resal, 'Mécanique Générale,' vol. iv., p. 288.

Hence,

$$w = \xi - o = e + \rho \left\{ \sin(\phi + \theta) - \sin\theta \right\}.$$

For admission and cut off, $w = O$, and we get

$$\text{Sin}(\phi + \theta) = \sin\theta - \frac{e}{\rho};$$

For admission, $\phi_1 = 2\pi - \dfrac{e}{\rho\cos\theta};$

For cut off, $\phi_2 = \pi - 2\theta + \dfrac{e}{\rho\cos\theta}.$

The angles are in circular measure in these equations, and can be reduced to degrees by multiplying by $\dfrac{180}{\pi}$ or by 57·3.

Similarly, since $i + e' = o + e$,

$$i = \rho\sin\theta - e'$$

$$w' = -\xi - i = e' - \rho \left\{ \sin(\phi + \theta) + \sin\theta \right\}.$$

For release and compression, $w' = O$; then

$$\text{Sin}(\phi + \theta) = -\sin\theta + \frac{e'}{\rho};$$

For release, $\phi_3 = \pi - \dfrac{e'}{\rho\cos\theta};$

For compression, $\phi_4 = 2\pi - 2\theta + \dfrac{e'}{\rho\cos\theta}.$

332. *Position of piston for given crank angles when the obliquity of the connecting rod is neglected.*—If l is the distance the piston has travelled from the beginning of its stroke, when the crank has revolved through the angle ϕ, measured from the dead point, then if the obliquity of the connecting rod is neglected

$$l = r(1 - \cos\phi) \quad . \quad . \quad . \quad (7)$$

where $\cos\phi$ is negative, if ϕ lies between 90° and 270°. By inserting the values of ϕ, obtained above, we obtain approximately the position of the piston for admission, cut off,

release and expansion. As, however, the obliquity of the connecting rod sensibly affects the position of the piston, it is better to set off the positions of the crank corresponding to the above values of ϕ on a diagram drawn to scale, and then by laying off the connecting-rod length, the position of the piston is found exactly.

333. *Crank angles corresponding to given ratios of expansion.*—Let l_2 be the travel of the piston corresponding to the crank angle ϕ_2. Then $\frac{l_2}{2r}$ is the ratio of cut off. The following table gives the relation between these quantities, when the obliquity of the connecting rod is neglected :—

$\frac{l_2}{2r} = 0{\cdot}4$	0·45	0·5	0·55	0·6	0·65
$\phi_2 = 78\frac{1}{2}°$	83	90	96	$101\frac{1}{2}$	$107\frac{1}{2}$
$\frac{l_2}{2r} = 0{\cdot}7$	0·75	0·8	0·85	0·9	0·95
$\phi_2 = 113\frac{1}{2}°$	120	127	$134\frac{1}{2}$	143	154

The ratio $\frac{2r}{l_2}$ is the ratio of expansion or number of times the steam is expanded.

334. *Zeuner's valve diagram for a simple slide valve.*— Most problems about slide valves are worked out very simply by graphic methods, of which several have been suggested, and more or less used in practice. Of these, that of Prof. Zeuner of Dresden is the simplest, and is applicable to the greatest variety of valve gears.

Let fig. 314 represent the mechanism of an engine, which, for definiteness, is supposed horizontal. o b, $b\,c$ are the positions of the crank and connecting rod, and o f, $f\,g$ the corresponding positions of the eccentric and eccentric

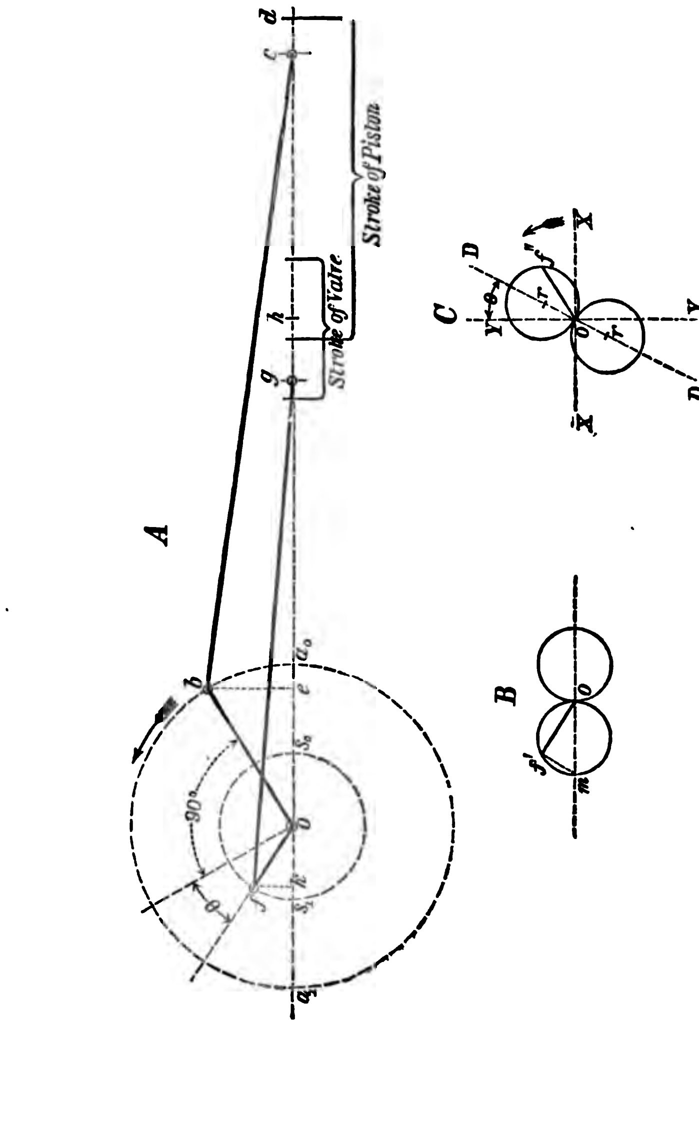


Fig. 314.

rod. The crank is supposed to be moving from a_0 to a_1. By setting off lengths equal to bc from a_0 and a_1 the length of piston stroke can be marked off; and by setting off lengths from $s_0\,s_1$ equal to fg the valve stroke can be marked off. Further, it may be noted that the angle $b\,o\,f$ is equal to $90°+$the angle of advance or $90°+\theta$.

Draw $b\,e$, $f\,k$ perpendicular to the line of stroke. For the position of the mechanism shown, the piston will have moved a distance $d\,c$ from the beginning of its stroke, and $d\,c$ will be approximately equal to $a_0\,e$, so that when much exactness is not required, $a_0\,e$ may be taken to represent the movement of the piston for an angular movement $a_0\,o\,b$ of the crank. Where exactness is required it is always easy to set off $b\,c$ equal to the connecting-rod length, and so to find $c\,d$; or conversely, if the position c of the piston is given, to find the position, b of the crank pin. Set off $o\,h=fg$. Then h is the middle point of the valve stroke, and in the position of the eccentric shown the valve has moved a distance $g\,h=\xi$ from its mid position. Since $o\,h=fg$ and $k\,g=fg$ nearly, it follows that $o\,k=g\,h=\xi$ nearly. And this approximation is for practical purposes almost always sufficient because of the smallness of the obliquity of the eccentric rod.

Now in diagram B, with centres on any horizontal line, and radii equal to half the eccentric radius, draw the two valve circles touching at O. Draw $o\,f'$ parallel to $o\,f$ in diagram A, and join $f'\,m$. In the triangles $o\,f\,k$, $o\,f'\,m$, $o\,m=o\,f$, the angle $m\,o\,f'=k\,o\,f$, and the right angle $o\,f'\,m=o\,k\,f$. Hence $o\,f'=o\,k$. Therefore the diagram B will answer as a valve diagram, because the radius vector $o\,f'$ drawn parallel to any position of the eccentric will be approximately enough equal to the travel of the valve from its mid position; and knowing the travel of the valve it will be easy to infer the condition of opening of the ports to steam and exhaust. The diagram B, however, will be more convenient

if it is rotated backwards through an angle $90° + \theta$, as shown at C. Then $\text{O}f''$ drawn parallel to the crank is the same line as $\text{O}f'$ in diagram B.

To draw the valve diagram C correctly, proceed thus. Take any rectangular axes X X, Y Y, the former being parallel to the line of stroke. From Y Y set off the angle of advance θ *towards the initial position of the crank.* On the line D D so

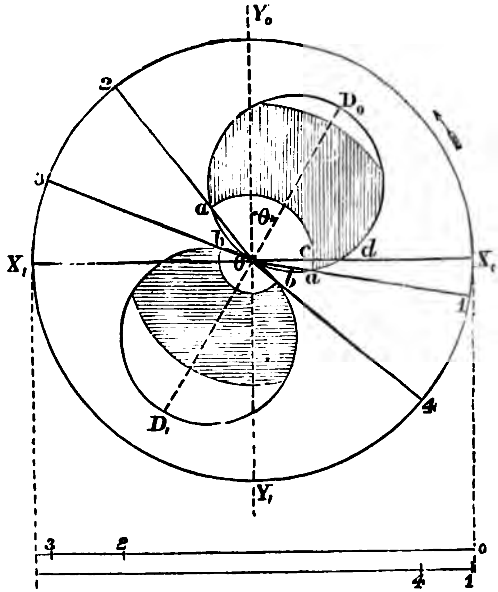


Fig. 315.

obtained take O r, O r = half the eccentric radius. Then $r\,r$ are the centres of the valve circles. Lastly, the radius vector of the valve circles, drawn from O parallel to any position of the crank, is the corresponding travel ξ of the valve from its mid position.

335. *Complete valve diagram with lap circles.*—Let $X_0\, X_1$,

$Y_0 Y_1$, be the rectangular axes, the motion of the crank being from X_0 to X_1 in the direction of the arrow. Draw $D_0 D_1$, making the angle of advance θ with $Y_0 Y_1$ on the side of X_0. Take O D_0, O D_1, each equal to the half travel of the valve, and on these lines as diameters describe the valve circles. Take O X_0 equal on any scale to the crank radius, and draw the crank circle X_0 2 3 X_1. With centre O and radius equal to the outside lap o, draw the outside lap circle $a\,a$; with centre O and radius equal to the inside lap i, draw the inside lap circle $b\,b$.

The port opens to steam when $\xi = o$. Hence O a 1 is the position of the crank when the valve opens. At the beginning of the stroke when the crank is at O X_0, the travel of the valve is O d. Hence $c\,d = \xi - o$ is the lead e. The valve closes to steam when ξ is again diminished to o. Hence O a 2 is the position of the crank when the valve closes and expansion begins. Similarly O b 3 is the position of the crank when the valve opens to exhaust, and O b 4 that when the valve closes to exhaust and compression begins.

By drawing circles with centres at O and radii equal to $a + o$ and $a + i$, where a is the width of port, we mark out the periods during which the valve is fully open to steam and to exhaust.

The lower figure is a horizontal projection of the points determined by the valve diagram on the crank-pin circle. It gives the proportionate lengths of stroke for each period, if the obliquity of the connecting rod is neglected. Thus O 2 is the period of admission; 23 the period of expansion; 34 the period of exhaust; and 41 the period of compression.

Suppose that in designing a valve gear there are given the ratio of cut off z, the eccentric radius or half valve travel ρ, and the lead e. In fig. 316 take $X_0 X_1$ parallel to the line of stroke. With radius ρ describe the circle D K A. Find the position O K 2 of the crank at which expansion

begins. This is found approximately by taking $\frac{BC}{AC} = z$, or more exactly by the method above. Join A K, and take K E equal to the given lead *e*. Bisect A E in F. Then A F=F E is the necessary outside lap *o*. Take O G=F K, and draw

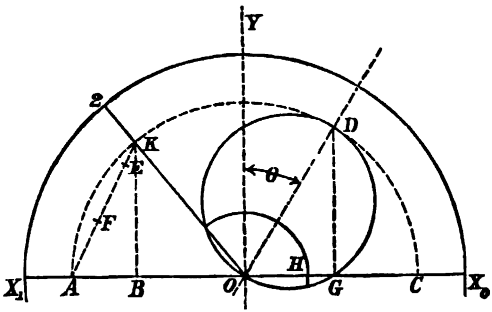


Fig. 316.

G D perpendicular to X X. Then D O Y is the required angle of advance. On O D describe the valve circle. Take O H=F E, and through H draw the lap circle. The valve diagram can then be completed by drawing the other valve circle and the inside lap circle.

336. *Mean pressure in the cylinder.*—Let p_2 be the absolute initial steam pressure, p_b the absolute back pressure; $z = \frac{l_2}{2r}$ the ratio of cut off in the cylinder; c the ratio of the clearance space at each end of the cylinder (including steam port) to the working volume of the cylinder or space described by the piston. Then the mean effective steam pressure during the stroke is

$$p_m = k p_2 - p_b, \quad . \quad . \quad . \quad (8)$$

where k is a coefficient depending on z and c. Let the ex-

pansion curve be assumed to be an hyperbola, which is accurate enough in calculating the mean steam pressure. Then

$$k = z + (z + c) \log_e \frac{1 + c}{z + c}.$$

The value of c varies for different types of engine. In ordinary slide-valve engines it is about 0·05. In Corliss

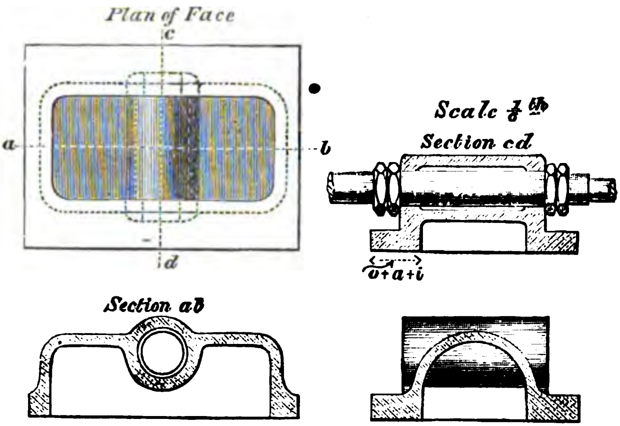


Fig. 317.

engines it is 0·015 to 0·025. Taking c=0·05, the following table gives values of k :

z=	k=	z=	k=	z=	k=
0·9	0·995	0·6	0·912	$\frac{1}{4}$	0·626
0·8	0·980	0·5	0·856	$\frac{1}{5}$	0·559
0·75	0·968	0·4	0·781	$\frac{1}{6}$	0·509
0·7	0·952	0·3	0·685	$\frac{1}{8}$	0·439

Fig. 317 shows an ordinary locomotive slide-valve.

Cocks.

337. The term cock is sometimes used for any valve opened or closed by hand, but it is more properly restricted

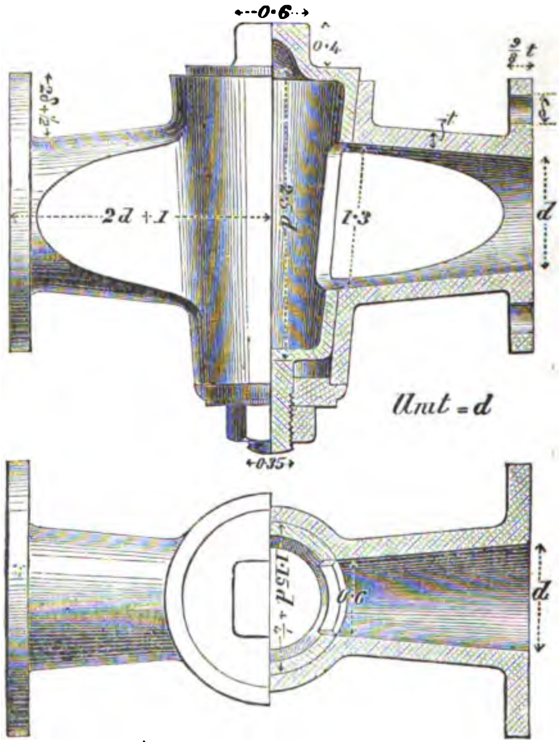


Fig. 318.

to valves which are nearly cylindrical, and which rotate in seatings of the same figure. In ordinary cocks the seating

is a hollow, slightly conical casing, and the valve, which is termed a plug, fits accurately in the seating. The passage-way for the fluid is formed through the plug. By rotating the plug in one direction its apertures are made to coincide with the entrance and discharge orifices of the casing. The cock is then open. By rotating it in the other direction the holes in the plug are brought over blank parts of the casing and the cock is closed. The slight taper given to the plug enables it to be accurately fitted, by turning and grinding, to its seating, and it can, from time to time, be refitted. Each time it is refitted the plug sinks a little lower in the casing. If the plug were cylindrical this refitting would be impossible. The objection to the use of cocks in many cases, especially for pipes of large size, is that a good deal of power is required to move then, and this is partly due to the conical form, which increases the friction.

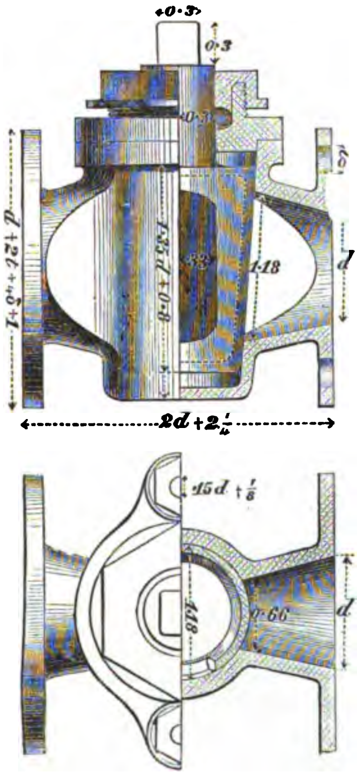


Fig. 319.

The simplest cocks have a solid plug, which is kept in place by a screwed end. When the cock is small, the casing has a

screwed socket on one side and a screwed end on the other, for the attachment of the cock to the pipes with which it is connected. But in larger cocks, the inflow and outflow orifices are provided with flanges.

For small brass cocks, with socket and spigot ends, the following proportions may be adopted:—

Diameter of waterway of cock $=d$
Diameter of plug at centre $=1\cdot15d+\frac{1}{4}$
Height of hole in plug $=1\cdot3d$
Width of hole in plug $=0\cdot6d$
Total length of tapered part of plug $=2\cdot5d$ to $3d$
Side of square for handle $=0\cdot7d$
Height of square for handle $=0\cdot4d$
Thickness of metal $=0\cdot2d+\frac{1}{16}$
Diameter of plug screw $=0\cdot35d$
Diameter of screwed end $=d+\frac{5}{16}$
Internal diameter of socket end $=d+\frac{3}{16}$
Total length $=3\cdot3d$
Taper of plug $=1$ in 12 to 1 in 9 on each side.

For cocks with flanged ends, like that shown in fig. 318, the proportions are the same. When the cock is not very small the thickness is best obtained from the rule—

$$t=\frac{1}{8}\sqrt{d}+\frac{3}{16} \text{ for cast iron.}$$

$$=\frac{1}{12}\sqrt{d}+\frac{3}{16} \text{ for brass.}$$

Some proportions are marked on the figure.

338. Large cocks connected with boilers, and in situations where failure would be dangerous, are best made with closed ends, as shown in fig. 319. The proportions of cocks of this description are a little different.

Diameter of waterway $=d$
Thickness of plug (brass) $=0\cdot12\sqrt{d}+\frac{1}{8}$
" " (cast-iron) $=0\cdot18\sqrt{d}+\frac{1}{4}$
Thickness of shell (brass) $=0\cdot18\sqrt{d}+\frac{1}{8}$
" " (cast-iron) $=0\cdot25\sqrt{d}+\frac{1}{4}$

The shell may be reduced to the same thickness as the plug in parts which do not require to be turned.

Diameter of plug at centre $= 1{\cdot}18d$

Size of openings in plug $= 1{\cdot}18d \times 0{\cdot}66d$

Overlap of plug at top and bottom $= 0{\cdot}08d + 0{\cdot}4$

Depth of stuffing-box $= \frac{1}{8}d + \frac{1}{2}$

Depth of gland $= \frac{1}{20}d + \frac{1}{4}$

Diameter of studs in cover, $\frac{1}{8}d + \frac{1}{8}$

Taper of plug $= 1$ in 12 on each side.

Some other proportions are marked on the figure.

CHAPTER XVII.

LUBRICATORS.

339. THE amount of frictional resistance of machine parts which slide on each other depends on the smoothness of the surfaces and on their lubrication. A lubricant is a substance which, interposed between the rubbing surfaces, reduces the friction. The diminution of friction diminishes the work wasted, the wear of the rubbing parts, and the amount of heat developed. Surfaces which run perfectly cool and well if properly lubricated, heat and even seize if the lubrication fails. Seizing is the cohering of the parts with force great enough to cause fracture or stoppage of the machine.

An efficient lubricant should possess the following qualities: (*a*) It should wet the rubbing surfaces. (*b*) It must not evaporate or decompose while in use. (*c*) At the temperature at which it is employed it should have enough, and only enough, viscosity to remain between the surfaces. (*d*) It must contain no acids or other constituents capable of acting on the rubbing surfaces. (*e*) It must be free from grit or other foreign matter.

Air or water are good lubricants when the velocity is great enough to carry in a layer between the rubbing surfaces.

Lubricants are sometimes solid at ordinary temperatures, as tallow or railway grease, more commonly fluid, as vegetable, animal, or mineral oils. Metaline and some other materials are used without lubricants, and act themselves as lubricants.

Of vegetable oils, olive, palm, rape, and others are used. Of animal oils, sperm is one of the very best, but lard, neat's

foot, seal, and other oils are used. Of mineral oils, some are derived from the distillation of shale, and others from petroleum wells. Railway grease is a mixture of tallow, palm oil, water, and a portion of caustic soda.

To provide for the proper lubrication of rubbing parts, a reservoir of the lubricant must be provided, so arranged, if possible, that it delivers the lubricant continuously in small quantities. The lubricant flows through an aperture to the rubbing surfaces, one of which is generally provided with channels for its suitable distribution. Lastly, in many

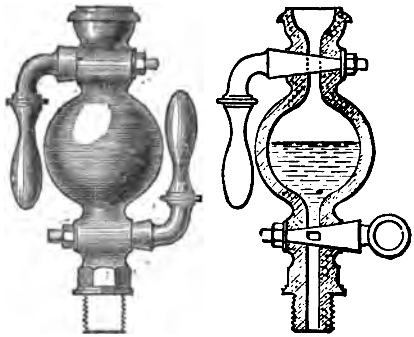


Fig. 320.

cases a vessel must be provided to catch the lubricant which flows off the rubbing surfaces after it has done its work.

340. *Cup lubricators.*—The simplest lubricator is a cup which can be filled from time to time, and which has a cock, by turning which the lubricant is permitted to flow to the rubbing surfaces.

Fig. 320 shows the ordinary form of cylinder oil cup or tallow cup. It consists of a vessel having two cocks. Closing the lower one and opening the upper one, it can be filled; closing the upper and opening the lower, the oil or melted tallow is admitted to the cylinder. It is often placed directly on the steam cylinder, sometimes on the steam

pipe, where the rapid current of steam carries forwards the oil to the working parts.

341. *Displacement lubricator.*—Fig. 321 shows the displacement lubricator invented by Ramsbottom. The steam, condensing on the surface of the oil, forms a drop which sinks down through the oil and displaces a small quantity of oil, which then flows down the bent pipe into the cylinder. The plug at the top serves for filling the cistern—that at bottom for removing the condensed water.

342. *Siphon lubricator.*—Fig. 322 shows an ordinary siphon lubricator for oil. This has a tube rising above the surface of the oil in which a cotton wick is placed. The oil is slowly siphoned by the wick and drops on to the bearing. Various

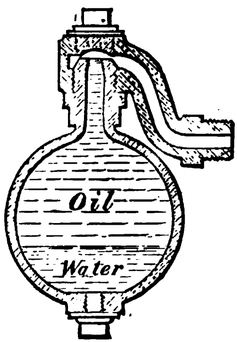


Fig. 321.

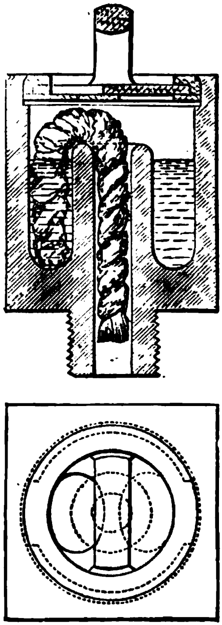


Fig. 322.

arrangements are adopted for closing the reservoir to keep out dirt. Sometimes there is a hinged or screwed cap. In the example shown there is a rotating plate with a hole. By rotating the plate this hole can be brought over a hole in a lower plate or over a blank part of the lower plate.

Clarendon Press Series.

The English Language and Literature.

HELPS TO THE STUDY OF THE LANGUAGE.

1. DICTIONARIES.

A NEW ENGLISH DICTIONARY ON HISTORICAL PRINCIPLES, founded mainly on the materials collected by the Philological Society. Imperial 4to. Parts I-IV, price 12*s.* 6*d.* each.

Vol. I (**A** and **B**), half-morocco, 2*l.* 12*s.* 6*d.*

Vol. II (**C** and **D**). *In the Press.*

Part IV, Section 2, **C—CASS**, beginning Vol. II, price 5*s.*

Part V, **CAST—CLIVY**, price 12*s.* 6*d.*

Part VI, **CLO—CONSIGNER**, price 12*s.* 6*d.*

Part VII, **CONSIGNIFICANT—CROUCHING**. Price 12*s.* 6*d.*

Edited by JAMES A. H. MURRAY, LL.D., sometime President of the Philological Society; with the assistance of many Scholars and Men of Science.

Vol. III (**E**, **F**, **G**), Part I, **E—EVERY**, Edited by HENRY BRADLEY, M.A., price 12*s.* 6*d.*

Bosworth and **Toller**. *An Anglo-Saxon Dictionary*, based on the MS. Collections of the late JOSEPH BOSWORTH, D.D. Edited and enlarged by Prof. T. N. TOLLER, M.A. Parts I-III, A-SAR. . . . [4to, 15*s.* each.

Part IV, Section I, SAR—SWÍÐRIAN. . . . . . [4to, 8*s.* 6*d.*

Mayhew and **Skeat**. *A Concise Dictionary of Middle English*, from A.D. 1150 to 1580. By A. L. MAYHEW, M.A., and W. W. SKEAT, Litt. D. [Crown 8vo, half-roan, 7*s.* 6*d.*

Skeat. *A Concise Etymological Dictionary of the English Language.* By W. W. SKEAT, Litt.D. *Fourth Edition.* . . [Crown 8vo, 5*s.* 6*d.*

2. GRAMMARS, READING BOOKS, &c.

Earle. *The Philology of the English Tongue.* By J. EARLE, M.A., Professor of Anglo-Saxon. *Fifth Edition.* . . [Extra fcap. 8vo, 8s. 6d.

——— *A Book for the Beginner in Anglo-Saxon.* By J. EARLE, M.A., Professor of Anglo-Saxon. *Third Edition.* . . [Extra fcap. 8vo, 2s. 6d.

Mayhew. *Synopsis of Old-English Phonology.* By A. L. MAYHEW, M.A. . . . . [Extra fcap. 8vo, bevelled boards, 8s. 6d.

Morris and **Skeat.** *Specimens of Early English:*—

Part I. From Old English Homilies to King Horn (A.D. 1150 to A.D. 1300). By R. MORRIS, LL.D. *Second Edition.* . . [Extra fcap. 8vo, 9s.

Part II. From Robert of Gloucester to Gower (A.D. 1298 to A.D. 1393). By R. MORRIS, LL.D., and W. W. SKEAT, Litt.D. *Third Edition.* 7s. 6d.

Skeat. *Specimens of English Literature,* from the 'Ploughmans Crede' to the 'Shepheardes Calender.' . . [Extra fcap. 8vo, 7s. 6d.

——— *The Principles of English Etymology:*

First Series. The Native Element. *Second Edition.* [Crown 8vo, 10s. 6d.

Second Series. The Foreign Element. . . . [Crown 8vo, 10s. 6d.

——— *A Primer of English Etymology.* [Extra fcap. 8vo, *stiff covers,* 1s. 6d.

——— *Twelve Facsimiles of Old-English Manuscripts.* . [4to, 7s. 6d.

Sweet. *A New English Grammar, Logical and Historical.* Part I. Introduction, Phonology, and Accidence. . . . . [Crown 8vo, 10s. 6d.

——— *A Short Historical English Grammar.* [Extra fcap. 8vo, 4s. 6d.

——— *A Primer of Historical English Grammar.* Extra fcap. 8vo, 2s.

——— *History of English Sounds from the Earliest Period.* With full Word-Lists. . . . . [8vo, 14s.

——— *An Anglo-Saxon Primer, with Grammar, Notes, and Glossary.* By HENRY SWEET, M.A. *Seventh Edition.* [Extra fcap. 8vo, 2s. 6d.

——— *An Anglo-Saxon Reader.* In Prose and Verse. With Grammatical Introduction, Notes, and Glossary. By the same Author. *Sixth Edition, Revised and Enlarged.* . . . . . [Extra fcap. 8vo, 8s. 6d.

——— *A Second Anglo-Saxon Reader.* By the same Author. [4s. 6d.

——— *Old English Reading Primers.* By the same Author:—

I. *Selected Homilies of Ælfric.* [Extra fcap. 8vo, *stiff covers,* 1s. 6d.

II. *Extracts from Alfred's Orosius.* [Extra fcap. 8vo, *stiff covers,* 2s.

——— *First Middle English Primer, with Grammar and Glossary.* By the same Author. *Second Edition.* . . . . . [Extra fcap. 8vo, 2s.

——— *Second Middle English Primer.* Extracts from Chaucer, with Grammar and Glossary. By the same Author. . [Extra fcap. 8vo, 2s. 6d.

——— *A Primer of Spoken English.* . . [Extra fcap. 8vo, 3s. 6d.

——— *A Primer of Phonetics.* . . . . [Extra fcap. 8vo, 3s. 6d.

——— *A Manual of Current Shorthand, Orthographic and Phonetic.* [4s. 6d.

Tancock. *An Elementary English Grammar and Exercise Book.* By O. W. TANCOCK, M.A. *Second Edition.* . . . [Extra fcap. 8vo, 1s. 6d.

——— *An English Grammar and Reading Book,* for Lower Forms in Classical Schools. By O. W. TANCOCK, M.A. *Fourth Edition.* [3s. 6d.

A SERIES OF ENGLISH CLASSICS.

(CHRONOLOGICALLY ARRANGED.)

Chaucer. I. *The Prologue to the Canterbury Tales.* (*School Edition.*) Edited by W. W. Skeat, Litt.D. . . [Extra fcap. 8vo, *stiff covers*, 1*s*.

——— II. *The Prologue; The Knightes Tale; The Nonne Prestes Tale.* Edited by R. Morris, LL.D. *A New Edition, with Collations and Additional Notes*, by W. W. Skeat, Litt.D. . . [Extra fcap. 8vo, 2*s*. 6*d*.

——— III. *The Prioresses Tale; Sir Thopas; The Monkes Tale; The Clerkes Tale; The Squieres Tale, &c.* Edited by W. W. Skeat, Litt.D. *Fourth Edition.* . . . . [Extra fcap. 8vo, 4*s*. 6*d*.

——— IV. *The Tale of the Man of Lawe; The Pardoneres Tale; The Second Nonnes Tale; The Chanouns Yemannes Tale.* By the same Editor. *New Edition, Revised.* . . . . . [Extra fcap. 8vo, 4*s*. 6*d*.

——— V. *Minor Poems.* By the same Editor. [Crown 8vo, 10*s*. 6*d*.

——— VI. *The Legend of Good Women.* By the same Editor. [Crown 8vo, 6*s*.

Langland. *The Vision of William concerning Piers the Plowman*, by William Langland. Edited by W. W. Skeat, Litt.D. *Fourth Edition.* [Extra fcap. 8vo, 4*s*. 6*d*.

Gamelyn, The Tale of. Edited by W. W. Skeat, Litt.D. [Extra fcap. 8vo, *stiff covers*, 1*s*. 6*d*.

Wycliffe. *The New Testament in English*, according to the Version by John Wycliffe, about A.D. 1380, and Revised by John Purvey, about A.D. 1388. With Introduction and Glossary by W. W. Skeat, Litt.D. [Extra fcap. 8vo, 6*s*.

——— *The Books of Job, Psalms, Proverbs, Ecclesiastes, and the Song of Solomon:* according to the Wycliffite Version made by Nicholas de Hereford, about A.D. 1381, and Revised by John Purvey, about A.D. 1388. With Introduction and Glossary by W. W. Skeat, Litt.D. [Extra fcap. 8vo, 3*s*. 6*d*.

Minot. *The Poems of Laurence Minot.* Edited, with Introduction and Notes, by Joseph Hall, M.A. . . . . . [Extra fcap. 8vo, 4*s*. 6*d*.

Spenser. *The Faery Queene.* Books I and II. Edited by G. W. Kitchin, D.D., with Glossary by A. L. Mayhew, M.A.

Book I. *Tenth Edition.* . . . . . . [Extra fcap. 8vo, 2*s*. 6*d*.

Book II. *Sixth Edition.* . . . . . . [Extra fcap. 8vo, 2*s*. 6*d*.

Hooker. *Ecclesiastical Polity*, Book I. Edited by R. W. Church, M.A., late Dean of St. Paul's. *Second Edition.* . . [Extra fcap. 8vo, 2*s*.

Marlowe and Greene. Marlowe's *Tragical History of Dr. Faustus*, and Greene's *Honourable History of Friar Bacon and Friar Bungay.* Edited by A. W. Ward, Litt.D. *New and Enlarged Edition.* [Crown 8vo, 6*s*. 6*d*.

Marlowe. *Edward II.* Edited by O. W. Tancock, M.A. *Second Edition.* . . . . . [Extra fcap. 8vo. *Paper covers*, 2*s*.; *cloth*, 3*s*.

Shakespeare. Select Plays. Edited by W. G. CLARK, M.A., and W. ALDIS WRIGHT, D.C.L. . . . . . [Extra fcap. 8vo, *stiff covers.*

The Merchant of Venice. 1s. *Macbeth.* 1s. 6d.
Richard the Second. 1s. 6d. *Hamlet.* 2s.

Edited by W. ALDIS WRIGHT, D.C.L.

The Tempest. 1s. 6d. *Coriolanus.* 2s. 6d.
As You Like It. 1s. 6d. *Richard the Third.* 2s. 6d.
A Midsummer Night's Dream. 1s. 6d. *Henry the Fifth.* 2s.
Twelfth Night. 1s. 6d. *King John.* 1s. 6d.
Julius Cæsar. 2s. *King Lear.* 1s. 6d.
Henry the Eighth. 2s.

Shakespeare as a Dramatic Artist; *a popular Illustration of the Principles of Scientific Criticism.* By R. G. MOULTON, M.A. [Cr. 8vo, 7s. 6d.

Bacon. *Advancement of Learning.* Edited by W. ALDIS WRIGHT, D.C.L. *Third Edition.* . . . . [Extra fcap. 8vo, 4s. 6d

—— *The Essays.* Edited, with Introduction and Illustrative Notes, by S. H. REYNOLDS, M.A. . . . . . [Demy 8vo, *half-bound*, 12s. 6d.

Milton. I. *Areopagitica.* With Introduction and Notes. By JOHN W. HALES, M.A. *Third Edition.* . . . . . [Extra fcap. 8vo, 3s.

—— II. *Poems.* Edited by R. C. BROWNE, M.A. In two Volumes. *Fifth Edition.*
[Extra fcap. 8vo, 6s. 6d. Sold separately, Vol. I. 4s., Vol. II. 3s

In paper covers:—

Lycidas, 3d. *L'Allegro*, 3d. *Comus*, 6d.

By OLIVER ELTON, B.A.

Lycidas, 6d. *L'Allegro*, 4d. *Il Penseroso*, 4d. *Comus*, 1s.

—— III. *Paradise Lost.* Book I. Edited with Notes, by H. C. BEECHING, M.A. . . . [Extra fcap. 8vo, 1s. 6d. *In Parchment*, 3s. 6d.

—— IV. *Paradise Lost.* Book II. Edited by E. K. CHAMBERS, B.A. . . . . [Extra fcap. 8vo, 1s. 6d. Books I and II together, 2s. 6d.

—— V. *Samson Agonistes.* Edited, with Introduction and Notes, by JOHN CHURTON COLLINS, M.A. . . . [Extra fcap. 8vo, *stiff covers*, 1s.

Milton's Prosody. By ROBERT BRIDGES. [Small 4to, 8s. 6d. net.

Bunyan. I. *The Pilgrim's Progress, Grace Abounding, Relation of the Imprisonment of Mr. John Bunyan.* Edited by E. VENABLES, M.A.
[Extra fcap. 8vo, 3s. 6d. *In Parchment*, 4s. 6d.

—— II. *The Holy War, and the Heavenly Footman.* Edited by MABEL PEACOCK. . . . . [Extra fcap. 8vo, 3s. 6d.

Clarendon. *Selections.* Edited by G. BOYLE, M.A., Dean of Salisbury. . . . . [Crown 8vo, 7s. 6d.

Dryden. *Select Poems.* (*Stanzas on the Death of Oliver Cromwell; Astræa Redux; Annus Mirabilis; Absalom and Achitophel; Religio Laici; The Hind and the Panther.*) Edited by W. D. CHRISTIE, M.A. *Fifth Edition.* Revised by C. H. FIRTH, M.A. . . . . . [Extra fcap. 8vo, 3s. 6d.

—— *Essay of Dramatic Poesy.* Edited, with Notes, by T. ARNOLD, M.A. . . . . [Extra fcap. 8vo, 3s. 6d.

Locke. *Conduct of the Understanding.* Edited, with Introduction, Notes, &c., by T. FOWLER, D.D. *Third Edition.* . [Extra fcap. 8vo 2s. 6d.

Addison. *Selections from Papers in the 'Spectator.'* By T. ARNOLD, M.A. *Sixteenth Thousand.* . [Extra fcap. 8vo, 4*s.* 6*d.* *In Parchment*, 6*s.*

Steele. *Selected Essays from the Tatler, Spectator, and Guardian.* By AUSTIN DOBSON. . . . [Extra fcap. 8vo, 5*s.* *In Parchment*, 7*s.* 6*d.*

Swift. *Selections from his Works.* Edited, with Life, Introductions, and Notes, by HENRY CRAIK. Two Vols. [Crown 8vo, cloth extra, price 15*s.*
Each volume may be had separately, price 7*s.* 6*d.*

Pope. I. *Essay on Man.* Edited by MARK PATTISON, B.D. *Sixth Edition.* . . . . [Extra fcap. 8vo, 1*s.* 6*d.*

——— II. *Satires and Epistles.* By the same Editor. *Second Edition.* [Extra fcap. 8vo, 2*s.*

Thomson. *The Seasons*, and *The Castle of Indolence.* Edited by J. LOGIE ROBERTSON, M.A. . . . . . . [Extra fcap. 8vo, 4*s.* 6*d.*

——— *The Castle of Indolence.* By the same Editor. [Extra fcap. 8vo, 1*s.* 6*d.*

Berkeley. *Selections.* With Introduction and Notes. By A. C. FRASER, LL.D. *Fourth Edition.* . . . . [Crown 8vo, 8*s.* 6*d.*

Johnson. I. *Rasselas.* Edited, with Introduction and Notes, by G. BIRKBECK HILL, D.C.L.
[Extra fcap. 8vo, *limp*, 2*s.*; *Bevelled boards*, 3*s.* 6*d.*; *in Parchment*, 4*s.* 6*d.*

——— II. *Rasselas; Lives of Dryden and Pope.* Edited by ALFRED MILNES, M.A. . . . . [Extra fcap. 8vo, 4*s.* 6*d.*
Lives of Dryden and Pope. . . [*Stiff covers*, 2*s.* 6*d.*

——— III. *Life of Milton.* Edited, with Notes, &c., by C. H. FIRTH, M.A. . . . [Extra fcap. 8vo, *stiff covers*, 1*s.* 6*d.*; *cloth*, 2*s.* 6*d.*

——— IV. *Vanity of Human Wishes.* With Notes, by E. J. PAYNE, M.A. . . . . [*Paper covers*, 4*d.*

Gray. *Selected Poems.* Edited by EDMUND GOSSE, M.A. [*In Parchment*, 3*s.*

——— *The same*, together with Supplementary Notes for Schools. By FOSTER WATSON, M.A. . . . . . [Extra fcap. 8vo, *stiff covers*, 1*s.* 6*d.*

——— *Elegy, and Ode on Eton College.* . . . . [*Paper covers*, 2*d.*

Goldsmith. *Selected Poems.* Edited, with Introduction and Notes, by AUSTIN DOBSON . . [Extra fcap. 8vo, 3*s.* 6*d.* *In Parchment*, 4*s.* 6*d.*

——— *The Traveller.* Edited by G. B. HILL, D.C.L. [*Stiff covers*, 1*s.*

——— *The Deserted Village.* . . . . [*Paper covers*, 2*d.*

Cowper. I. *The Didactic Poems of* 1782, with Selections from the Minor Pieces, A.D. 1779–1783. Edited by H. T. GRIFFITH, B.A. [Extra fcap. 8vo, 3*s.*

——— II. *The Task, with Tirocinium,* and Selections from the Minor Poems, A.D. 1784–1799. By the same Editor. [Extra fcap. 8vo, 3*s.*

Burke. I. *Thoughts on the Present Discontents; the two Speeches on America.* Edited by E. J. PAYNE, M.A. . . . [Extra fcap. 8vo, 4*s.* 6*d.*

——— II. *Reflections on the French Revolution.* By the same Editor. *Second Edition.* . . . . [Extra fcap. 8vo, 5*s.*

——— III. *Four Letters on the Proposals for Peace with the Regicide Directory of France.* By the same Editor. [Extra fcap. 8vo, 5*s.*

Burns. *Selected Poems.* Edited by J. LOGIE ROBERTSON, M.A. [Crown 8vo, 6s.

Keats. *Hyperion,* Book I. With Notes, by W. T. ARNOLD, B.A. 4d.

Byron. *Childe Harold.* With Introduction and Notes, by H. F. TOZER, M.A. . . . . . . [Extra fcap. 8vo, 3s. 6d. *In Parchment,* 5s.

Shelley. *Adonais.* With Introduction and Notes. By W. M. ROSSETTI. . . . . [Crown 8vo, 5s.

Scott. *Lady of the Lake.* Edited, with Preface and Notes, by W. MINTO, M.A. With Map. . . . . . [Extra fcap. 8vo, 3s. 6d.

——— *Lay of the Last Minstrel.* Edited by W. MINTO, M.A. With Map. . . . [Extra fcap. 8vo, *stiff covers,* 2s. *In Parchment,* 3s. 6d.

——— *Lay of the Last Minstrel.* Introduction and Canto I, with Preface and Notes, by W. MINTO, M.A. . . . . . [*Paper covers,* 6d.

——— *Lord of the Isles.* Edited, with Introduction and Notes, by THOMAS BAYNE . . . . [Extra fcap. 8vo, 3s. 6d.

——— *Marmion.* Edited by T. BAYNE. . [Extra fcap. 8vo, 3s. 6d.

Campbell. *Gertrude of Wyoming.* Edited, with Introduction and Notes, by H. MACAULAY FITZGIBBON, M.A. *Second Edition* . [Extra fcap. 8vo, 1s.

Wordsworth. *The White Doe of Rylstone.* Edited by WILLIAM KNIGHT, LL.D., University of St. Andrews. . . [Extra fcap. 8vo, 2s. 6d.

Typical Selections *from the best English Writers. Second Edition.* In Two Volumes. . . . . [Extra fcap. 8vo, 3s. 6d. each.

HISTORY AND GEOGRAPHY, &c.

Freeman. *A Short History of the Norman Conquest of England.* By E. A. FREEMAN, M.A. *Second Edition.* . . [Extra fcap. 8vo, 2s. 6d.

Greswell. *History of the Dominion of Canada.* By W. PARR GRESWELL, M.A. . . . . [Crown 8vo, 7s. 6d.

——— *Geography of the Dominion of Canada and Newfoundland.* By the same Author. . . . . [Crown 8vo, 6s.

——— *Geography of Africa South of the Zambesi.* By the same Author. . . . . [Crown 8vo, 7s. 6d.

Hughes (Alfred). *Geography for Schools.* Part I, *Practical Geography.* With Diagrams. . . . . [Extra fcap. 8vo, 2s. 6d.

Kitchin. *A History of France.* With Numerous Maps, Plans, and Tables. By G. W. KITCHIN, D.D., Dean of Winchester. *Second Edition.*
Vol. I. To 1453. Vol. II. 1453–1624. Vol. III. 1624–1793. Each 10s. 6d.

Lucas. *Introduction to a Historical Geography of the British Colonies.* By C. P. LUCAS, B.A. . . . . . [Crown 8vo, with 8 maps, 4s. 6d.

——— *Historical Geography of the British Colonies:—*

I. *The Mediterranean and Eastern Colonies* (exclusive of India). [Crown 8vo, with 11 maps, 5s.

II. *The West Indian Dependencies.* With Twelve Maps. [Crown 8vo, 7s. 6d.

MATHEMATICS AND PHYSICAL SCIENCE.

Aldis. *A Text Book of Algebra (with Answers to the Examples).* By W. STEADMAN ALDIS, M.A. . . . . [Crown 8vo, 7*s.* 6*d.*

Emtage. *An Introduction to the Mathematical Theory of Electricity and Magnetism.* By W. T. EMTAGE, M.A. . . . . [7*s.* 6*d.*

Fisher. *Class-Book of Chemistry.* By W. W. FISHER, M.A., F.C.S. *Second Edition* . . . . [Crown 8vo, 4*s.* 6*d.*

Hamilton and **Ball.** *Book-keeping.* By Sir R. G. C. HAMILTON, K.C.B., and JOHN BALL (of the firm of Quilter, Ball, & Co.). *New and Enlarged Edition.* . . . . [Extra fcap. 8vo, 2*s.*

*** *Ruled Exercise Books adapted to the above*; fcap. folio, 1*s.* 6*d.* *Ruled Book adapted to the Preliminary Course*; small 4to, 4*d.*

Harcourt and **Madan.** *Exercises in Practical Chemistry.* Vol. I. *Elementary Exercises.* By A. G. VERNON HARCOURT, M.A., and H. G. MADAN, M.A. *Fourth Edition.* Revised by H. G. MADAN, M.A. [Crown 8vo, 10*s.* 6*d.*

Hensley. *Figures made Easy: a first Arithmetic Book.* By LEWIS HENSLEY, M.A. . . . . [Crown 8vo, 6*d.* *Answers*, 1*s.*

——— *The Scholar's Arithmetic.* By the same Author. [Crown 8vo, 2*s.* 6*d.* *Answers*, 1*s.* 6*d.*

——— *The Scholar's Algebra.* An Introductory work on Algebra. By the same Author. . . . . [Crown 8vo, 2*s.* 6*d.*

Minchin. *Hydrostatics and Elementary Hydrokinetics.* By G. M. MINCHIN, M.A. . . . . [Crown 8vo, 10*s.* 6*d.*

Nixon. *Euclid Revised.* Containing the essentials of the Elements of Plane Geometry as given by Euclid in his First Six Books. Edited by R. C. J. NIXON, M.A. *Second Edition.* . . . . [Crown 8vo, 6*s.*

*** May likewise be had in parts as follows:—

Book I, 1*s.* Books I, II, 1*s.* 6*d.* Books I–IV, 3*s.* Books V, VI, 3*s.*

——— *Supplement to Euclid Revised.* By the same Author. . [6*d.*

——— *Geometry in Space.* Containing parts of Euclid's Eleventh and Twelfth Books. By the same Author. . . . . [Crown 8vo, 3*s.* 6*d.*

——— *Elementary Plane Trigonometry; that is, Plane Trigonometry without Imaginaries.* By the same Author . . . [Crown 8vo, 7*s.* 6*d.*

Selby. *Elementary Mechanics of Solids and Fluids.* By A. L. SELBY, M.A. . . . . [Crown 8vo, 7*s.* 6*d.*

Williamson. *Chemistry for Students.* By A. W. WILLIAMSON, Phil. Doc., F.R.S. . . . . [Extra fcap. 8vo, 8*s.* 6*d.*

Woollcombe. *Practical Work in Heat.* For use in Schools and Colleges. By W. G. WOOLLCOMBE, M.A., B.Sc. . . . [Crown 8vo, 3*s.*

Fowler. *The Elements of Deductive Logic*, designed mainly for the use of Junior Students in the Universities. By T. FOWLER, D.D. *Ninth Edition*, with a Collection of Examples. [Extra fcap. 8vo, 3s. 6d.

——— *The Elements of Inductive Logic*, designed mainly for the use of Students in the Universities. *Fifth Edition.* [Extra fcap. 8vo, 6s.

Music.—Farmer. *Hymns and Chorales for Schools and Colleges.* Edited by JOHN FARMER, Organist of Balliol College. . . . . . [5s.

☞ *Hymns without the Tunes*, 2s.

Hullah. *The Cultivation of the Speaking Voice.* By JOHN HULLAH. [Extra fcap. 8vo, 2s. 6d.

Maclaren. *A System of Physical Education: Theoretical and Practical.* With 346 Illustrations drawn by A. MACDONALD, of the Oxford School of Art. By ARCHIBALD MACLAREN, the Gymnasium, Oxford. *Second Edition.* [Extra fcap. 8vo, 7s. 6d.

Troutbeck and Dale. *A Music Primer for Schools.* By J. TROUTBECK, D.D., formerly Music Master in Westminster School, and R. F. DALE, M.A., B.Mus., late Assistant Master in Westminster School. [Crown 8vo, 1s. 6d.

Tyrwhitt. *A Handbook of Pictorial Art.* By R. St. J. TYRWHITT, M.A. With coloured Illustrations, Photographs, and a chapter on Perspective, by A. MACDONALD. *Second Edition.* . . . . [8vo, *half-morocco*, 18s.

Upcott. *An Introduction to Greek Sculpture.* By L. E. UPCOTT, M.A. . . . . [Crown 8vo, 4s. 6d.

Student's Handbook to the University and Colleges of Oxford. *Twelfth Edition.* . . . . [Crown 8vo, 2s. 6d.

Helps to the Study of the Bible, taken from the *Oxford Bible for Teachers.* A new, enlarged, and illustrated edition, comprising Introductions to the several Books, the History and Antiquities of the Jews, the results of Modern Discoveries, and the Natural History of Palestine, with copious Tables, Concordance and Indices, and a series of Maps. . . . [Crown 8vo, 4s. 6d.

Helps to the Study of the Book of Common Prayer. Being a Companion to Church Worship. . . . . [Crown 8vo, 3s. 6d.

*** A READING ROOM *has been opened at the* CLARENDON PRESS WAREHOUSE, AMEN CORNER, *where visitors will find every facility for examining old and new works issued from the Press, and for consulting all official publications.*

London: HENRY FROWDE,
OXFORD UNIVERSITY PRESS WAREHOUSE, AMEN CORNER.
Edinburgh: 12 FREDERICK STREET.